MPC40S Date: Answer key

OUTCOME T6 – Review

1. Simplify the expression $\frac{\cot^2\theta}{1+\cot^2\theta}$

a) $\cos^2\theta$ b) $\sin^2\theta$ c) $\tan^2\theta$ d) $\sec^2\theta$

$$\frac{\cot^2\theta}{1+\cot^2\theta} = \frac{\frac{\cos^2\theta}{\sin^2\theta}}{\csc^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} \div \frac{1}{\sin^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} \times \frac{\sin^2\theta}{1} = \cos^2\theta$$

2. The value of $(\sin x - \cos x)^2 + \sin 2x$ is

a) -1 b) 0 c) 1 d) 2

$\sin^2 x - \cancel{2\sin x\cos x} + \cos^2 x + \cancel{2\sin x\cos x}$

$= \sin^2 x + \cos^2 x$

$= 1$

3. The expression $\frac{1-\tan^2\theta}{1+\tan^2\theta}$ is equivalent to

a) $\cos 2\theta$ b) $\sin 2\theta$ c) $\cos^2\theta$ d) $\sin^2\theta$

$$\frac{\frac{\cos^2\theta}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}} = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{\cos^2\theta + \sin^2\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{1} = \cos^2\theta - \sin^2\theta$$

4. If you simplify $\sin(\pi + x) + \sin(\pi - x)$

a) -2 b) 0 c) 2 d) not possible

$= \cancel{\sin\pi\cos x} + \cancel{\cos\pi\sin x} + \cancel{\sin\pi\cos x} - \cancel{\cos\pi\sin x}$

$= 0$

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5. Which of the following is **not** an identity?

a) $\sec\theta - \cos\theta = \sin\theta\tan\theta$ ✓

b) $1 - \cos^2\theta = \cos^2\theta\tan^2\theta$ ✓

c) $\csc\theta - \cos\theta\tan\theta = \frac{\cos\theta}{\tan\theta}$ ✓

d) $\cos^2\theta = \frac{1-\cos 2\theta}{2}$

a) $\frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta}$

$= \frac{1-\cos^2\theta}{\cos\theta}$

$= \frac{\sin^2\theta}{\cos\theta}$

$= \sin\theta \cdot \frac{\sin\theta}{\cos\theta}$

$= \sin\theta\tan\theta$

LHS = RHS

b) $\cos^2\theta\tan^2\theta$

$= \cancel{\cos^2\theta}\frac{\sin^2\theta}{\cancel{\cos^2\theta}}$

$= \sin^2\theta$

$= 1-\cos^2\theta$

LHS = RHS

c) $\frac{1}{\sin\theta} - \cancel{\cos\theta}\frac{\sin\theta}{\cancel{\cos\theta}}$

$= \frac{1}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta}$

$= \frac{1-\sin^2\theta}{\sin\theta}$

$= \frac{\cos^2\theta}{\sin\theta}$

$= \cos\theta \cdot \frac{\cos\theta}{\sin\theta}$

$= \cos\theta\cot\theta$

$= \frac{\cos\theta}{\tan\theta}$

LHS = RHS

d) $\frac{1-(1-2\sin^2\theta)}{2}$

$= \frac{1-1+2\sin^2\theta}{2}$

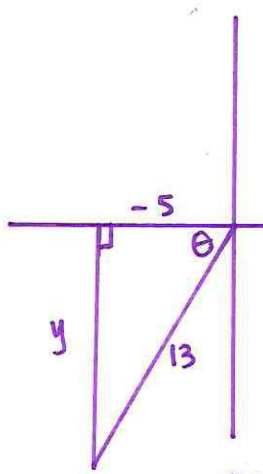
$= \frac{2\sin^2\theta}{2}$

$= \sin^2\theta$

$= 1-\cos^2\theta$

LHS ≠ RHS

6. If $\cos\theta = -\frac{5}{13}$ where $\pi \le \theta \le \frac{3\pi}{2}$, determine the exact value of $\sin\left(\theta - \frac{\pi}{2}\right)$



$(-5)^2 + y^2 = (13)^2$

$25 + y^2 = 169$

$y^2 = 144$

$y = \pm\sqrt{144}$

Q3: $y = -12$

$\therefore \cos\theta = -\frac{5}{13}$

$\sin\theta = -\frac{12}{13}$

$\sin(\theta - \pi/2) = \sin\theta\cos(\pi/2) - \cos\theta\sin(\pi/2)$

$= \left(-\frac{12}{13}\right)(0) - \left(-\frac{5}{13}\right)(1)$

$= 0 + \frac{5}{13}$

$= \frac{5}{13}$

7. What single trigonometric function is equivalent to $\sin(\overset{\alpha}{3y})\cos\left(\overset{\beta}{\frac{y}{2}}\right) - \cos(\overset{\alpha}{3y})\sin\left(\overset{\beta}{\frac{y}{2}}\right)$?

$$\sin(3y)\cos\left(\frac{y}{2}\right) - \cos(3y)\sin\left(\frac{y}{2}\right) = \sin\left(3y - \frac{y}{2}\right)$$

$$= \sin\left(\frac{6y}{2} - \frac{y}{2}\right)$$

$$= \sin(5y/2)$$

8. Consider the equation $\sin\left(x + \frac{\pi}{2}\right) = \csc x - 1$

a) Verify the equation is true for $x = \frac{\pi}{2}$.

LHS	RHS
$\sin\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$	$\csc(\pi/2) - 1$
$= \sin(\pi)$	$= 1 - 1$
$= 0$	$= 0$
LHS = RHS	

∴ yes $x = \pi/2$ is a solution

b) Is the equation an identity? Explain.

LHS	RHS
$\sin(x + \pi/2)$	$\csc x - 1$
$= \sin x \cos(\pi/2) + \cos x \sin(\pi/2)$	
$= \sin x(0) + \cos x(1)$	
$= 0 + \cos x$	
$= \cos x$	
LHS ≠ RHS	

↳ Not an identity

↳ This is an equation

9. Consider the equation $\frac{\tan x+\sec x}{\cot x}=\frac{\sin x}{1-\sin x}$.

a) State the non-permissible values on the domain $0° \leq x \leq 360°$

$$\frac{\frac{\sin x}{\cos x}+\frac{1}{\cos x}}{\frac{\cos x}{\sin x}} = \frac{\sin x}{1-\sin x}$$

$\cos x \neq 0$

$\therefore x \neq 90°, 270°$

$1-\sin x \neq 0$

$1 \neq \sin x$

$x \neq 90°$

b) Prove the equation is an identity algebraically.

LHS	RHS
$\frac{\tan x + \sec x}{\cot x}$	$\frac{\sin x}{1-\sin x}$

$$= \frac{\frac{\sin x}{\cos x}+\frac{1}{\cos x}}{\frac{\cos x}{\sin x}}$$

$$= \frac{\sin x+1}{\cos x} \cdot \frac{\sin x}{\cos x}$$

$$= \frac{(\sin x+1)(\sin x)}{\cos^2 x}$$

$$= \frac{(\sin x+1)(\sin x)}{(1-\sin^2 x)}$$

$$= \frac{\cancel{(\sin x+1)}(\sin x)}{(1-\sin x)\cancel{(1+\sin x)}}$$

$$= \frac{\sin x}{1-\sin x}$$

LHS = RHS

10. Simplify each of the following trigonometric expressions.

a) $\tan x \cos^2 x$

$= \frac{\sin x}{\cos x} \cdot \cos^2 x$

$= \sin x \cos x$

b) $\frac{1}{\sec^2 x} + \frac{1}{\csc^2 x}$

$= \cos^2 x + \sin^2 x$

$= 1$

c) $\frac{\sin 2x}{2 \sin x}$

$= \frac{2 \sin x \cos x}{2 \sin x}$

$= \cos x$

d) $\frac{\cos 2\theta - 1}{2 \sin \theta}$

$= \frac{1 - 2\sin^2\theta - 1}{2 \sin\theta}$

$= \frac{-2\sin^2\theta}{2\sin\theta}$

$= -\sin\theta$

e) $\frac{\sin^3 x}{\cos 2x - \cos^2 x}$

$= \frac{\sin^3 x}{1 - 2\sin^2 x - (1 - \sin^2 x)}$

$= \frac{\sin^3 x}{1 - 2\sin^2 x - 1 + \sin^2 x}$

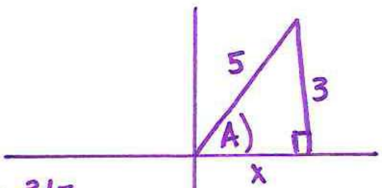
$= \frac{\sin^3 x}{-\sin^2 x}$

$= -\sin x$

11. Angle A and angle B are in Quadrant I.

If $\sin A = \frac{3}{5}$ and $\cos B = \frac{5}{13}$, determine the value of each expression.

a) $\sin(A+B)$ b) $\cos 2A$



$\sin A = 3/5$

$\cos A = 4/5$

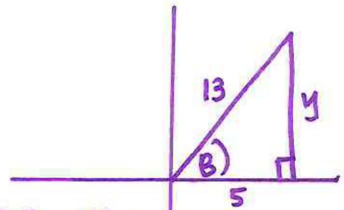
$x^2 + (3)^2 = (5)^2$

$x^2 + 9 = 25$

$x^2 = 16$

$x = \pm\sqrt{16}$

Q1 $\therefore x = 4$



$\cos B = 5/13$

$\sin B = 12/13$

$(5)^2 + y^2 = (13)^2$

$25 + y^2 = 169$

$y^2 = 144$

$y = \pm\sqrt{144}$

Q1 $\therefore y = 12$

a) $\sin(A+B)$

$= \sin A \cos B + \cos A \sin B$

$= \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$

$= \frac{15}{65} + \frac{48}{65}$

$= +\frac{63}{65}$

b) $\cos 2A$

$= 2\cos^2 A - 1$

$= 2\left(\frac{4}{5}\right)^2 - 1$

$= 2\left(\frac{16}{25}\right) - 1$

$= \cancel{\frac{50}{169} - 1}$

$= \cancel{\frac{-119}{169}}$

$\frac{32}{25} - \frac{25}{25}$

$\frac{7}{25}$

12. Determine the exact value of each of the following:

a) $\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right)$

$= \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$= \sin(\pi/4)\cos(\pi/6) + \cos(\pi/4)\sin(\pi/6)$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

b) $\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$

$= \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= \cos(\pi/3)\cos(\pi/4) + \sin(\pi/3)\sin(\pi/4)$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4}$

c) $\tan\left(\frac{7\pi}{12}\right) = \tan\left(\frac{4\pi}{12} + \frac{3\pi}{12}\right)$

$= \tan(\pi/3 + \pi/4)$

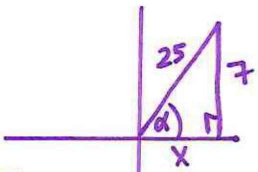
$$= \frac{\tan(\pi/3) + \tan(\pi/4)}{1 - \tan(\pi/3)\tan(\pi/4)}$$

$$= \frac{\frac{\sqrt{3}}{1} + 1}{1 - \left(\frac{\sqrt{3}}{1}\right)(1)}$$

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}}$$

13. Given that $\sin\alpha = \frac{7}{25}$ and $\cos\beta = \frac{9}{41}$ and ~~[illegible]~~ are in Quadrant I, find:

a) $\sin(\alpha+\beta)$ b) $\cos(\alpha+\beta)$ c) $\sec(\alpha+\beta)$



$\sin\alpha = 7/25$

$\cos\alpha = 24/25$

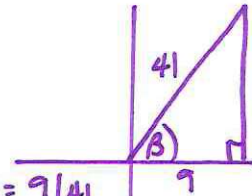
$x^2 + (7)^2 = (25)^2$

$x^2 + 49 = 625$

$x^2 = 576$

$x = \pm\sqrt{576}$

Q1 $\therefore x = 24$



$\cos\beta = 9/41$

$\sin\beta = 40/41$

$(9)^2 + y^2 = (41)^2$

$81 + y^2 = 1681$

$y^2 = 1600$

$y = \pm\sqrt{1600}$

Q1 $\therefore y = 40$

a) $\sin(\alpha + B)$

$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \left(\frac{7}{25}\right)\left(\frac{9}{41}\right) + \left(\frac{24}{25}\right)\left(\frac{40}{41}\right)$

$= \frac{63}{1025} + \frac{960}{1025}$

$= \frac{1023}{1025}$

b) $\cos(\alpha + B)$

$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \left(\frac{24}{25}\right)\left(\frac{9}{41}\right) - \left(\frac{7}{25}\right)\left(\frac{40}{41}\right)$

$= \frac{216}{1025} - \frac{280}{1025}$

$= \frac{-64}{1025}$

c) $\sec(\alpha + B)$

$= \frac{1}{\cos(\alpha + B)}$

$= \frac{1}{\frac{-64}{1025}}$

$= \frac{-1025}{64}$

14. Prove each of the following identities.

a) $\frac{\sin x}{1+\cos} = \frac{1-\cos}{\sin x}$

LHS	RHS
$\frac{\sin x}{1+\cos x}$	$\frac{1-\cos x}{\sin x}$
$= \frac{\sin x}{(1+\cos x)} \cdot \frac{(1-\cos x)}{(1-\cos x)}$	
$= \frac{\sin x(1-\cos x)}{1-\cos^2 x}$	
$= \frac{\sin x(1-\cos x)}{\sin^2 x}$	
$= \frac{1-\cos x}{\sin x}$	
LHS = RHS	

b) $\frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = 1$

LHS	RHS
$\frac{\sin x}{\csc x} + \frac{\cos x}{\sec x}$	1
$= \frac{\sin x}{\frac{1}{\sin x}} + \frac{\cos x}{\frac{1}{\cos x}}$	
$= \sin^2 x + \cos^2 x$	
$= 1$	
LHS = RHS	

c) $\frac{1}{\csc x - \sin x} = \tan x \sec x$

LHS	RHS
$\frac{1}{\csc x - \sin x}$	$\tan x \sec x$
$= \frac{1}{\frac{1}{\sin x} - \sin x}$	
$= \frac{1}{\frac{1-\sin^2 x}{\sin x}}$	
$= \frac{\sin x}{1-\sin^2 x}$	
$= \frac{\sin x}{\cos^2 x}$	
$= \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$	
$= \tan x \sec x$	
LHS = RHS	

15. Determine the exact value for each of the following:

a) $\sin\frac{5\pi}{16}\cos\frac{\pi}{16} - \cos\frac{5\pi}{16}\sin\frac{\pi}{16}$ $= \sin\left(\frac{5\pi}{16} - \frac{\pi}{16}\right)$

$= \sin(4\pi/16)$

$= \sin(\pi/4)$

$= \frac{\sqrt{2}}{2}$

b) $\cos 33° \cos 27° - \sin 33° \sin 27°$ $= \cos(33° + 27°)$

$= \cos(60°)$

$= 1/2$

16. Prove this identity $\frac{\csc^4 x - \cot^4 x}{\csc^2 x + \cot^2 x} + \cot^2 x = \csc^2 x$

LHS	RHS
$\frac{\csc^4 x - \cot^4 x}{\csc^2 x + \cot^2 x} + \cot^2 x$	$\csc^2 x$
$= \frac{\cancel{(\csc^2 x + \cot^2 x)}(\csc^2 x - \cot^2 x)}{\cancel{(\csc^2 x + \cot^2 x)}} + \cot^2 x$	
$= \csc^2 x - \cot^2 x + \cot^2 x$	
$= \csc^2 x$	
LHS = RHS	