MPC40S Date: ____________________

6.2 – Sum, Difference, and Double Angle Identities

1. **Simplify** each expression using identities and then give an exact value for each expression.

a) $\cos 25° \cos 5° - \sin 25° \sin 5°$

$= \cos(25° + 5°)$

$= \cos(30°)$

$= \frac{\sqrt{3}}{2}$

b) $\sin 40° \cos 20° + \cos 40° \sin 20°$

$= \sin(40° + 20°)$

$= \sin(60°)$

$= \frac{\sqrt{3}}{2}$

c) $\sin\frac{\pi}{3}\cos\frac{\pi}{6} + \cos\frac{\pi}{3}\sin\frac{\pi}{6}$

$= \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)$

$= \sin\left(\frac{2\pi}{6} + \frac{\pi}{6}\right)$

$= \sin\left(\frac{3\pi}{6}\right)$

$= \sin\left(\frac{\pi}{2}\right)$

$= 1$

d) $\cos\frac{7\pi}{12}\cos\frac{\pi}{3} + \sin\frac{7\pi}{12}\sin\frac{\pi}{3}$

$= \cos\left(\frac{7\pi}{12} - \frac{\pi}{3}\right)$

$= \cos\left(\frac{7\pi}{12} - \frac{4\pi}{12}\right)$

$= \cos\left(\frac{3\pi}{12}\right)$

$= \cos\left(\frac{\pi}{4}\right)$

$= \frac{\sqrt{2}}{2}$

2. **Simplify** each expression below.

a) $\sin(90° + A)$

$= \sin(90°)\cos(A) + \cos(90°)\sin(A)$

$= (1)\cos A + (0)(\sin A)$

$= \cos A$

b) $\cos\left(A - \frac{3\pi}{2}\right)$

$= \cos(A)\cos\left(\frac{3\pi}{2}\right) + \sin(A)\sin\left(\frac{3\pi}{2}\right)$

$= \cos(A)(0) + \sin(A)(-1)$

$= 0 - \sin(A)$

$= -\sin(A)$

3. **Simplify** each expression to a single trigonometric function and then evaluate.

a) $2\sin\frac{\pi}{6}\cos\frac{\pi}{6}$

$= \sin\left(2\left(\frac{\pi}{6}\right)\right)$

$= \sin\left(\frac{2\pi}{6}\right)$

$= \sin\left(\frac{\pi}{3}\right)$

$= \frac{\sqrt{3}}{2}$

b) $\cos^2\frac{\pi}{3} - \sin^2\frac{\pi}{3}$

$= \cos\left(2\left(\frac{\pi}{3}\right)\right)$

$= \cos\left(\frac{2\pi}{3}\right)$

$= \frac{-1}{2}$

c) $1 - 2\sin^2 15°$

$= \cos(2(15°))$

$= \cos(30°)$

$= \frac{\sqrt{3}}{2}$

d) $\frac{2\tan\frac{\pi}{6}}{1-\tan^2\frac{\pi}{6}}$

$= \tan\left(2\left(\frac{\pi}{6}\right)\right)$

$= \tan\left(\frac{2\pi}{6}\right)$

$= \tan\left(\frac{\pi}{3}\right)$

$= \sqrt{3}$

4. Determine the **exact value** of each trigonometric expression below.

a) $\cos\frac{\pi}{12}$ $= \cos\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$

$= \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$= \frac{\sqrt{2}+\sqrt{6}}{4}$

b) $\tan 15°$ $= \tan(45° - 30°)$

$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$

$= \frac{1 - 1/\sqrt{3}}{1 + (1)(1/\sqrt{3})}$

$= \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$

$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$

c) $\sin 105°$ $= \sin(60° + 45°)$

$= \sin(60°)\cos(45°) + \cos(60°)\sin(45°)$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$

d) $\sin\frac{23\pi}{12}$ $= \sin\left(\frac{15\pi}{12} + \frac{8\pi}{12}\right)$

$= \sin\left(\frac{5\pi}{4} + \frac{2\pi}{3}\right)$

$= \sin\left(\frac{5\pi}{4}\right)\cos\left(\frac{2\pi}{3}\right) + \cos\left(\frac{5\pi}{4}\right)\sin\left(\frac{2\pi}{3}\right)$

$= \left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

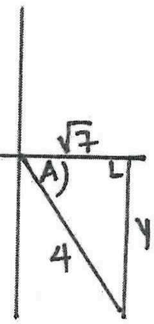
$= \frac{\sqrt{2}-\sqrt{6}}{4}$

MPC40S   Date: ____________________

5. If $\cos A = \frac{\sqrt{7}}{4}$ and $\sin B = \frac{2}{5}$ and angles A and B are not in the first quadrant, determine the following values:

a) $\cos(A+B)$ b) $\tan 2A$ c) $\sin(A-B)$

Angle A



$(\sqrt{7})^2 + y^2 = (4)^2$

$7 + y^2 = 16$

$y = 9$

$y = \pm\sqrt{9}$

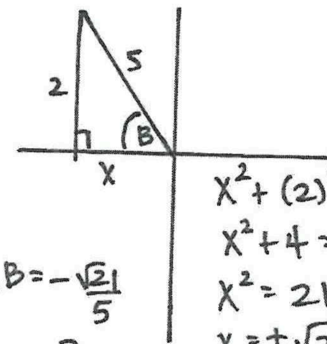
$y = -3$

$\therefore \cos A = \frac{\sqrt{7}}{4}$

$\sin A = -\frac{3}{4}$

$\tan A = -\frac{3}{\sqrt{7}}$

Angle B



$x^2 + (2)^2 = (5)^2$

$x^2 + 4 = 25$

$x^2 = 21$

$x = \pm\sqrt{21}$

$x = -\sqrt{21}$

$\cos B = -\frac{\sqrt{21}}{5}$

$\sin B = \frac{2}{5}$

a) $\cos(A+B)$

$= \cos A \cos B - \sin A \sin B$

$= \left(\frac{\sqrt{7}}{4}\right)\left(-\frac{\sqrt{21}}{5}\right) - \left(-\frac{3}{4}\right)\left(\frac{2}{5}\right)$

$= -\frac{\sqrt{147}}{20} + \frac{6}{20}$

$= \frac{-\sqrt{147}+6}{20}$

b) $\tan 2A$

$= \frac{2\tan A}{1-\tan^2 A}$

$= \frac{2\left(\frac{-3}{\sqrt{7}}\right)}{1-\left(\frac{-3}{\sqrt{7}}\right)^2}$

$= \frac{\frac{-6}{\sqrt{7}}}{1-\frac{9}{7}}$

$= \frac{\frac{-6}{\sqrt{7}}}{\frac{-2}{7}} = \frac{+21}{\sqrt{7}}$

c) $\sin(A-B)$

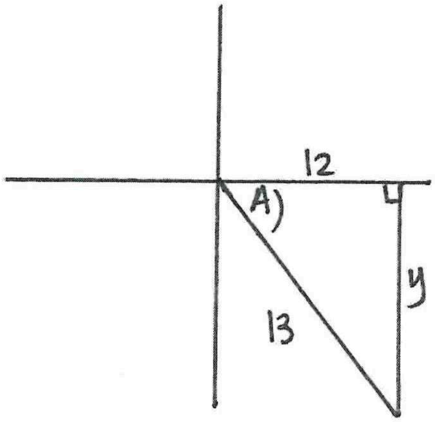
$= \sin A \cos B - \cos A \sin B$

$= \left(-\frac{3}{4}\right)\left(-\frac{\sqrt{21}}{5}\right) - \left(\frac{\sqrt{7}}{4}\right)\left(\frac{2}{5}\right)$

$= \frac{3\sqrt{21}}{20} - \frac{2\sqrt{7}}{20}$

$= \frac{3\sqrt{21} - 2\sqrt{7}}{20}$

6. If $\cos \angle A = \frac{12}{13}$ and $\angle A$ is in Quadrant IV, determine the **exact value** of $\sin 2A$



$(12)^2 + (y)^2 = (13)^2$

$144 + y^2 = 169$

$y^2 = 25$

$y = -5$

$\cos A = \frac{12}{13}$

$\sin A = -\frac{5}{13}$

$\sin 2A = 2\sin A\cos A$

$= (2)(-\frac{5}{13})(\frac{12}{13})$

$= \frac{-120}{169}$