P. 8 Solutions Review of Limits

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(a) $\lim_{x\to-\infty} \dfrac{\frac{x^2}{x^3} - \frac{2x^3}{x^3}}{\frac{4x^3}{x^3} - \frac{6}{x^3}}$

$\lim_{x\to-\infty} \dfrac{\frac{1}{x} - 2}{4 - \frac{6}{x^3}}$

$= \dfrac{-2}{4}$

(b) $\lim_{x\to\infty} \dfrac{\frac{x^2}{x^4} - \frac{2x^4}{x^4}}{\frac{4x^3}{x^4} - \frac{6}{x^4}}$

$\lim_{x\to\infty} \dfrac{\frac{1}{x^2} - 2}{\frac{1}{x} - \frac{6}{x^4}}$

DNE $\quad -\infty$

(c) $\lim_{x\to-\infty} \dfrac{\frac{x^2}{x^5} + \frac{x^3}{x^5}}{\frac{4x^5}{x^5} + \frac{x^2}{x^5}}$

$\lim_{x\to-\infty} \dfrac{\frac{1}{x^3} + \frac{1}{x^2}}{4x + \frac{1}{x^3}}$

$= 0$

(d) $\lim_{x\to\infty} \dfrac{\sqrt{x^2}}{2x+1}$ $\rightarrow |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

$\lim_{x\to\infty} \dfrac{|x|}{2x+1}$

$\lim_{x\to\infty} \dfrac{x}{2x+1}$

$\lim_{x\to\infty} \dfrac{x/x}{\frac{2x}{x} + \frac{1}{x}} = \dfrac{1}{2}$

(e) $\lim_{x\to-\infty} \dfrac{|x|}{2x+1}$

$\lim_{x\to-\infty} \dfrac{-x}{2x+1} = \dfrac{-1}{2}$

(f) $\lim_{x\to0} \dfrac{\frac{1}{x+2} + \frac{1}{x-2}}{x} \cdot \dfrac{(x-2)(x+2)}{(x-2)(x+2)}$

$\lim_{x\to0} \dfrac{x-2+x+2}{x(x-2)(x+2)}$

$\lim_{x\to0} \dfrac{2x}{x(x-2)(x+2)}$

$= \dfrac{2}{-4} = \dfrac{-1}{2}$

(g) $\lim_{x\to2^-} \dfrac{x-3}{x-2}$

$= +\infty$

note: $x = 2$ is an asymptote.
So we approach either $\pm\infty$
for $x = 1.9$ we have + value.

(h) $\lim_{x\to2^+} \dfrac{x-3}{x-2} = -\infty$

for $x = 2.1$ we have − value

(i) $\lim_{x\to-4^-} \dfrac{(x-3)}{(x+4)^2}$

for $x = -4.1$ $\dfrac{(-)}{(+)}$

$-\infty$

(j) $\lim_{x\to-5^-} \dfrac{|x+5|}{x+5}$

$\lim_{x\to-5^-} \dfrac{-(x+5)}{x+5}$

$= -1$

(k) $\lim_{x\to4^+} \dfrac{+(x-4)}{x-4}$

(l) $\lim_{x\to-2^-} \dfrac{3|x+2|}{x+2}$

$= 1$

$$\lim_{x \to -2^-} \frac{-3(x+2)}{x+2}$$

$$= -3$$

(m) $$\lim_{x \to 10} \frac{(x-10)(x+10)}{x-10}$$

$$\lim_{x \to 10} \quad x+10$$

$$= 20$$

(n) $$\lim_{x \to -3} \frac{(x^2-9)(x^2+9)}{x+3}$$

$$\lim_{x \to -3} \frac{(x-3)(x+3)(x^2+9)}{x+3}$$

$$\lim_{x \to -3} \quad (x-3)(x^2+9)$$

$$= -6(18)$$

$$= -108$$

(o) $$\lim_{x \to 25} \frac{\sqrt{x}-5}{x-25} \cdot \frac{\sqrt{x}+5}{\sqrt{x}+5}$$

$$\lim_{x \to 25} \frac{x-25}{(x-25)(\sqrt{x}+5)}$$

$$= \frac{1}{10}$$

(p) $$\lim_{x \to \infty} \frac{\sqrt{x^2-6x}-x}{\sqrt{x^2-6x}+x} \cdot \frac{\sqrt{x^2-6x}+x}{\sqrt{x^2-6x}+x}$$

$$\lim_{x \to \infty} \frac{x^2-6x-x^2}{\sqrt{x^2+6x}+x}$$

$$\lim_{x \to \infty} \frac{-6x}{\sqrt{x^2+6x}+x}$$

$$\lim_{x \to \infty} \frac{-6x}{\sqrt{x^2+\frac{6}{x}}+x}$$

$$\lim_{x \to \infty} \frac{-6x}{|x|\sqrt{1+\frac{6}{x}}+x}$$

$$\lim_{x \to \infty} \frac{\frac{-6x}{x}}{\frac{x\sqrt{1+\frac{6}{x}}}{x}+\frac{x}{x}}$$

$$\lim_{x \to \infty} \frac{-6}{\sqrt{1}+1} = -3$$

(q) $$\lim_{x \to -4^-} \frac{x+5}{(x+4)^3}$$

$$-\infty$$

(r) $$\lim_{x \to \infty} \frac{\left(\sqrt{3x^2+8x}-x\right)\left(\sqrt{3x^2+8x}+x\right)}{\sqrt{3x^2+8x}+x}$$

$$\lim_{x \to \infty} \frac{3x^2+8x-x^2}{\sqrt{3x^2+8x}+x}$$

$$\lim_{x \to \infty} \frac{2x^2+8x}{|x|\sqrt{3+\frac{8}{x}}+x}$$

$$\frac{2x^2+8x}{}$$

$$\lim_{x\to\infty} \frac{\frac{2x^2}{x^2} + \frac{8x}{x^2}}{\frac{x}{x^2}\sqrt{3 + 8x} + \frac{x}{x^2}}$$

$$\lim_{x\to\infty} \frac{2 + \frac{8}{x}}{\frac{1}{x}\sqrt{3+8x} + \frac{1}{x}}$$

DNE approaches ∞