Date: **Answer key**

OUTCOME T5 – Review

1. **Solve** the following trigonometric equations over the given interval.

a) $2\cos\theta = 2$ where $0° \le \theta \le 360°$

$$\cos\theta = 1$$

$$\theta = 0°$$

$$\theta = 360°$$

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b) $4\tan\theta - 7 = 5\tan\theta - 6$ where $-2\pi \le \theta \le 2\pi$

$$-1 = \tan\theta$$

$$\theta = -\frac{\pi}{4} \qquad \theta = \frac{3\pi}{4}$$

$$\theta = -\frac{5\pi}{4} \qquad \theta = \frac{7\pi}{4}$$

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c) $4\sin^2 x - 1 = 0$ where $-4\pi \le x \le \pi$

$-\frac{21\pi}{6} \qquad \frac{6\pi}{6}$

$$\sin^2 x = \frac{1}{4}$$

$$\sin x = \pm\frac{1}{2}$$

$X =$

$$\sin x = -\frac{1}{2} \qquad \Big| \qquad \sin x = \frac{1}{2}$$

$$X = -\frac{\pi}{6}, -\frac{5\pi}{6} \qquad \Big| \qquad X = \frac{\pi}{6}, \frac{5\pi}{6}$$
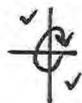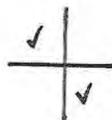
$$-\frac{13\pi}{6}, -\frac{17\pi}{6} \qquad \Big| \qquad -\frac{7\pi}{6}, -\frac{11\pi}{6},$$

$$-\frac{19\pi}{6}, -\frac{23\pi}{6}$$

2. **Solve** the following trigonometric equations over the given interval.

a) $2\cos^2\theta + \cos\theta = 0$ where $[0, 2\pi]$

$\cos\theta(2\cos\theta + 1) = 0$

$\cos\theta = 0$

$\theta = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

$2\cos\theta + 1 = 0$

$\cos\theta = -\dfrac{1}{2}$

$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

$\therefore \theta = \dfrac{\pi}{2}$

$\theta = \dfrac{3\pi}{2}$

$\theta = \dfrac{2\pi}{3}$

$\theta = \dfrac{4\pi}{3}$

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b) $2\sin^2\theta - 5\sin - 3 = 0$ where $[-3\pi, 3\pi]$

$(2\sin\theta + 1)(\sin\theta - 3) = 0$

$2\sin\theta + 1 = 0$

$\sin\theta = -\dfrac{1}{2}$

$\theta = -\dfrac{\pi}{6}, -\dfrac{5\pi}{6}, -\dfrac{13\pi}{6},$

$-\dfrac{17\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

$\sin\theta - 3 = 0$

$\sin\theta = 3$

no solution
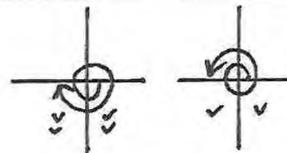
$\therefore \theta = -\dfrac{\pi}{6}, -\dfrac{5\pi}{6}, -\dfrac{13\pi}{6}$

$-\dfrac{17\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

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c) $\sec\theta + 10 = 2 - 4\sec\theta$ where $\theta \in \mathbb{R}$ (in radians)

$\dfrac{5\sec\theta}{5} = \dfrac{-8}{5}$

$\sec\theta = -\dfrac{8}{5}$

$\cos\theta = -\dfrac{5}{8}$

$\theta_R = \cos^{-1}\left(\dfrac{5}{8}\right)$

$\theta_R = 0.896$

$\dfrac{Q2}{}$

$\theta = \pi - \theta_R$

$\theta = 2.246$

$\dfrac{Q3}{}$

$\theta = \pi + \theta_R$

$\theta = 4.038$

$\therefore \theta = 2.246 + 2\pi k, \quad k \in \mathbb{Z}$

$\theta = 4.038 + 2\pi k, \quad k \in \mathbb{Z}$

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3. **Solve** the following trigonometric equations over the given interval.

a) $\tan^2 x - 3\tan x = 0$ where the domain is all real numbers (in degrees).

$\tan x (\tan x - 3) = 0$

$\tan x = 0$

$x = 0°, 180°,$
$360°$

$\tan x - 3 = 0$

$\tan x = 3$

$x_R = \tan^{-1}(3)$

$x_R = 71.565°$

$\underline{QI}$
$\theta = \theta_R$
$\theta = 71.565°$

$\underline{Q3}$
$\theta = 180 + \theta_R$
$\theta = 251.565°$

$\theta = 0° + 360°k, k \in \mathbb{Z}$

$\theta = 180° + 360°k, k \in \mathbb{Z}$

$\theta = 71.565° + 360°k, k$

$\theta = 251.565° + 360°k, k$

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b) $\csc^2\theta + 4\csc\theta - 12 = 0$ where $[0, 2\pi]$

$(\csc\theta + 6)(\csc\theta - 2) = 0$

$\csc\theta + 6 = 0$
$\csc\theta = -6$
$\sin\theta = -\frac{1}{6}$

$\theta_R = \sin^{-1}(1/6)$

$\theta_R = 0.167$

$\underline{Q3}$
$\cdot = \pi + \theta_R$
$? = 3.309$

$\underline{Q4}$
$\theta = 2\pi - \theta_R$
$\theta = 6.116$

$\csc\theta - 2 = 0$
$\csc\theta = 2$
$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}$

$\theta = \frac{5\pi}{6}$

$\therefore \theta = 3.309$
$\theta = 6.116$
$\theta = \frac{\pi}{6}$
$\theta = \frac{5\pi}{6}$

c) $3\sin^2\theta + \sin\theta - 1 = 0$ where $0° \leq \theta \leq 360°$

$$\sin\theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sin\theta = \frac{-(1) \pm \sqrt{(1)^2 - 4(3)(-1)}}{2(3)}$$

$$\sin\theta = \frac{-1 \pm \sqrt{13}}{6}$$

$$\sin\theta = \frac{-1 + \sqrt{13}}{6} \qquad\qquad \sin\theta = \frac{-1 - \sqrt{13}}{6}$$

$$\sin\theta = 0.434 \qquad\qquad \sin\theta = -0.768$$

$\theta_R = \sin^{-1}(0.434)$ $\qquad\qquad$ $\theta_R = \sin^{-1}(0.768)$

$\theta_R = 25.722°$ $\qquad\qquad$ $\theta_R = 50.175°$

Q1	Q2	Q3	Q4
$\theta = \theta_R$	$\theta = 180° - \theta_R$	$\theta = 180° + \theta_R$	$\theta = 360° - \theta_R$
$\theta = 25.722°$	$\theta = 154.278°$	$\theta = 129.825°$	$\theta = 309.825°$

$\therefore \theta = 25.722°$

$\theta = 154.278°$

$\theta =$ ' $\qquad$ 230.175°

$\theta = 309.825°$