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6.4 – Solving Trigonometric Equations Using Identities

1. **Solve** each equation algebraically over the interval $0° \leq x \leq 360°$

a) $\sin 2x - \cos x = 0$

$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ | $2\sin x - 1 = 0$

$x = 90°, 270°$ | $\sin x = \frac{1}{2}$

$x = 30°, 150°$

$\therefore x = 90°, 270°, 30°, 150°$

b) $\cos 2x = 0$

$2\cos^2 x - 1 = 0$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm\sqrt{\frac{1}{2}}$

$\cos x = \pm\frac{1}{\sqrt{2}}$

$x = 45°, 135°, 225°, 315°$

2. **Solve** each equation algebraically over the interval $0 \leq x \leq 2\pi$

a) $2\cos^2 x - 5\sin x - 5 = 0$

$2(1 - \sin^2 x) - 5\sin x - 5 = 0$

$2 - 2\sin^2 x - 5\sin x - 5 = 0$

$0 = 2\sin^2 x + 5\sin x + 3$

$0 = (2\sin x + 3)(\sin x + 1)$

$0 = 2\sin x + 3$ | $\sin x + 1 = 0$

$-3/2 = \sin x$ | $\sin x = -1$

no solution | $x = \frac{3\pi}{2}$

$\therefore x = \frac{3\pi}{2}$

b) $\cos 2x - 5\cos x = 2$

$2\cos^2 x - 1 - 5\cos x = 2$

$2\cos^2 x - 5\cos x - 3 = 0$

$(2\cos x + 1)(\cos x - 3) = 0$

$2\cos x + 1 = 0$	$\cos x - 3 = 0$
$\cos x = -1/2$	$\cos x = 3$
$x = \frac{2\pi}{3}, \frac{4\pi}{3}$	no solution

$\therefore x = \frac{2\pi}{3}$ & $\frac{4\pi}{3}$

c) $1 + \cos x = 2\sin^2 x$

$1 + \cos x = 2(1 - \cos^2 x)$

$1 + \cos x = 2 - 2\cos^2 x$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0$	$\cos x + 1 = 0$
$\cos x = 1/2$	$\cos x = -1$
$x = \frac{\pi}{3}, \frac{5\pi}{3}$	$x = \pi$

$\therefore x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi$

3. Determine the error that the student made in the following work, then write the correct answer.

$\sin 2x = 1$

$\boxed{\sin x = \frac{1}{2}}$ → $\sin 2x \neq 2\sin x$

Cannot divide by 2 on both sides.

$x = 60°$ & $120°$

let $2x = \theta$

$\sin\theta = 1$

$\theta = 90°$

$\therefore \frac{2x}{2} = \frac{90°}{2}$

$x = 45°$

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4. **Solve** the equation $2\cos 2x + 1 = 0$. Indicate the general solution expressed in radians.

$2(2\cos^2 x - 1) + 1 = 0$

$4\cos^2 x - 2 + 1 = 0$

$4\cos^2 x - 1 = 0$

$\cos^2 x = \frac{1}{4}$

$\cos x = \pm \frac{1}{2}$

$\cos x = \frac{1}{2}$ | $\cos x = -\frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ | $x = 2\pi/3, 4\pi/3$

$\therefore x = \frac{\pi}{3} + 2\pi k, k \in Z$

~~$x = \frac{5\pi}{3} + 2\pi k, k \in Z$~~

$x = \frac{2\pi}{3} + 2\pi k, k \in Z$

$x = \frac{4\pi}{3} + 2\pi k, k \in Z$

OR

$x = \frac{\pi}{3} + \pi n$

$\frac{2\pi}{3} + \pi n$

$n \in Z.$