June 2017

Outcome T5

Question 5 — 3 marks

Solve the following equation algebraically over the interval $[0, 2\pi]$.

$6\sin^2\theta + \sin\theta - 1 = 0$

$(3\sin\theta - 1)(2\sin\theta + 1) = 0$

$\sin\theta = \frac{1}{3}$ or $\sin\theta = -\frac{1}{2}$

$\theta = 0.33983869$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\theta = 0.340, 2.802$

Question 32 — 4 marks

Solve $\cos 2\theta = 0$, where $\theta \in \mathbb{R}$.

$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$2\theta = \frac{\pi}{2} + \pi n \quad n \in I$

$\theta = \frac{\pi}{4} + \frac{\pi}{2} n \quad n \in I$

Question 43 — 2 marks

Solve $\sec\theta + 2 = 0$ over the interval $[0, 2\pi]$.

$\sec\theta = -2$

$\cos\theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

January 2017

Question 5 — 3 marks

Solve the following equation algebraically over the interval $0 \le \theta \le 2\pi$.

$2\cos^2\theta + 9\cos\theta - 5 = 0$

$(2\cos\theta - 1)(\cos\theta + 5) = 0$

$\cos\theta = \frac{1}{2}$ $\quad \cos\theta = -5$

NO SOLUTION.

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

Question 2 — 3 marks

Solve the following equation over the interval $[0, 2\pi]$:

$$3\sin^2\theta - 10\sin\theta - 8 = 0$$

$(3\sin\theta + 2)(\sin\theta - 4) = 0$

$\sin\theta = -\frac{2}{3}$ $\quad$ $\sin\theta = 4$ NO SOLUTION!

$\theta_r = 0.729728$

$\theta = 3.871, 5.553$

Question 9 — 4 marks

Solve the following equation algebraically for θ, where $0 \le \theta \le 2\pi$:

$$2\cos 2\theta = 1$$

$\cos 2\theta = \frac{1}{2}$

$2\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Question 19 — 1 mark

Identify the equation that has a general solution of $\left.\begin{array}{l}\theta = \frac{\pi}{6} + 2\pi k \\ \theta = \frac{5\pi}{6} + 2\pi k\end{array}\right\}$ where $k \in \mathbb{Z}$.

a) $\sin\theta = \frac{1}{2}$

b) $\cos\theta = \frac{1}{2}$

c) $\sin\theta = \frac{\sqrt{3}}{2}$

d) $\cos\theta = \frac{\sqrt{3}}{2}$

Question 33 — 1 mark

Describe the error that was made when solving the following equation:

$$\sin^2\theta + \sin\theta - 2 = 1$$

$\sin^2\theta + \sin\theta = 3$

$\sin\theta(\sin\theta + 1) = 3$

$\sin\theta = 3$ $\quad \sin\theta + 1 = 3$

$\sin\theta = 2$

$\therefore$ No solution $\quad \therefore$ No solution

The equation must $= 0$ before we factor. All terms must be grouped on one side.

Zero Product Property

January 2016

Question 3 — 3 marks

Solve the following equation over the interval $[0, 2\pi]$:

$$\sin^2\theta + 6\sin\theta - 2 = 0$$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Let $\sin\theta = x$

$x = \frac{-6 \pm \sqrt{36 - 4(1)(-2)}}{2(1)}$

$x = \frac{-6 \pm \sqrt{44}}{2}$

$\sin\theta = 0.316625$

$\sin\theta = -6.316$

No solution

$\theta = 0.322$

$\theta = 0.322, 2.8190$

Question 5 — 4 marks

Solve $(2\sin\theta - 1)(\sin\theta + 1) = 0$ where $\theta \in \mathbb{R}$.

$\sin\theta = \frac{1}{2}$ $\quad \sin\theta = -1$

$\theta = \frac{\pi}{6} + 2k\pi \quad \frac{5\pi}{6} + 2k\pi \quad \frac{3\pi}{2} + 2k\pi$

$k \in I$

June 2015

Question 2 — 4 marks

Solve $\tan^2\theta - 5\tan\theta + 4 = 0$ where $\theta \in \mathbb{R}$.

$\tan\theta - 4 = 0$ $\quad$ $\tan\theta - 1 = 0$

$\tan\theta = 4$ $\quad$ $\tan\theta = 1$

$\theta_r = \tan^{-1}(4)$ $\quad$ $\theta = \frac{\pi}{4} + \pi n$ $\quad n \in \mathbb{I}$

$\theta_r = 1.3258176$

$\theta = 1.326 + \pi n$

January 2015

Question 4 — a) 1 mark b) 2 marks

Talla incorrectly solved the following trigonometric equation:

Solve: $2\sec x - 5 = 0;\ 0^\circ \le x \le 360^\circ$.

Talla's work:

$2\sec x - 5 = 0$

~~$\sec x = \frac{5}{2}$~~

No solution, sec x cannot be greater than 1.

a) **Explain her error.** She mistakenly thought that.

$-1 \le \sec x \le 1$ $\quad$ but $\quad \sec x \ge 1$

$\sec x \le -1$

$|\sec x| \ge 1$

b) **Determine the correct solution.**

$\sec x = \frac{5}{2}$

$\cos x = \frac{2}{5}$

$x_r = 66.421812^\circ$ [redacted]

$x = 66.422^\circ$, 293.578°, [redacted]

Question 2 — 2 marks

Solve the following equation over the interval $[0, 2\pi)$.

$$\tan^2\theta + 2.8\tan\theta + 1.96 = 0$$

Use the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for $ax^2 + bx + c = 0$.

$\tan\theta = \dfrac{-2.8 \pm \sqrt{(-2.8)^2 - 4(1)(1.96)}}{2(1)}$.

$\tan\theta = -1.4$

$\theta_r = \tan^{-1}(1.4)$

$\theta_r = 0.950547$

$\theta = 2.191, 5.333$

Question 25 — 1 mark

The general solution to the equation $\cos\theta = -\dfrac{1}{2}$ is:

a) $\left.\begin{array}{l}\theta = \dfrac{\pi}{3} + 2\pi k \\ \theta = \dfrac{5\pi}{3} + 2\pi k\end{array}\right\}$ where $k \in I$

b) $\left.\begin{array}{l}\theta = \dfrac{\pi}{3} + \pi k \\ \theta = \dfrac{5\pi}{3} + \pi k\end{array}\right\}$ where $k \in I$

c) $\left.\begin{array}{l}\theta = \dfrac{2\pi}{3} + 2\pi k \\ \theta = \dfrac{4\pi}{3} + 2\pi k\end{array}\right\}$ where $k \in I$

d) $\left.\begin{array}{l}\theta = \dfrac{2\pi}{3} + \pi k \\ \theta = \dfrac{4\pi}{3} + \pi k\end{array}\right\}$ where $k \in I$

Question 32 — 1 mark

Given the equation $2\sin^2\theta - 3\sin\theta + 1 = 0$, verify that $\theta = \dfrac{\pi}{2}$ is a solution.

LHS $2\left[\sin\left(\frac{\pi}{2}\right)\right]^2 - 3\sin\frac{\pi}{2} + 1$

$2(1)^2 - 3(1) + 1$

$2 - 3 + 1$

0

LHS = RHS.

$\theta = \frac{\pi}{2}$ is a solution.

Question 39 — 1 mark

Explain why the equation $\sec\theta = \frac{1}{4}$ has no solution.

Since $\sec\theta = \frac{1}{\cos\theta}$ then $\cos\theta = 4$

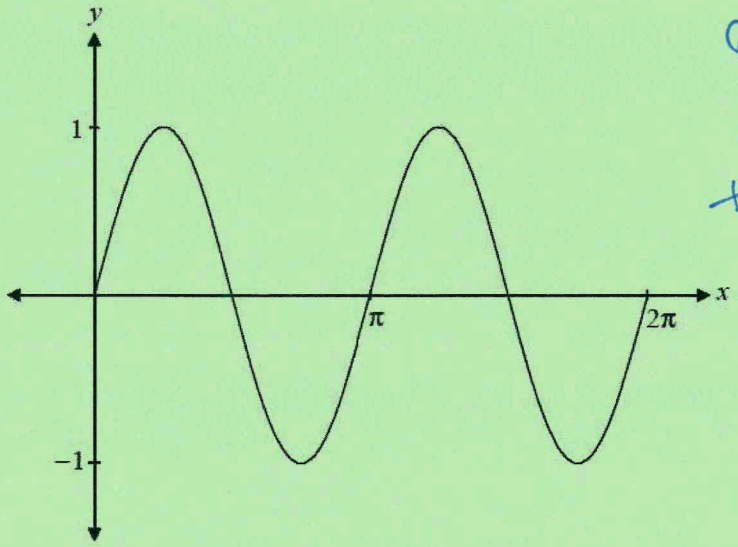
And $-1 \leq \cos\theta \leq 1$

$\therefore$ NO SOLUTION

Question 40 — 1 mark

The graph of $y = \sin 2x$ is sketched below.

Explain how to use this graph to solve the equation $\sin 2x = \frac{1}{2}$ over the interval $[0, 2\pi]$.



Graph $y = \frac{1}{2}$ and determine the x coordinate of the points of intersection in the interval $[0, 2\pi]$

January 2014

Question 2 — 3 marks

Solve the following equation over the interval $0 \leq \theta < 2\pi$.

$$(\tan\theta - 3)(\tan\theta + 1) = 0$$

$\tan\theta = 3$ $\quad$ $\tan\theta = -1$

$\theta_r = \tan^{-1}(3)$

$\theta_r = 1.249045$

$\theta = 1.250, 4.391, \frac{3\pi}{4}, \frac{7\pi}{4}$

June 2013

Question 2 — 4 marks

Solve the equation $\csc^2\theta + 3\csc\theta - 4 = 0$ over the interval $[0, 2\pi]$.
Express your answers as exact values or correct to 3 decimal places.

$(\csc\theta + 4)(\csc\theta - 1) = 0$

$\csc\theta = -4$ $\quad$ $\csc\theta = 1$

$\sin\theta = -\frac{1}{4}$ $\quad$ $\sin\theta = 1$

$\theta_r = \sin^{-1}\left(\frac{1}{4}\right)$

$\theta_r = 0.252683$

$\theta = 3.394,\ 6.0305$

$\frac{\pi}{2}$

Question 21 — 1 mark

Which of the following represents the general solution to the equation $\tan\theta = -1$?

a) $\theta = \frac{\pi}{4} + 2k\pi,\ k \in \mathrm{I}$

b) $\theta = \frac{\pi}{4} + k\pi,\ k \in \mathrm{I}$

c) $\theta = \frac{3\pi}{4} + 2k\pi,\ k \in \mathrm{I}$

d) $\theta = \frac{3\pi}{4} + k\pi,\ k \in \mathrm{I}$

January 2013

Question 1 — 3 marks

Gina correctly started to answer the following question. Complete her solution.

Question: Solve the following equation for all real values of θ.
Express your answer in radians correct to 3 decimal places.

$3\sin^2\theta - 14\sin\theta - 5 = 0$

Gina's solution: $3\sin^2\theta - 14\sin\theta - 5 = 0$

$(3\sin\theta + 1)(\sin\theta - 5) = 0$

$\sin\theta = -\frac{1}{3}$ $\quad$ $\sin\theta = 5$

no solution.

$\theta_r = \sin\left(\frac{1}{3}\right)$

$\theta_r = 0.339837$

$\theta = 3.481 + 2\pi n$

$5.943 + 2\pi n$ $\quad n \in \mathrm{I}$