Pre-Calc 40s Past Achievement Test Questions on Polynomials

June 2015

## Question 8 — 1 mark

Explain how the value of $n$ affects the behaviour of the graph of the polynomial function $p(x) = (x+3)(x-1)^n$, as $p(x)$ approaches the $x$-intercept at $x = 1$.

- If $n=1$ the graph will cross through $x=1$
- If $n$ is odd and greater than 1 the graph will level off as it crosses the x-axis
- If $n$ is even the graph will bounce off the x intercept

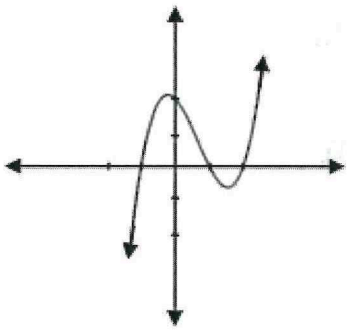
## Question 19 — 1 mark

Identify the graph of the function $f(x) = -(x-2)(x-1)^2(x+1)$.

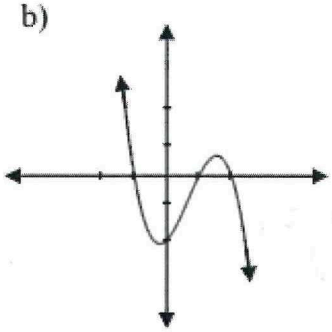
4th degree

negative leading coefficient

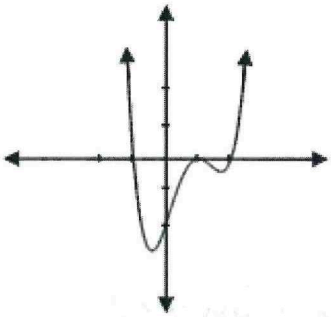
a)



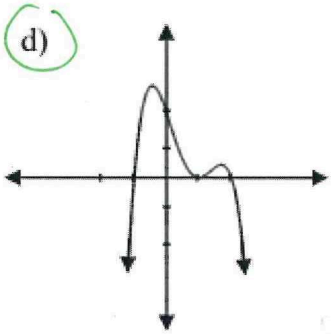
b)



c)

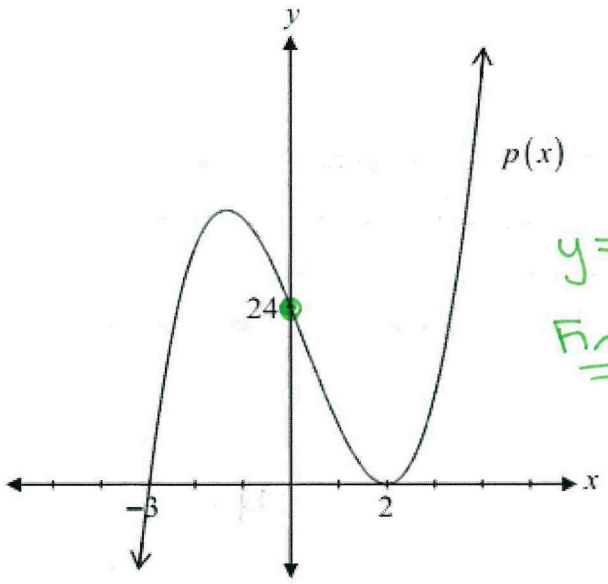


d)



## Question 28 — 3 marks

Determine the equation of the polynomial function represented by the graph.




$y = a(x+3)(x-2)^2$

Find a : subs. (0, 24)

$24 = a(0+3)\cdot(0-2)^2$

$24 = a(3)(4)$

$24 = 12a$

$\frac{24}{12} = a$

$2 = a$

$p(x) = 2(x+3)(x-2)^2$

## Question 39 — a) 2 marks b) 1 mark c) 1 mark

Christine dives off a diving board.

Her dive is modelled by the function $h(t) = t^3 - 3t^2 - t + 3$, where $h$ is her height in metres, relative to the water surface and $t$ is the time in seconds after diving off the diving board.

a) Given that $(t+1)$ is a factor for the function $h(t)$, determine the other factors.

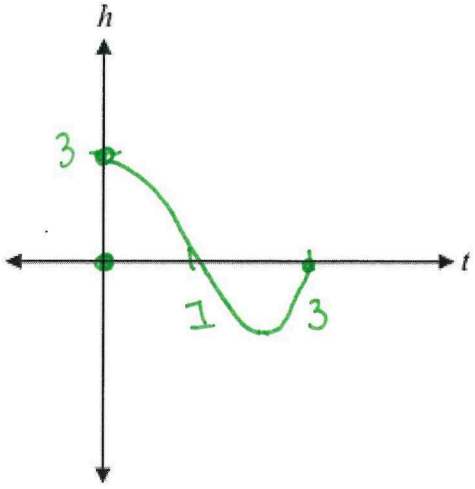
| −1 | 1 | −3 | −1 | 3 |
|---|---|---|---|---|
| | | −1 | 4 | −3 |
| | 1 | −4 | 3 | 0 |

$h(t) = (t+1)(t^2 - 4t + 3)$

$= (t+1)(t-3)(t-1)$

The other factors are $(t-3)$ and $(t-1)$

b) Sketch the graph of the function $h(t)$ for the time interval $t = 0$ to $t = 3$.




when x = 0

y when t = 0

h(0) = 3

c) Determine how many seconds Christine is underwater.

2 seconds

from 1sec → 3 sec.

January 2015

## Question 9 — 2 marks

Is $(x-2)$ a factor of the polynomial $p(x) = -x^4 - 3x^3 + 11x^2 + 3x - 10$?

Justify your response.

$p(2) = -(2)^4 - 3(2)^3 + 11(2)^2 + 3(2) - 10$

$= -16 - 3(8) + 11(4) + 6 - 10$

$= -16 - 24 + 44 - 4$

$= 0$

$(x-2)$ is a factor of $P(x)$

Since $P(2) = 0$ the factor theorem states

## Question 13 — 1 mark

Explain how the end behaviours of the graphs of polynomial functions with an even degree and with an odd degree are different.

Even degree polynomials go up into QI and II or down into QIII and IV.

While odd degree polynomials either go down into QIII and up into QI

or up into QII and down into QIV

## Question 24 — 1 mark

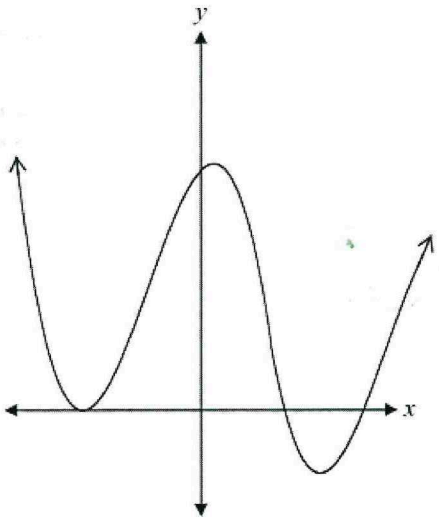
What is the degree of the polynomial function represented by the graph below?

a) 2

b) 3

c) 4

d) 5




## Question 27 — 2 marks

Determine all of the zeroes of the function $p(x) = x^3 - 5x^2 - 2x + 24$, given one of the factors of $p(x)$ is $(x-3)$.

$$\begin{array}{c|cccc} 3 & 1 & -5 & -2 & 24 \\ & & 3 & -6 & -24 \\ \hline & 1 & -2 & -8 & 0 \end{array}$$

$0 = (x-3)(x^2-2x-8)$

$0 = (x-3)(x-4)(x+2)$

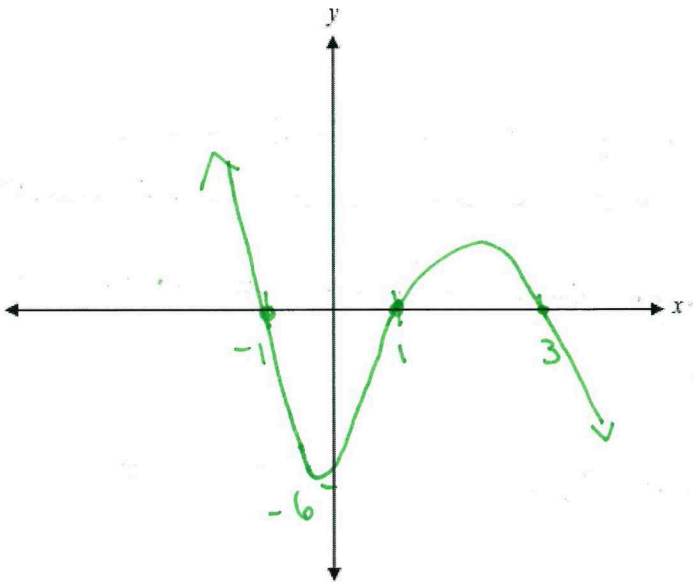
$x = 3, 4, -2$

## Question 33 — 3 marks

Sketch the graph of $y = -2(x-1)(x-3)(x+1)$.

y-int: $y = -2(-1)(-3)(1)$

$y = -6$



## Question 8

a) 1 mark b) 1 mark

Let $P(x) =$

a) Determine the remainder when $x^4 - 3x^2 + 1$ is divided by $x + 2$.

$P(-2) = (-2)^4 - 3(-2)^2 + 1$

$= 16 - 3(4) + 1$

$= 5$

## Question 16

2 marks

Divide $\left(x^3 - 5x - 4\right)$ by $(x + 1)$.

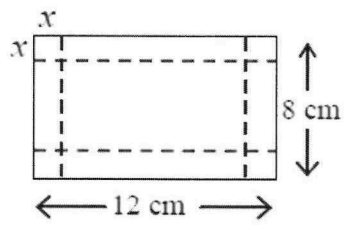
| | | | | |
|---|---|---|---|---|
| $-1$ | $1$ | $0$ | $-5$ | $-4$ |
| | | $-1$ | $1$ | $4$ |
| | $1$ | $-1$ | $-4$ | $0$ |

$\frac{x^3 - 5x - 4}{x+1} = x^2 - x - 4$

## Question 28

1 mark

A sheet of paper 12 cm long and 8 cm wide is used to make a box with no lid. Equal squares of side length $x$ cm are cut from each of the corners and the sides are folded up to make the box.




Which of the following expresses the volume of the box?

a) $V(x) = x(12 + x)(8 + x)$

b) $V(x) = x(12 - x)(8 - x)$

c) $V(x) = x(12 + 2x)(8 + 2x)$

d) $V(x) = x(12 - 2x)(8 - 2x)$

## Question 42 — 2 marks

Write the equation for $f(x)$ that satisfies all of the following conditions:

- $f(x)$ is a polynomial function of degree 4
- $f(x)$ has a zero at 2 with a multiplicity of 3
- $f(x)$ has a zero at $-5$
- $f(x)$ has a $y$-intercept of 80

$y = a(x+5)(x-2)^3$

$80 = a(5)(-8)$

$-2 = a$

$y = -a(x+5)(x-2)^3$

## Question 45 — 3 marks

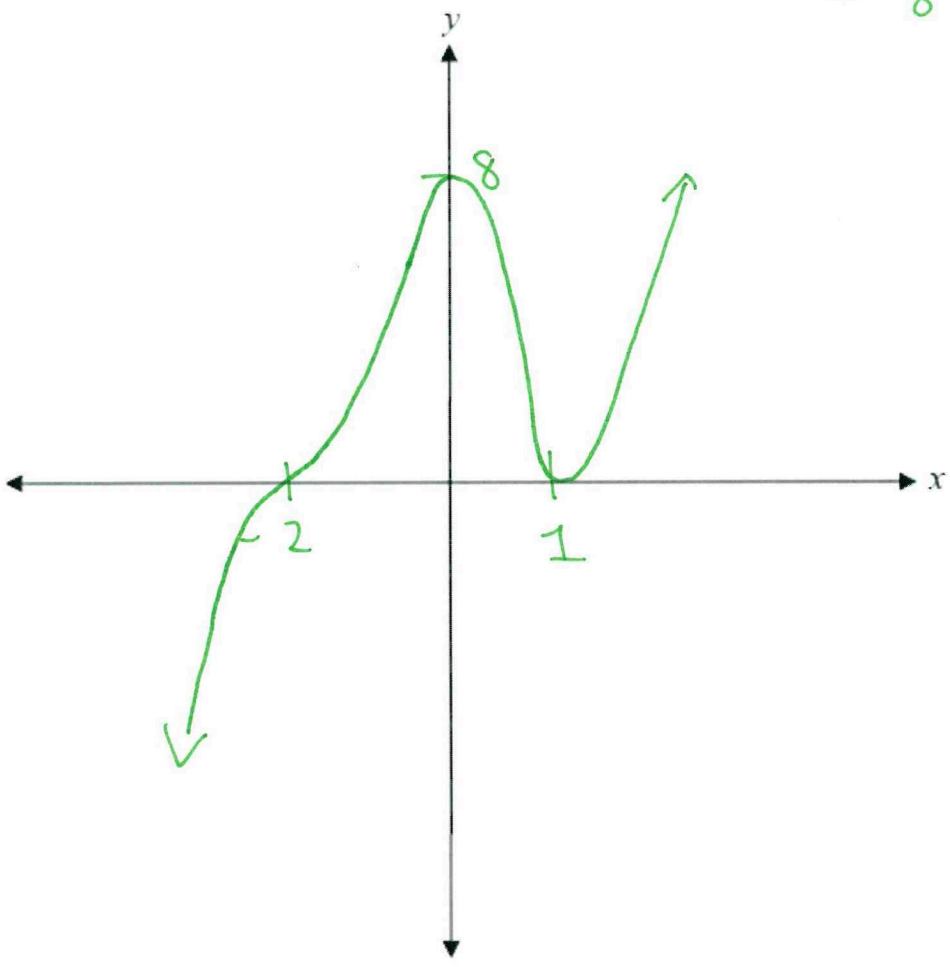
Sketch the graph of $f(x) = (x-1)^2(x+2)^3$.

Label the $x$-intercepts and the $y$-intercept.

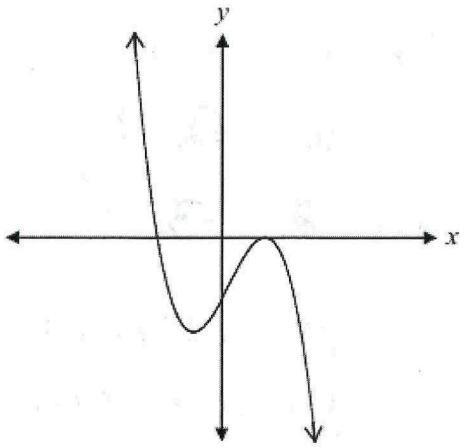
y int:

$f(0) = (-1)^2(+2)^3$

$= 8$



## Question 20 — 1 mark




Given the above graph of a polynomial function, which one of the following statements can be true?

a) The function has a degree of 4 with a positive leading coefficient.

b) The function has a degree of 4 with a negative leading coefficient.

c) The function has a degree of 3 with a positive leading coefficient.

d) The function has a degree of 3 with a negative leading coefficient.

## Question 21 — 1 mark

Given that $(x+3)$ is a factor of polynomial $P(x)$, which of the following is true?

a) $P(-3)=0$ b) $P(0)=-3$ c) $P(0)=3$ d) $P(3)=0$

## Question 26 — 2 marks

One of the factors of $P(x)=x^3-kx^2-7x+10$ is $(x-2)$. $P(+2)=0$

Find the value of $k$.

$0=(+2)^3-k(+2)^2-7(+2)+10$

$0=+8-4k-14+10$

$-4=-4k$

$1=k$

## Question 35 — 1 mark

When $P(x)$ is divided by $x-3$, it has a quotient of $2x^2+x-6$ and a remainder of 4.

Determine $P(x)$.

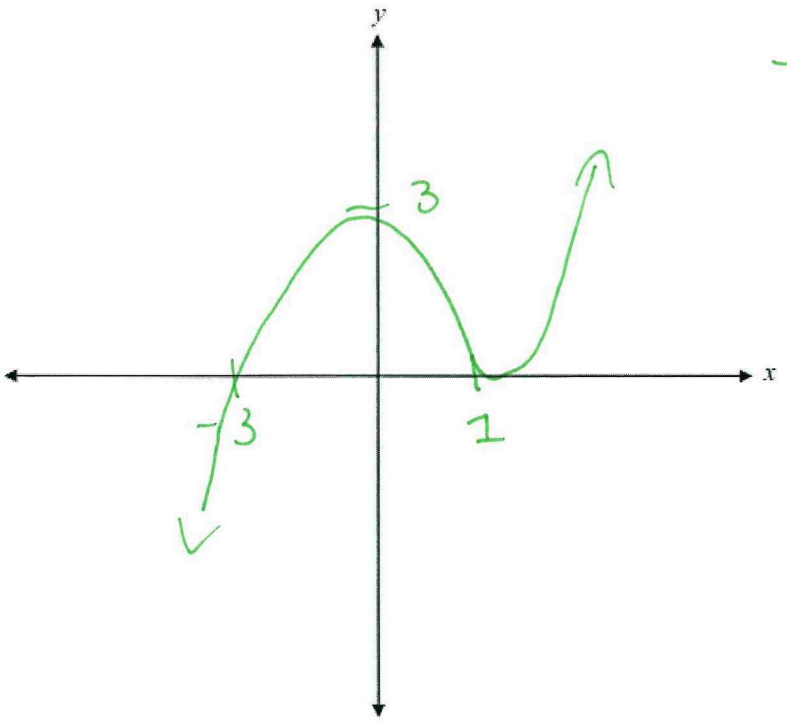
$\frac{P(x)}{x-3}=2x^2+x-6+\frac{4}{x-3}$

$P(x)=(2x^2+7x-6)(x-3)+4$

## Question 40 — 4 marks

Sketch the graph of $y = x^3 + x^2 - 5x + 3$ given that one of the $x$-intercepts is 1.

Identify the $x$-intercepts and $y$-intercept.




$+1 \mid 1 \quad 1 \quad -5 \quad 3$

$\quad\quad +1 \quad +2 \quad -3$

$\quad 1 \quad 2 \quad -3 \quad 0$

$y = (x-1)(x^2+2x-3)$

$y = (x-1)(x+3)(x-1)$

y int:

$y = -1(3)(-1)$

$y = 3$

June 2013

## Question 24 — 1 mark

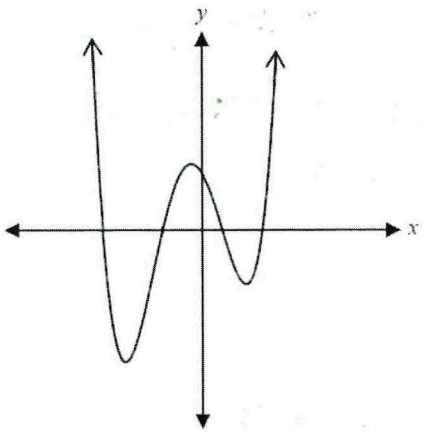
What is the degree of the polynomial represented below?

a) 2

b) 3

c) 4

d) 5

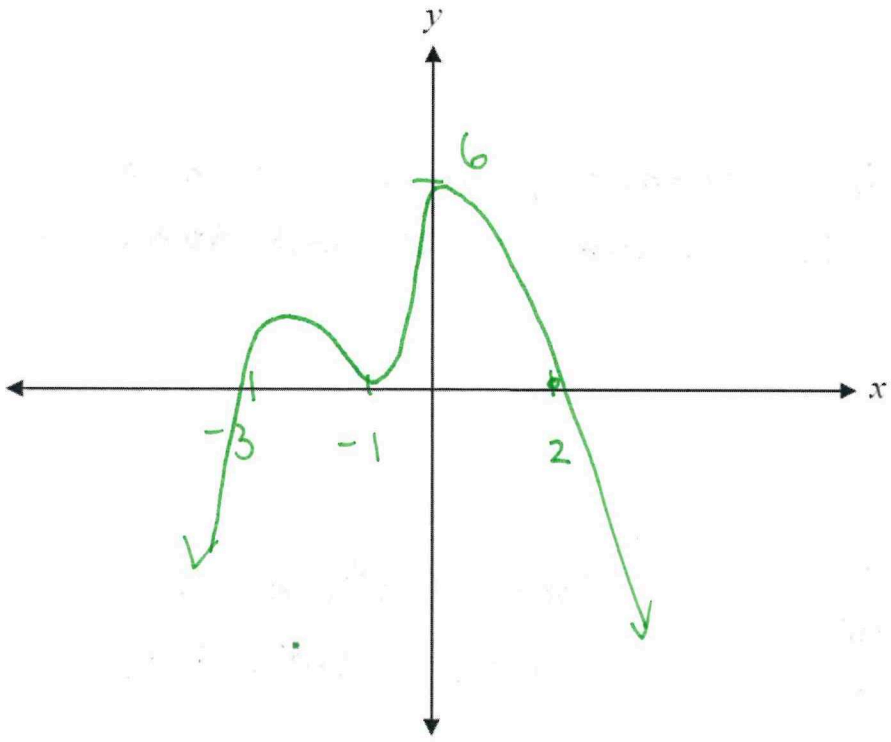


## Question 29 — 3 marks

Sketch the graph of:

$$f(x) = (2 - x)(x + 3)(x + 1)^2$$

Label the $x$-intercepts and $y$-intercept.

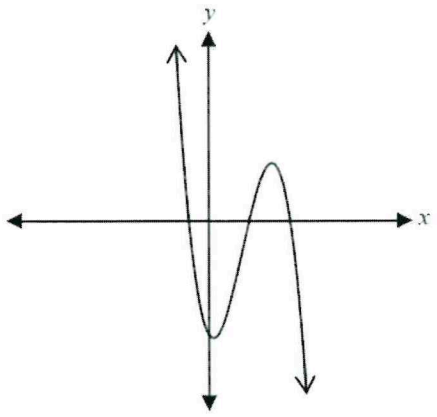



y int: $f(0) = -(-2)(3)(1)$

$= 6.$

## Question 31 — 1 mark

The graph below represents the equation $y = ax^3 + 6x^2 + 5x - 10$.




What must be true about the value of $a$? Explain your reasoning.

a is negative, since the end behavior is up into Q II and down into Q IV

## Question 35 — 2 marks

Given that $(x - 1)$ is one of the factors, express $x^3 - 57x + 56$ as a product of factors.

| 1 | 1 | 0 | -57 | 56 |
|---|---|---|---|---|
| | | 1 | 1 | -56 |
| | 1 | 1 | -56 | 0 |

$(x-1)(x^2+x-56)$

$(x-1)(x+8)(x-7)$

## Question 44 — 2 marks

Is $(x - 3)$ a factor of $x^4 - x^3 - 3x^2 + x - 1$?

Justify your answer.

Let $P(x) = x^4 - x^3 - 3x^2 + x - 1$

$P(3) = 3^4 - 3^3 - 3(3)^2 + 3 - 1$

$= 81 - 27 - 27 + 3 - 1$

$= 29$

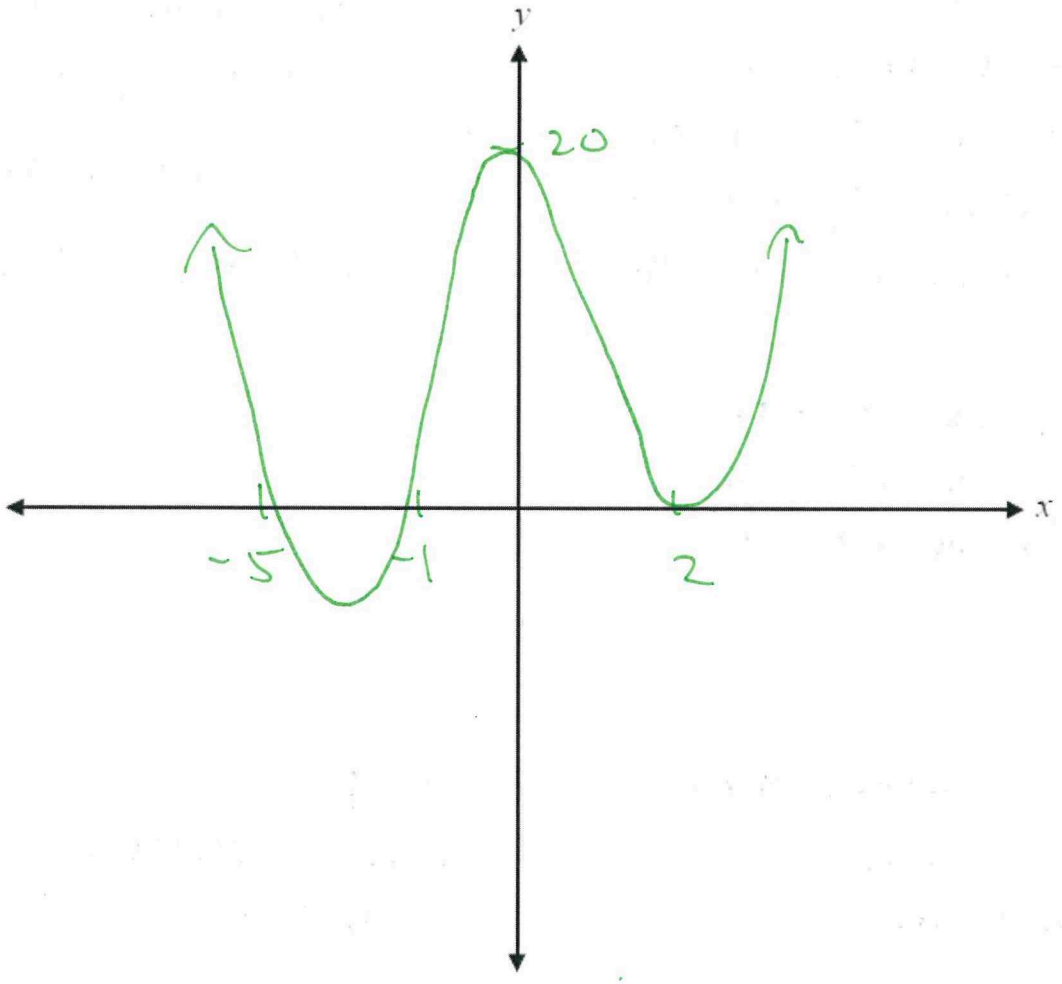
Since $P(3) \neq 0$ $(x-3)$ is not a factor of $P(x)$

## Question 27

3 marks

Sketch the graph of $y = (x+1)(x-2)^2(x+5)$.

Identify the $x$-intercepts and $y$-intercept.




$x$-intercepts: $-5, -1, 2.$

$y$-intercept: _____

$y = (1)(-2)^2(5)$

$y = 20$

## Question 31 — 5 marks

A box in the shape of a rectangular prism has side lengths $x$, $x+2$, and $x+10$.

Write a function, $V(x)$, to express the volume of the box in terms of $x$.

Find all possible values of $x$, given that the volume of the box is $96\ \text{cm}^3$.

State the dimensions of the box.

$V(x) = x(x+2)(x+10)$

$96 = x(x+2)(x+10)$

$96 = x(x^2+12x+20)$

$0 = x^3+12x^2+20x-96$

Let $P(x) = x^3+12x^2+20x-96$

$P(2) = 0$

| 2 | 1 | 12 | 20 | −96 |
|---|---|---|---|---|
| | | 2 | 28 | 96 |
| | 1 | 14 | 48 | 0 |

$P(x) = (x-2)(x^2+14x+48)$

$P(x) = (x-2)(x+8)(x+6)$

$0 = (x-2)(x+8)(x+6)$

$x = 2, -8, -6$

## Question 40 — 1 mark

Given that $h(x) = 2x^2+5x-3$ and that $h(x) = f(x) \cdot g(x)$, determine $f(x)$ and $g(x)$.

$h(x) = (2x-1)(x+3)$

$f(x) = (2x-1)$

$g(x) = (x+3)$

Also

$f(x) = 2x^2+5x-3$

$g(x) = 1$

## Question 47 — 1 mark

If $p(x) = x^5 - 12x + 1$, determine the remainder when $p(x)$ is divided by $(x+2)$.

$p(-2) = (-2)^5 - 12(-2) + 1$

$= -32 + 24 + 1$

$= -8 + 1$

$= -7$

Remainder $-7$.