June 2015

Question 17

1 mark

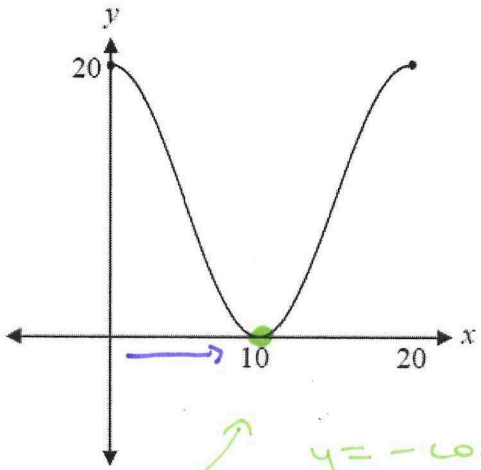
Using $y = -10\cos\left[B(x - C)\right] + D$, the value of C that corresponds to the following graph is:

a) 5

b) 10

c) 15

d) 20



phase shift

$y = -\cos x$ "begins" on the minimum value.

Question 25

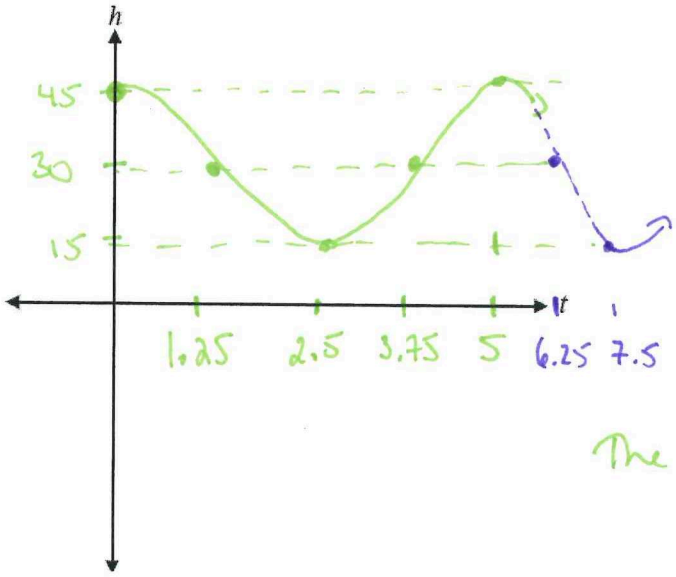
a) 3 marks b) 1 mark

The height of a bicycle pedal as the bicycle is moving at a constant speed can be represented by the following function:

$$h(t) = 15\cos\frac{2\pi}{5}t + 30$$

where h is the height of the pedal above the ground, in cm, and t is the time, in seconds.

a) Sketch a graph of at least one period of this function, where $t \geq 0$.



period $= \frac{2\pi}{b}$

$= \frac{2\pi}{\frac{2\pi}{5}}$

$= 2\pi\left(\frac{5}{2\pi}\right)$

$= 5$ seconds

The points are $\frac{5}{4} = 1.25$ apart

b) Determine the height of the bicycle pedal at 7.5 seconds.

15 cm

January 2015

Question 10

1 mark

Determine the period of the sinusoidal function $y = \frac{1}{2}\sin\left(\frac{1}{3}x\right)$.

State your answer in radians.

period $= \frac{2\pi}{b}$

$= \frac{2\pi}{\frac{1}{3}}$

$= 2\pi(3)$

$= 6\pi$

Question 21

1 mark

Identify a non-permissible value of x for the expression $\frac{1}{\cos 2x}$.

a) 0

b) $\frac{\pi}{4}$

c) $\frac{\pi}{2}$

d) π

$\cos 2x \neq 0$

$\cos 2\left(\frac{\pi}{4}\right)$

$= \cos \frac{\pi}{2}$

$= 0$

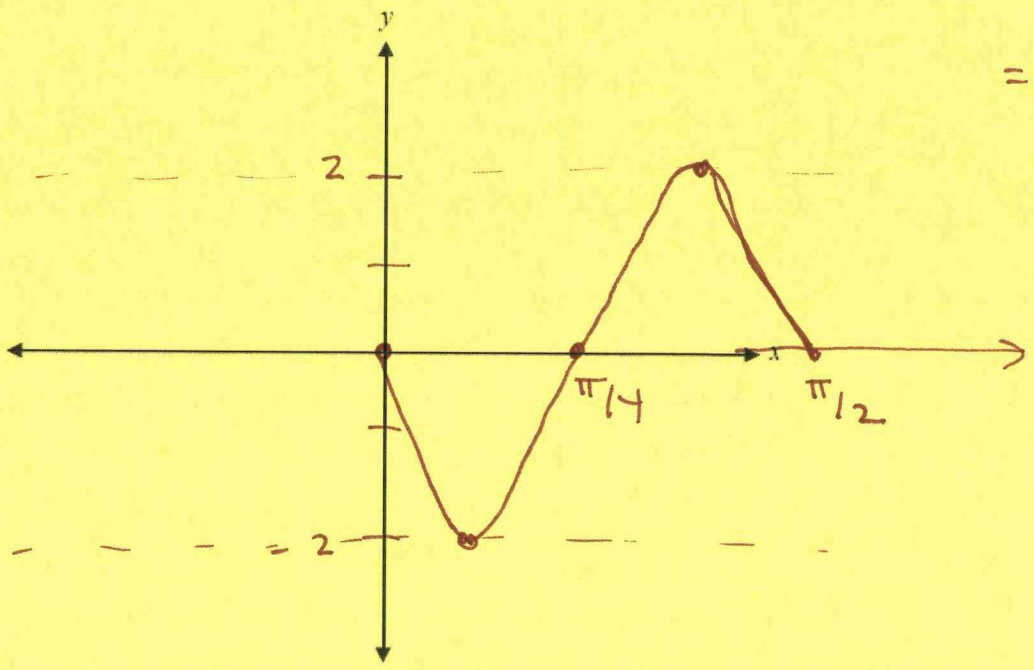
Question 29

3 marks

Sketch the graph of at least one period of the function $y = -2\sin(4x)$.

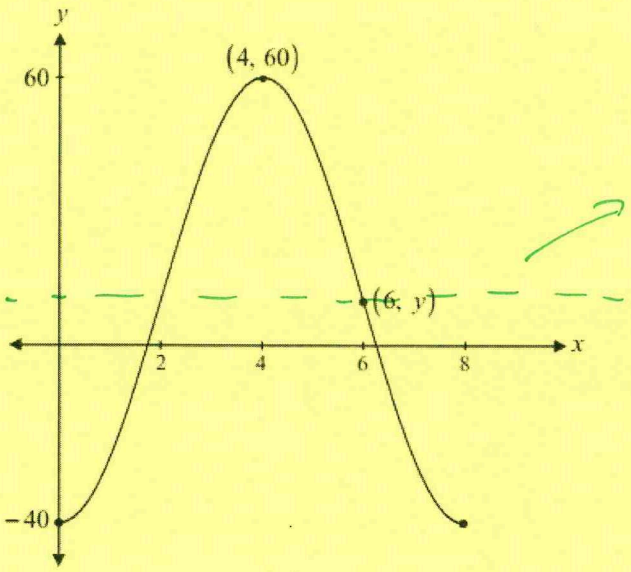
period $= \frac{2\pi}{4}$

$= \frac{\pi}{2}$



Question 11 — 1 mark

Using the graph of the sinusoidal function below, find the value of y in the point $(6,\ y)$.



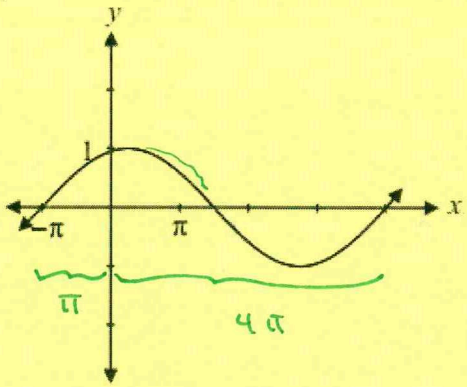
median

$\frac{60 + (-40)}{2} = 10$

$y = 10.$

Question 26 — 1 mark

If the equation $y = \sin\left(b\left(x + \pi\right)\right)$ is represented by the following graph, what is the value of b?



π

4π

$4\pi + \pi = 5\pi$

a) $\frac{2}{5}$ b) $\frac{5}{2}$ c) $\frac{2\pi}{5}$ d) 5π

$b = \frac{2\pi}{\text{period}} = \frac{2\pi}{5\pi}$

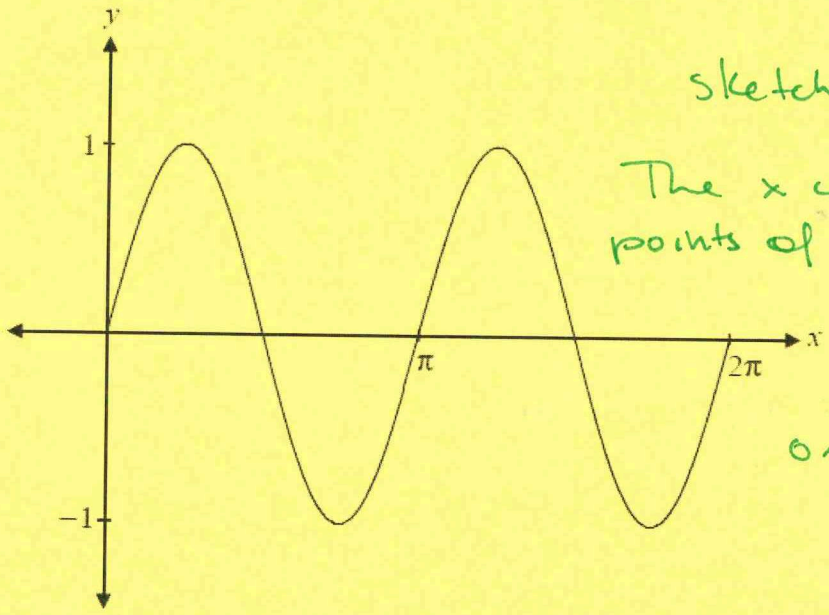
$= \frac{2}{5}$

Question 40

1 mark

The graph of $y = \sin 2x$ is sketched below.

Explain how to use this graph to solve the equation $\sin 2x = \frac{1}{2}$ over the interval $[0,\ 2\pi]$.



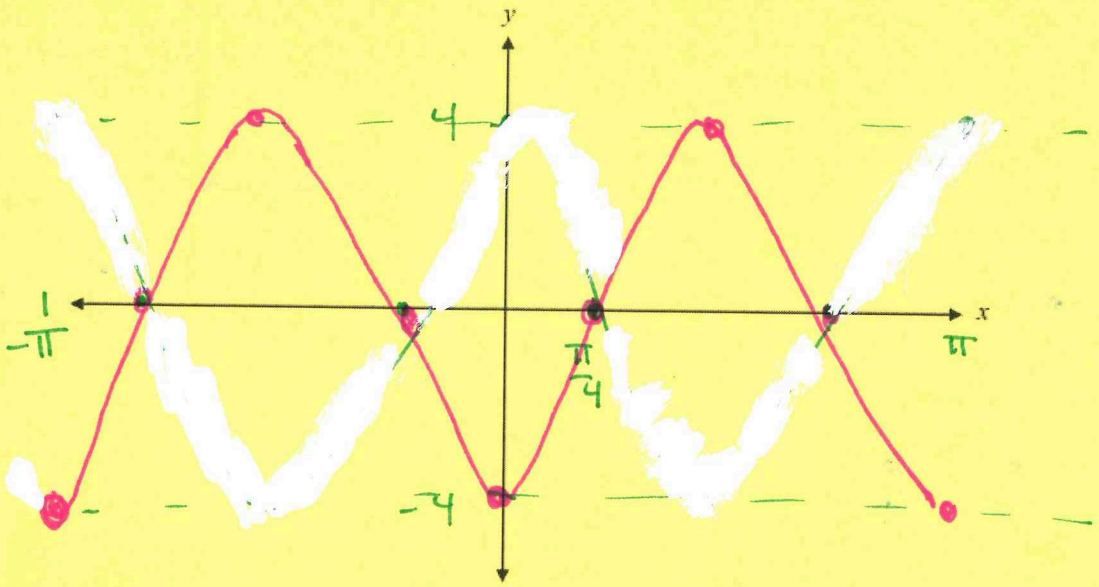
sketch in $y = \frac{1}{2}$

The x coordinate of the points of intersection of $y = \sin 2x$ and $y = \frac{1}{2}$ on the interval $[0, 2\pi]$ are the solutions

Question 41

3 marks

Sketch the graph of $y = -4\cos(2x)$ over the interval $[-\pi,\ \pi]$.



period $= \frac{2\pi}{2}$

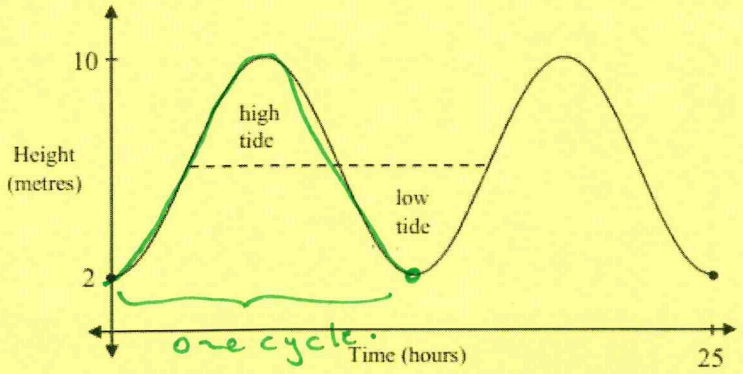
$= \pi$

5 points are $\frac{\pi}{4}$ apart

Question 33

a) 1 mark b) 2 marks

The following graph represents tidal levels in the Bay of Fundy over a 25-hour period.



a) What is the average height of the water?

median : $\frac{10+2}{2} = 6\text{m}$

b) What is the period of the graph above?

Explain what the period represents in this situation.

12.5 hours. It represents the time it takes for the water level to go through one full cycle

Question 43

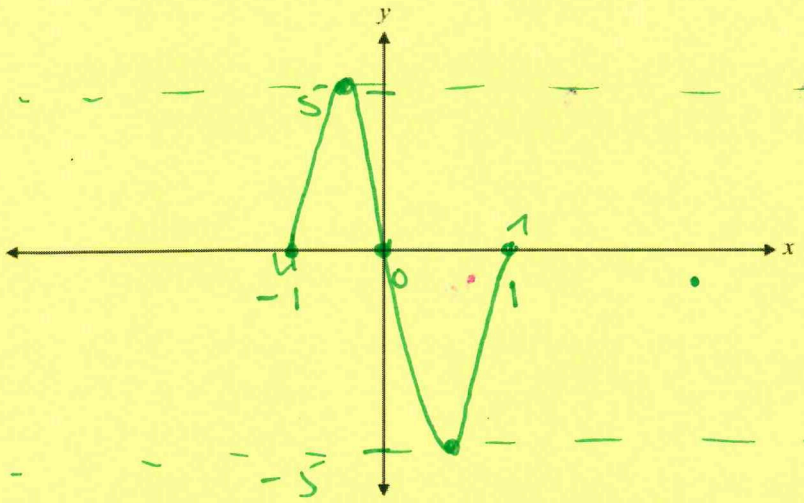
4 marks

Sketch a graph of at least one period of the function $y = 5\sin\left[\pi(x+1)\right]$.

Clearly indicate the x-intercepts.

period $= \frac{2\pi}{\pi} = 2$

point are $\frac{2}{4} =$ 0.5 apart



Question 20

1 mark

Which of the following is true about the periods of the three functions below?

$f(\theta) = 2\sin 3\left(\theta - \frac{\pi}{2}\right)$ $\frac{2\pi}{3}$

$g(\theta) = \sin 3\theta + 6$ $\frac{2\pi}{3}$

$k(\theta) = 3\sin\theta + 6$ 2π

a) The graphs of $f(\theta)$ and $g(\theta)$ have the same period. ✓

b) The graphs of $g(\theta)$ and $k(\theta)$ have the same period.

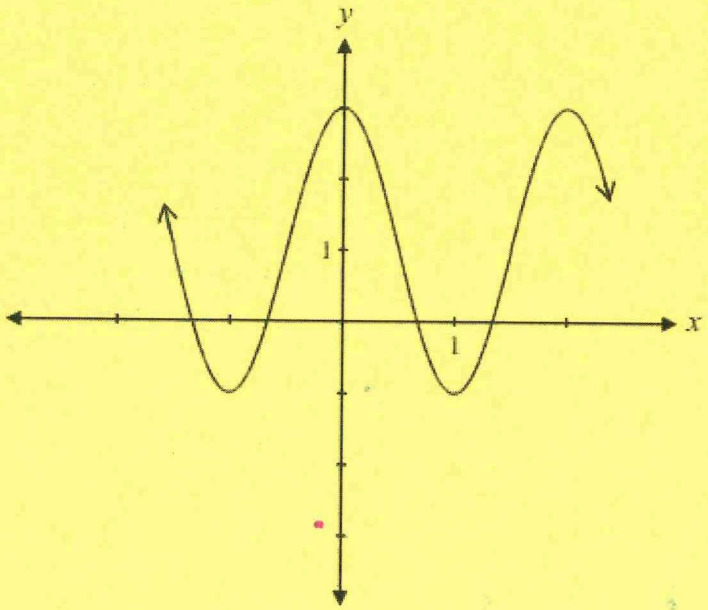
c) All of the graphs have the same period.

d) None of the graphs have the same period.

Question 25

1 mark

Given the graph of $y = 2\cos\pi x + 1$ below, determine another equation that will produce the same graph.



$y = -2\cos(\pi(x-1)) + 1$

$y = 2\cos(\pi(x-2)) + 1$

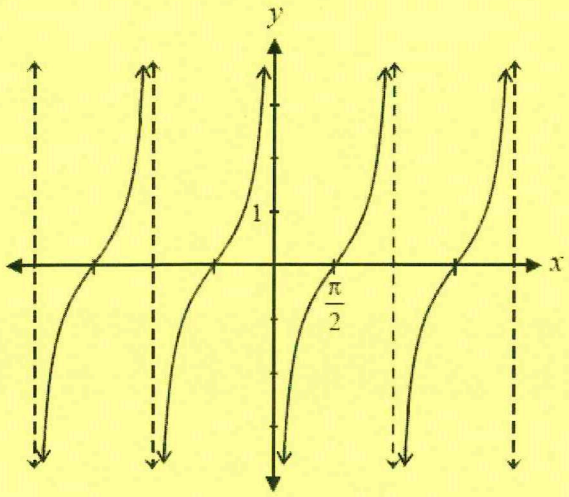
$y = -2\cos(\pi(x+1)) + 1$

$y = -2\sin(\pi(x-0.5)) + 1$

etc!

Question 38 — 1 mark

Mohamed is asked to sketch the graph of $y = \tan x$.
His graph is shown below.



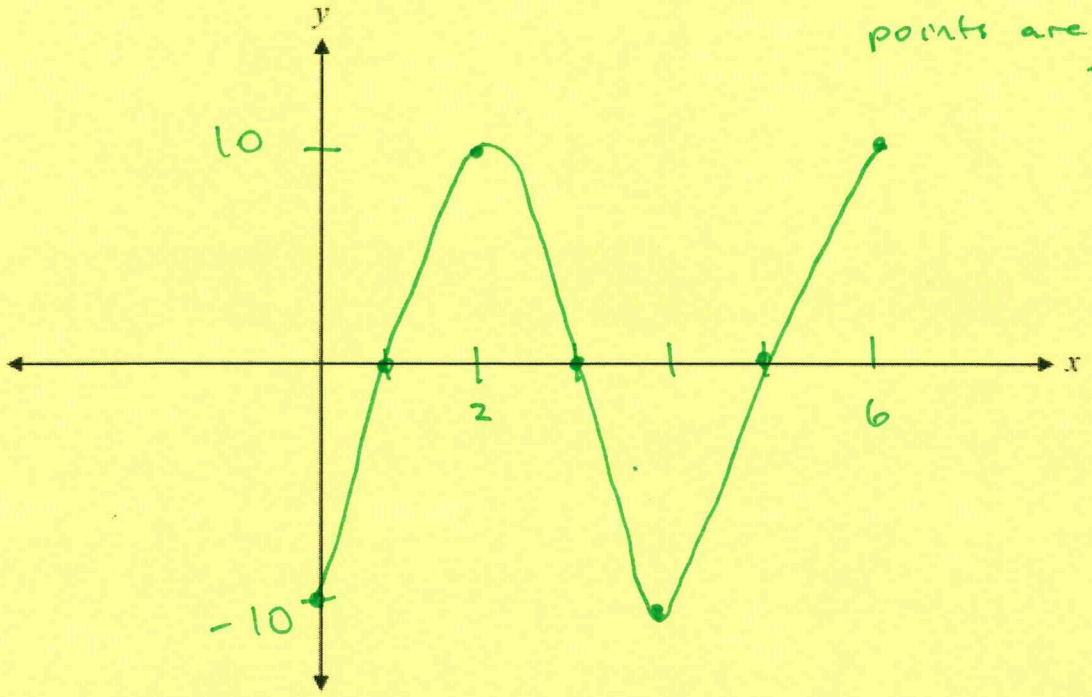
Explain why his graph is incorrect. It should be translated π/2 to the left/right

Question 42 — 3 marks

Sketch the graph of $y = 10\cos\left[\frac{\pi}{2}(x-2)\right]$ over the interval $[0, 6]$.

period = $\frac{2\pi}{\frac{\pi}{2}}$ = 4

points are 1 radian apart.



Question 28 — 2 marks

The graph of the function $y = \sin x$ has been transformed to create a new graph.

The range of this new graph is $[-4, 4]$ and the zeros are $x = k\frac{\pi}{2}$, where k is an integer.

Write the equation that corresponds to this new graph.

$y = -4\sin 2x$

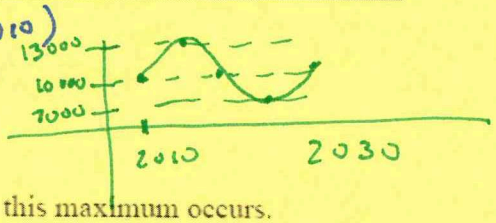
Question 34 — 3 marks

Given the following sinusoidal equation:

$$P(t) = 3000\sin\left[\frac{\pi}{10}(t - 2010)\right] + 10\,000$$

period $= \frac{2\pi}{\frac{\pi}{10}} = 20$ yrs

(10,000, 2010)



Determine the maximum value of $P(t)$ and a value of t at which this maximum occurs.

median + amplitude.

Maximum value of $P(t)$: 13 000

Value of t: 2015

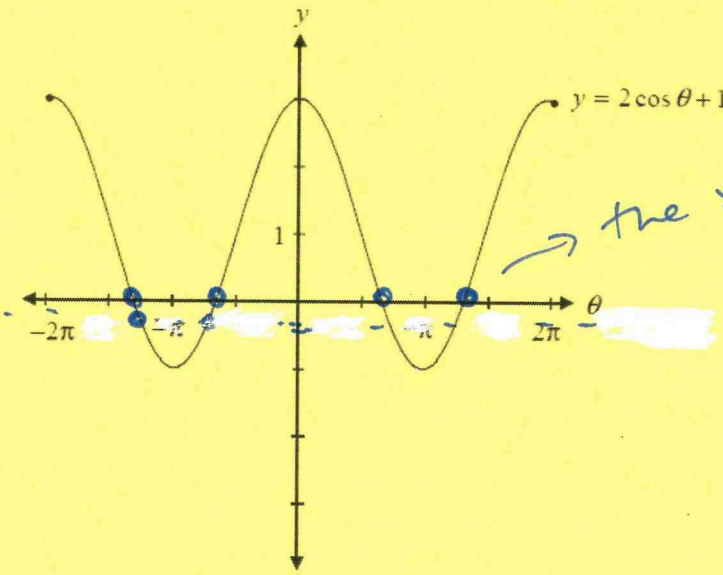
period = 20 yrs.

Question 41 — 1 mark

$0 = 2\cos\theta + 1$

$-\frac{1}{2} = \cos\theta$

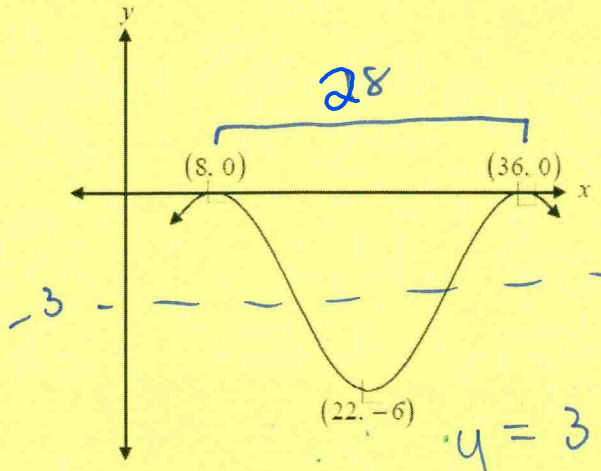
The graph of $y = 2\cos\theta + 1$ below can be used to solve the equation $\cos\theta = -\frac{1}{2}$ over the interval $[-2\pi, 2\pi]$. Indicate on the graph where to find the solutions to the equation $\cos\theta = -\frac{1}{2}$.



the x intercepts!

June 2012

44. The graph of a sinusoidal function is sketched below.



$d = -3$

$a = 3$

$b = \frac{2\pi}{\text{period}}$

$= \frac{2\pi}{28} = \frac{\pi}{14}$

$c = 8$

note: c can change and "a" might be negative

$y = 3\cos\left(\frac{\pi}{14}(x-8)\right) - 3$

Using the form $y = A\cos[B(x - C)] + D$, write an equation that this graph could represent.

January 2012

24. What is the range of the function $y = -2\sin x + 1$?

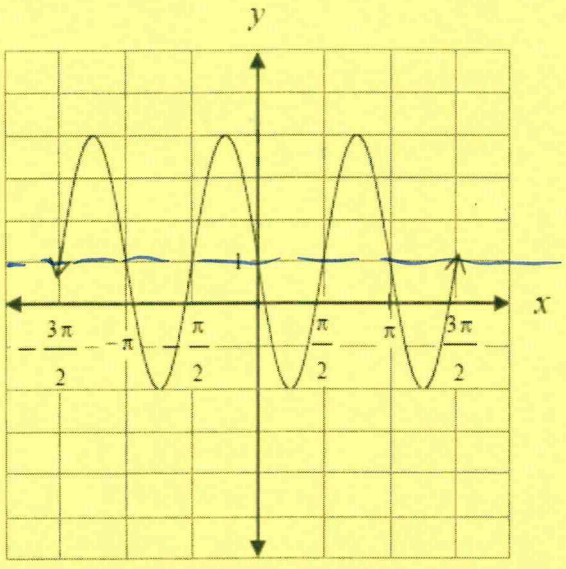
a) $[-2, 2]$

b) $[-1, 3]$

c) $[-1, 1]$

d) $[0, 2]$

36. What is the amplitude of the following graph?



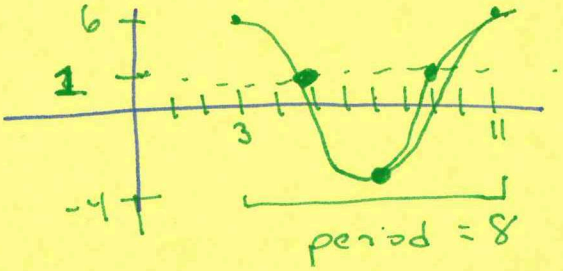
3.

49. A sinusoidal curve has a maximum at (3, 6). The next maximum on the curve is at (11, 6). The range of this function is $[-4, 6]$.

Find the values of A, B, C, and D if the sinusoidal equation for this curve is $y = A\sin[B(x - C)] + D$.

d: median: $\frac{\text{max} + \text{min}}{2}$

$\frac{6 - 4}{2} = 1$



$b = \frac{2\pi}{8} = \frac{\pi}{4}$

A =	-5	5
B =	$\pi/4$	$\frac{\pi}{4}$
C =	5	7
D =	1	1

June 2011

16. What is the y-intercept of $y = \cos x$?

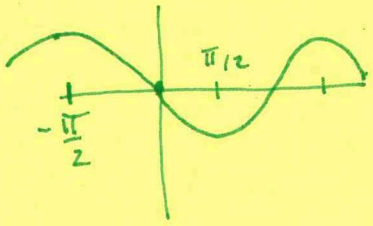
a) 0

b) 1

c) $\frac{\pi}{2}$

d) π

18. Which one of the following equations represents the same graph as $y = -\sin x$?



a) $y = \cos(-x)$

b) $y = -\cos x$

c) $y = \cos\left(x - \frac{\pi}{2}\right)$

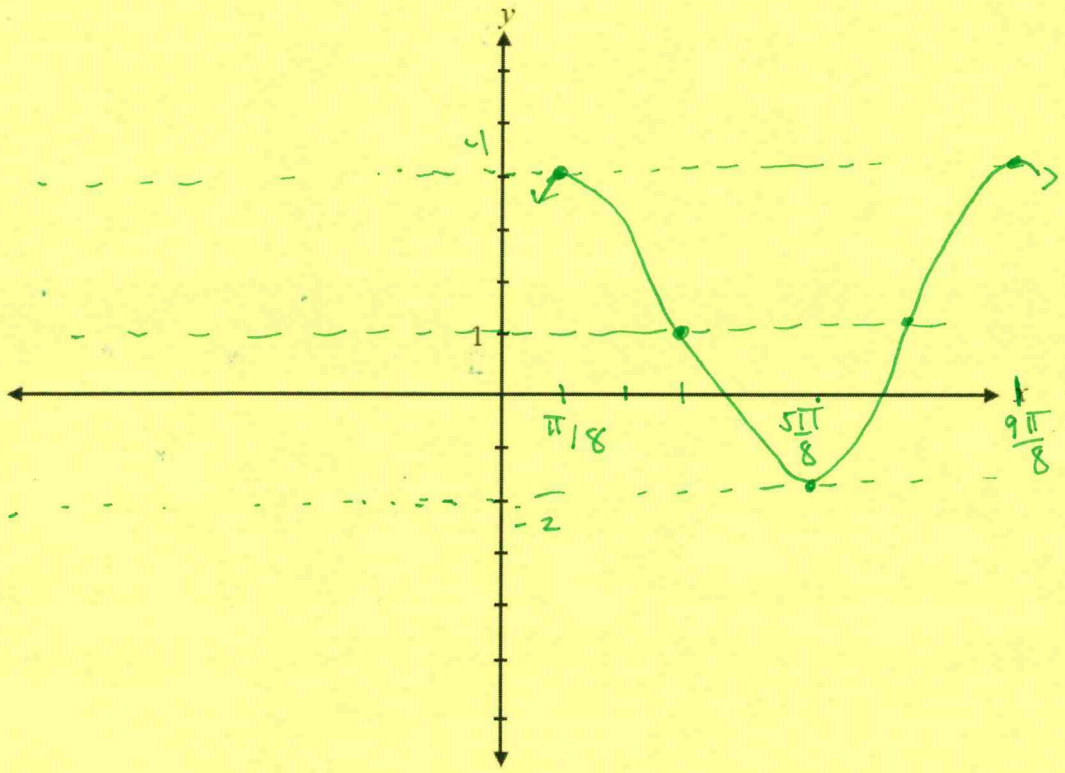
d) $y = \cos\left(x + \frac{\pi}{2}\right)$

46. Sketch a clearly labelled graph of at least one period of the following function:

$$y = 3\cos\left[2\left(x - \frac{\pi}{8}\right)\right] + 1$$

period $= \frac{2\pi}{2} = \pi$

points are $\frac{\pi}{4}$ apart



47. A certain population was studied over a period of time. It was determined that this population varied sinusoidally as a function of time.

At the start of year 4, the population reached its maximum of 27 000. The population gradually declined and, at the start of year 10, it reached its minimum of 13 000.

This situation can be modelled by the equation $y = A\sin\left[B(x - C)\right] + D$.

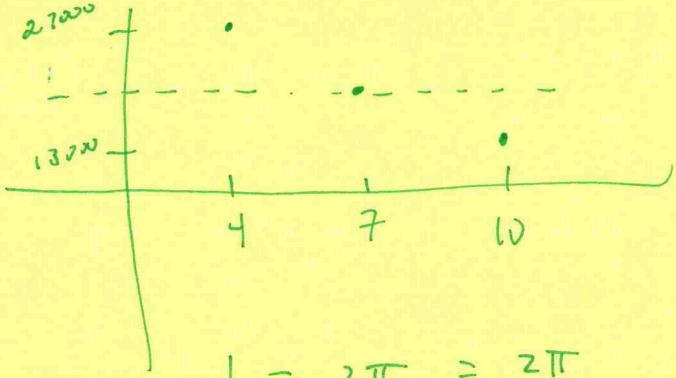
Determine the values of A, B, C, and D, where y represents the population and x represents time in years.

A = -7000

B = $\frac{\pi}{6}$

C = 7

D = 20000



$b = \frac{2\pi}{\text{period}} = \frac{2\pi}{12}$

January 2011

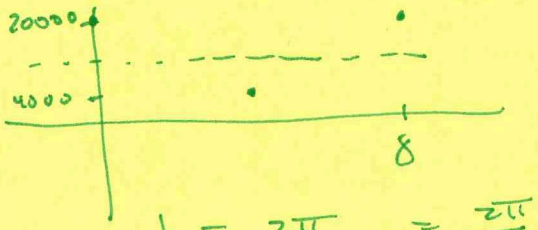
50. The population of rabbits in a park increases and decreases sinusoidally as a function of time. The initial population of rabbits is 20 000. Every 8 years, the population of rabbits returns to its maximum of 20 000. The minimum population of rabbits is 4000. This situation can be modelled by the equation $y = A\cos[B(x - C)] + D$.

State the values of A, B, and D.

A = 8000

B = $\pi/4$

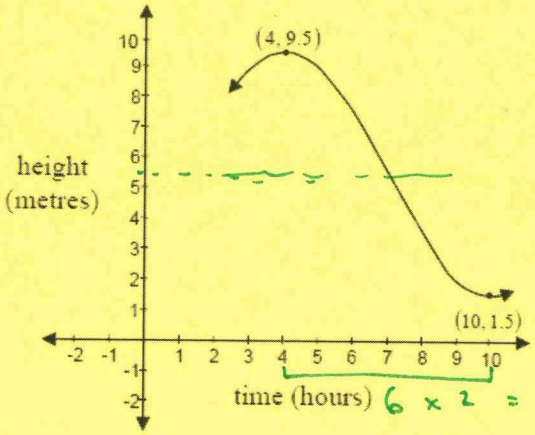
D = 12000



$b = \frac{2\pi}{\text{period}} = \frac{2\pi}{8}$

June 2010

6. At 4 a.m. on a typical day in Churchill, the height of the water at high tide is 9.5 metres. At 10 a.m. that same day, the height of the water at low tide is 1.5 metres. The height, h, of the water varies sinusoidally with time, t.



median $\frac{9.5 + 1.5}{2} = 5.5$

$d = 5.5$

$a = 4$

$b = \frac{2\pi}{\text{period}} = \frac{2\pi}{12} = \frac{\pi}{6}$

6 x 2 = 12 period

a) Write a sinusoidal equation in the form $h = A\cos[B(t - C)] + D$ to represent this function.

$h = 4\cos\left[\frac{\pi}{6}(t-4)\right] + 5.5$

or

$h = -4\cos\left[\frac{\pi}{6}(t-10)\right] + 5.5$

b) Determine the height in metres of the water at 11 a.m. that same day.

Express your answer correct to 3 decimal places.

$h = 4\cos\left(\frac{\pi}{6}(11-4)\right) + 5.5$

$= 2.036$ m

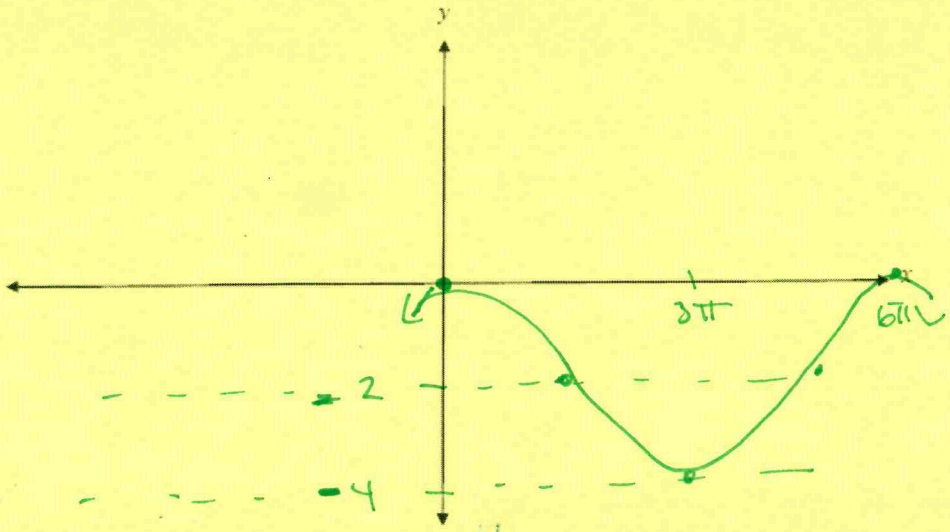
44. Given that $f(x) = 2\cos\left(\frac{1}{3}x\right) - 2$,

a) state the period of $f(x)$.

period $= \frac{2\pi}{\frac{1}{3}}$

$= 6\pi$

b) sketch a clearly labelled graph of at least one period of $f(x)$.



January 2010

49. Sketch a clearly labelled graph of at least one period of the trigonometric function $y = -3\cos(2x)$.

period $= \pi$