June 2015

Question 32   a) 2 marks   b) 1 mark

Given the functions $f(x) = x + 2$ and $g(x) = \dfrac{1}{x-5}$:

a) Determine the equation of the composite function $f(g(x))$ and its domain.

$$f(g(x)) = \dfrac{\dfrac{1}{x-5} + 2}{}$$

$$f(x) = x + 2$$

$$f(g(x)) = g(x) + 2$$

$$= \dfrac{1}{x-5} + 2$$

domain: $x \neq 5 \, ; \, x \in \mathbb{R}$

b) Determine the x-intercept and y-intercept of $f(g(x))$.

x-intercept: ~~305~~ $9/2$

y-intercept: 1.8

$\underline{\text{x-int}:}$ $\quad 0 = \dfrac{1}{x-5} + 2$

$(y=0)$ $\quad -2 = \dfrac{1}{x-5}$

$x - 5 = \dfrac{1}{-2}$

$x = +5 - \dfrac{1}{2}$

$x = \cancel{-5.5}$ $\quad 9/2$

y-int: $(x = 0)$

$y = \dfrac{1}{-5} + 2$

$= -0.2 + 2$

$= 1.8$

## Question 11   2 marks

The domain of $f(x)$ is $x \le 2$. The domain of $g(x)$ is $x \ge -7$.

State the domain of $f(x) + g(x)$.

Justify your answer.   ① $[-7, 2]$ is defined in **both** f and g.

Question 14   2 marks

Given the graphs of $f(x)$ and $g(x)$, sketch the graph of $g(x) - f(x)$.

$f(x)$

$g(x)$
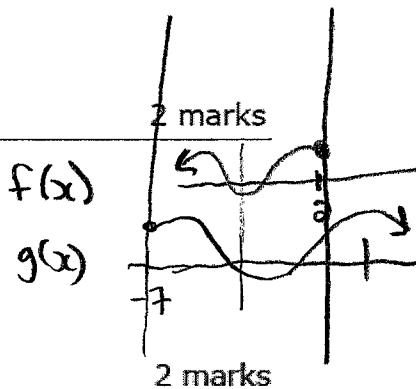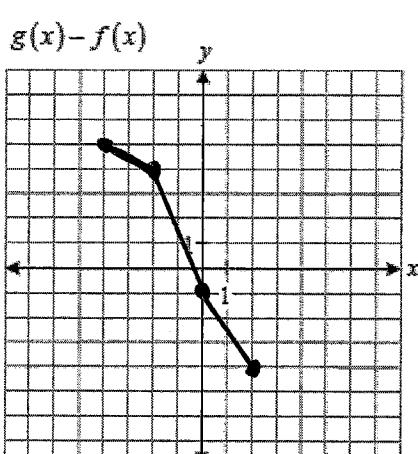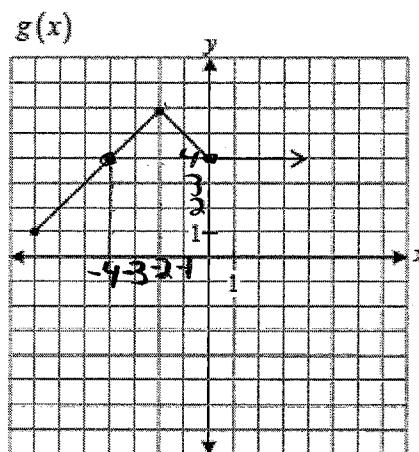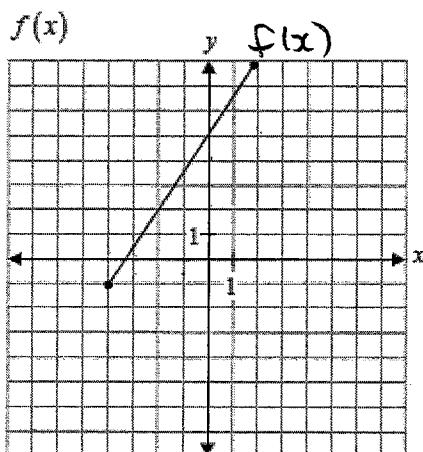
$g(x) - f(x)$

x	$g(x) - f(x)$
-4	$4 - (-1) = 5$
-2	$6 - 2 = 4$
0	$4 - 5 = -1$
2	$4 - 8 = -4$

Question 42   a) 2 marks   b) 1 mark

a) Given the functions $f(x) = \sqrt{4 + x}$ and $g(x) = |3x - 6|$, evaluate $f(g(-5))$.

b) Is it possible to evaluate $g(f(-5))$?

Justify your answer.

$f(-5) = \sqrt{4 + (-5)}$
$= \sqrt{-1}$ undefined.

↳ **No.** -5 is **not** in domain of f.

$g(-5) = |3(-5) - 6|$
$= |-15 - 6|$
$= |-21|$
$= 21$

$f(21) = \sqrt{4 + 21}$
$= \sqrt{25}$
$= 5$

$\therefore f(g(-5)) = 5$

Question 6   1 mark

Given the following two functions, $f(x) = \sqrt{x-1}$ and $g(x) = x^2 + 1$, evaluate $\boxed{g(f(3)) = 3}$

$f(3) = \sqrt{3-1} = \sqrt{2}$

$g(\sqrt{2}) = (\sqrt{2})^2 + 1 = 2 + 1 = 3$

Question 37   a) 1 mark   b) 1 mark

Given $f(x) = x^2 - 2x - 3$ and $g(x) = x + 1$:

a) Write the equation of $y = f(g(x))$.

$$f(g(x)) = (g(x))^2 - 2(g(x)) - 3$$

$$= (x+1)^2 - 2(x+1) - 3$$

$$= x^2 + 2x + 1 - 2x - 2 - 3$$

$$= x^2 - 4$$

b) Sketch the graph of $y = f(g(x))$.
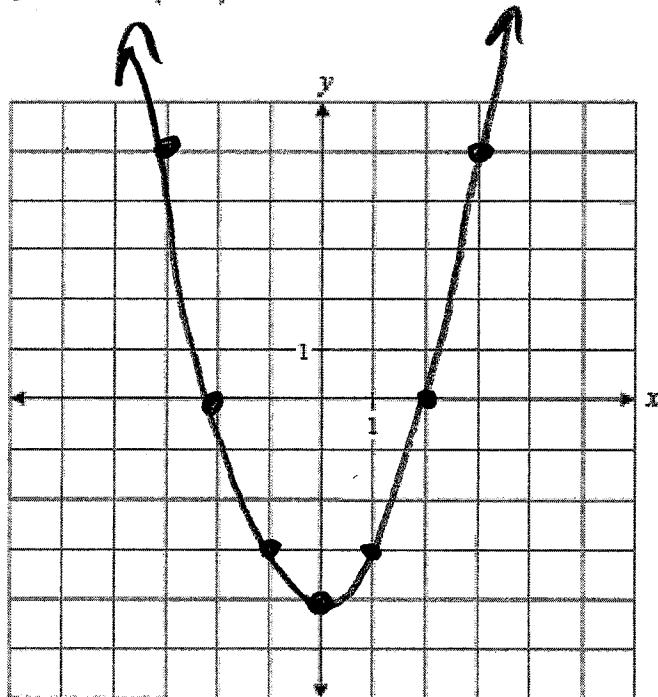
$y = x^2 - 4$

$y = (x-2)(x+2)$

x-int: ± 2

y-int -4

3

Given the graphs of $f(x)$ and $g(x)$, sketch the graph of $(f \cdot g)(x)$.

$f(x)$

$g(x)$

$y = 2$

$(f \cdot g)(x)$

x	$f(x) \cdot g(x)$
-3	$0(2) = 0$
0	$3(2) = 6$
1	$4(2) = 8$

No arrow head!
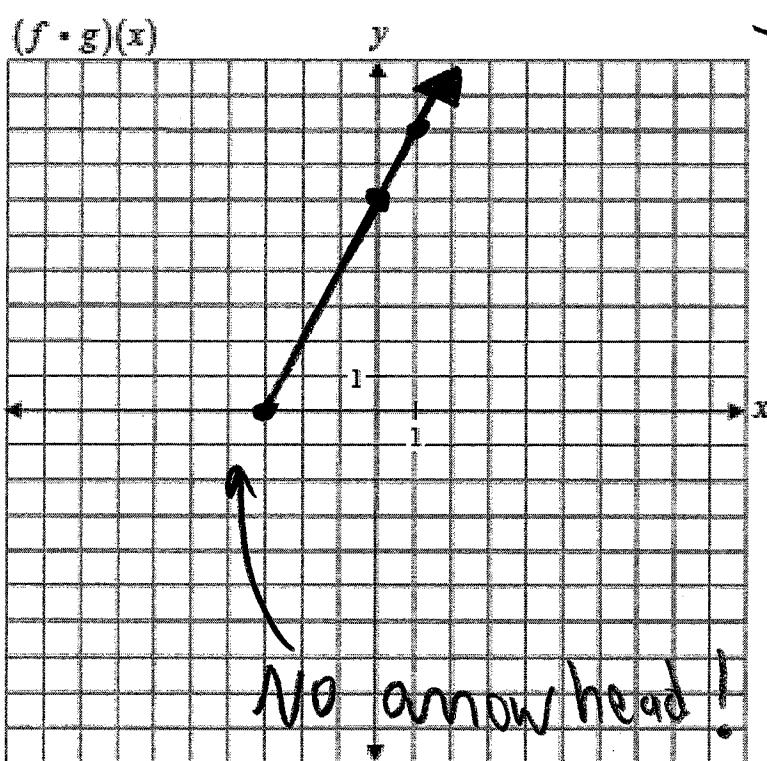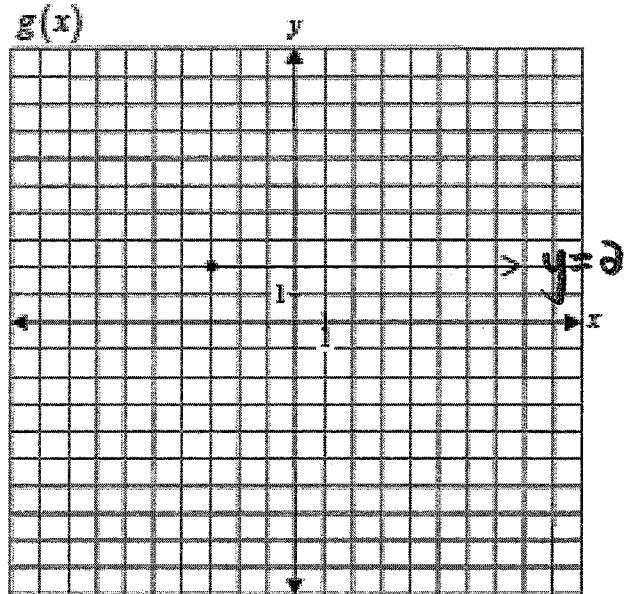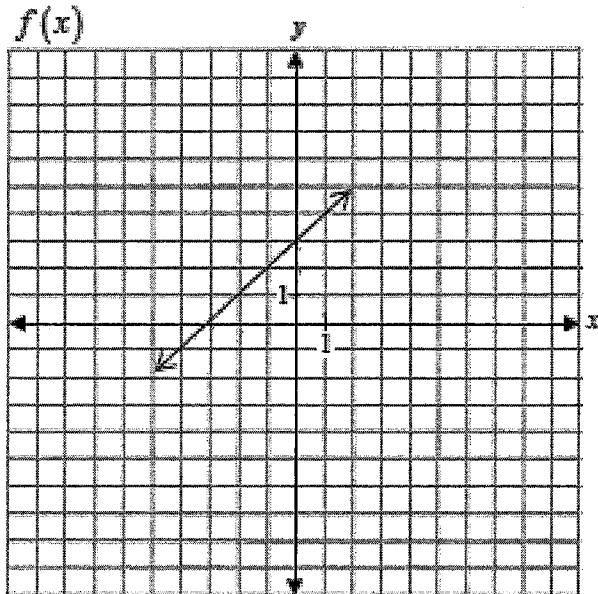
Question 10   2 marks

Given the graphs of $f(x)$ and $(f-g)(x)$, sketch the graph of $g(x)$.

$f(x) - g(x)$

$f(x)$
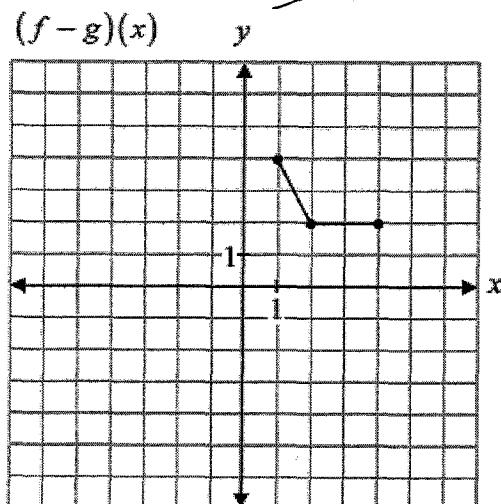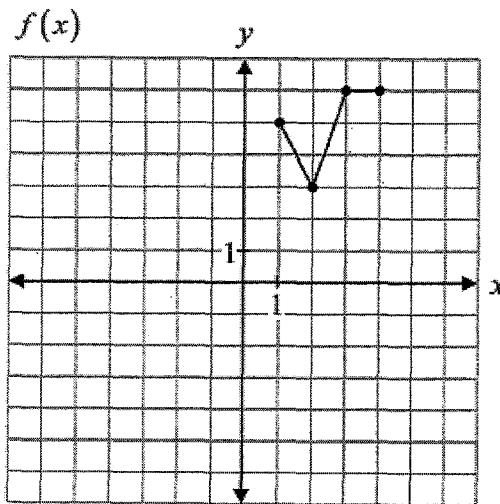
$(f-g)(x)$

Note:

$(f-g)x = f(x) - g(x)$

$\therefore g(x) = f(x) - (f-g)(x)$

$\left(\begin{array}{c} \text{order} \\ \text{matters} \end{array}\right)$

$g(x)$
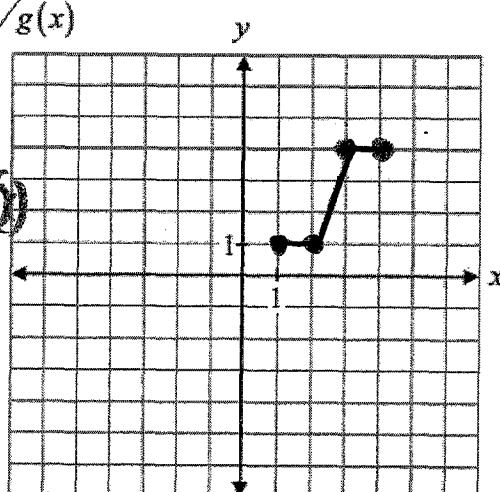
x	$f(x) - (f-g)(x)$
1	$5 - 4 = 1$
2	$3 - 2 = 1$
3	$6 - 2 = 4$
4	$6 - 2 = 4$

Question 15   1 mark

If $f(x) = x^3$ and $g(x) = 2x - 3$, what is the value of $\left(\dfrac{f}{g}\right)(-1)$? $= \dfrac{-1}{-5} = \dfrac{1}{5}$

$f(-1) = -1 \quad g(-1) = -5$

Given $f(x) = x^2 - 1$ and $g(x) = \sqrt{x+1}$, sketch the graph of $y = f(g(x))$ and state its domain.
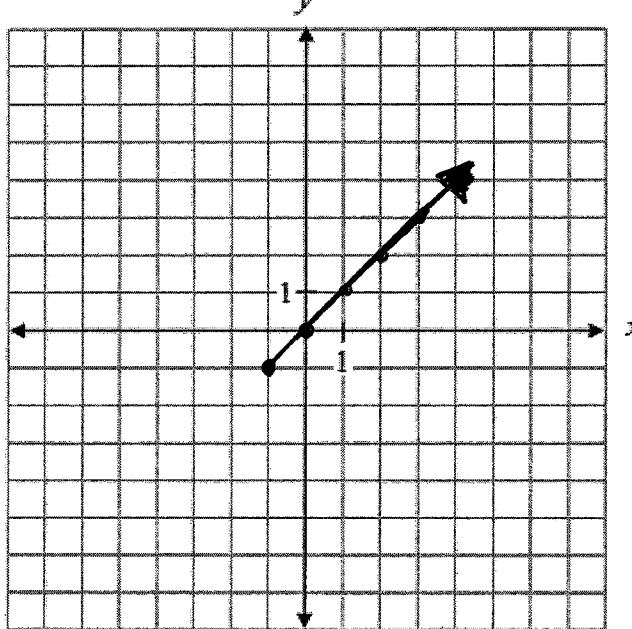
$f(x) = x^2 - 1$

$f(g(x)) = \left(\sqrt{x+1}\right)^2 - 1$

$= x + 1 - 1 \; ; \; x \geq -1$

$= x \; ; \; x \geq -1$ ↑

Need this

Careful!

Domain: $x \geq -1 \; ; \; x \in \mathbb{R}$

The x-intercept of $f(x)$ is 4 and the x-intercept of $g(x)$ is 4.   → $f(4) = 0$

Benjamin concludes that the x-intercept of $f(x) + g(x)$ is 8. $g(4) = 0$

Do you agree with Benjamin? Justify your answer.

No! $f(x) + g(x)$ means you add the y-values together

So $f(4) + g(4) = 0 + 0 = 0$ There will be an x-intercept at 4.

June 2013

Question 9   1 mark

Given that $f(x) = \{(1, 3), (2, 5), (3, 4), (4, 2)\}$, find $f(f(3))$. $= 2$

$f(3) = 4$

$f(4) = 2$

Given the graphs of $f(x)$ and $g(x)$ below,

$f(x)$

$g(x)$
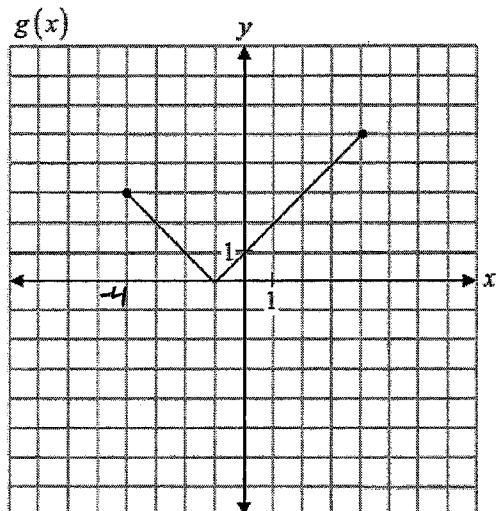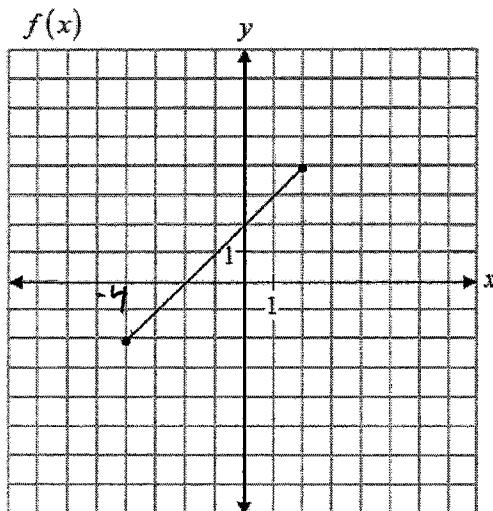
sketch the graph of $y = f(x) - g(x)$.
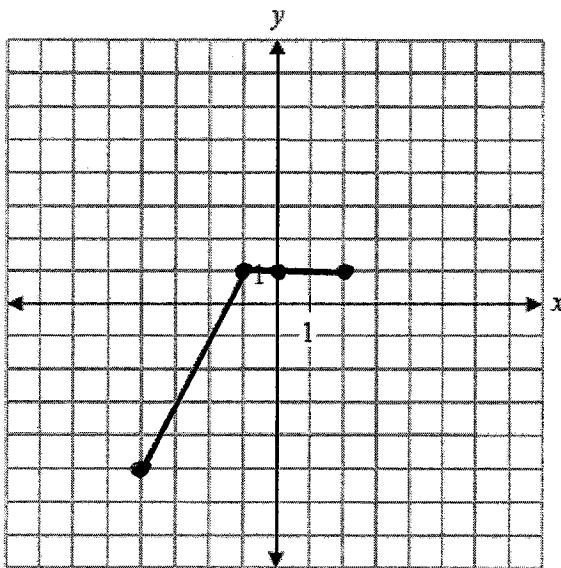
x	$f(x) - g(x)$
-4	$-2 - (3) = -5$
-1	$1 - 0 = 1$
0	$2 - 1 = 1$
2	$4 - 3 = 1$

Given $f(x) = \sqrt{x - 2}$ and $g(x) = 3x$, write the equation for $h(x) = f(g(x))$.

What are the restrictions on the domain of $h(x)$?
Explain your reasoning.

$$3x - 2 \geq 0$$
$$3x \geq 2$$
$$x \geq 2/3$$

$$h(x) = f(3x)$$
$$= \sqrt{3x - 2}$$

You can not square root a $(-)$.

Given $f(x) = x - 1$ and $g(x) = x^2$, write the equation of $y = f\big(g(x)\big)$ and sketch the graph.
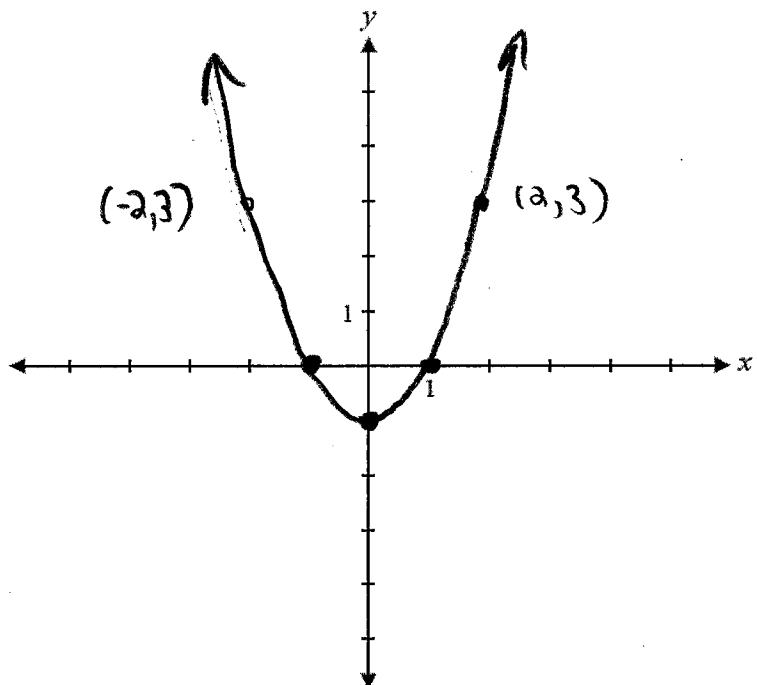
$(-2,3)$ $(2,3)$

$f(x) = x - 1$

$f(g(x)) = g(x) - 1$

$= x^2 - 1$

$= (x-1)(x+1)$

x-int ± 1

y-int -1

Question 15

Given the following graphs:

$f(x)$

$g(x)$
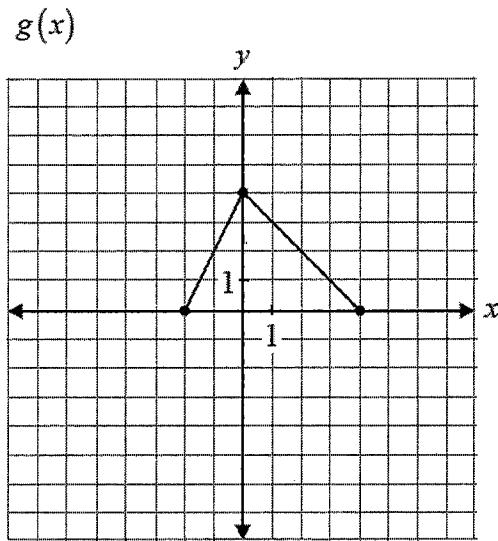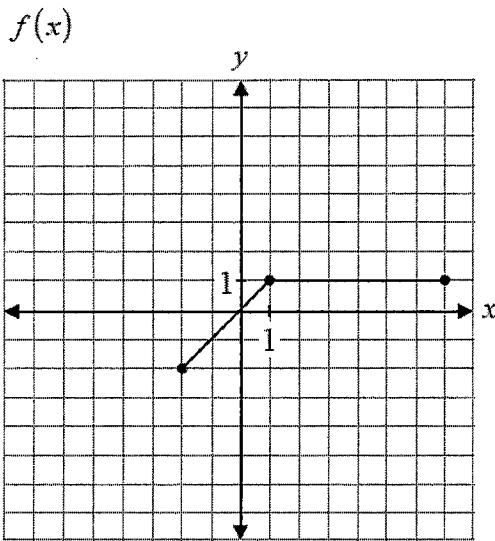
Sketch the graph of $f(x) + g(x)$.
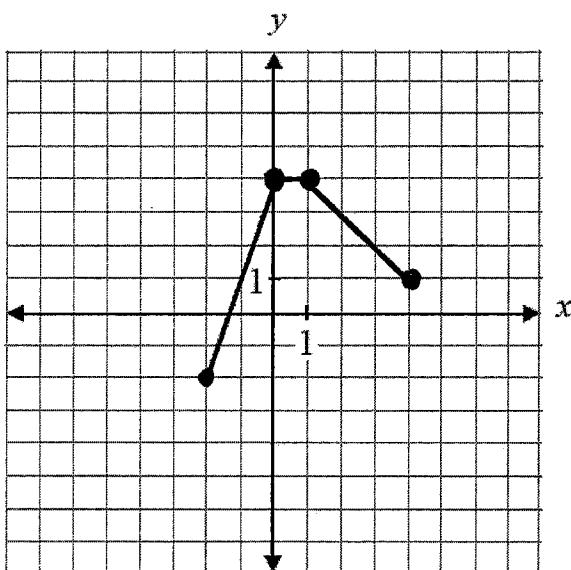
x	$f(x) + g(x)$
-2	$-2 + 0 = -2$
0	$0 + 4 = 4$
1	$1 + 3 = 4$
4	$1 + 0 = 1$

Given the following graphs:

$f(x)$

$g(x)$
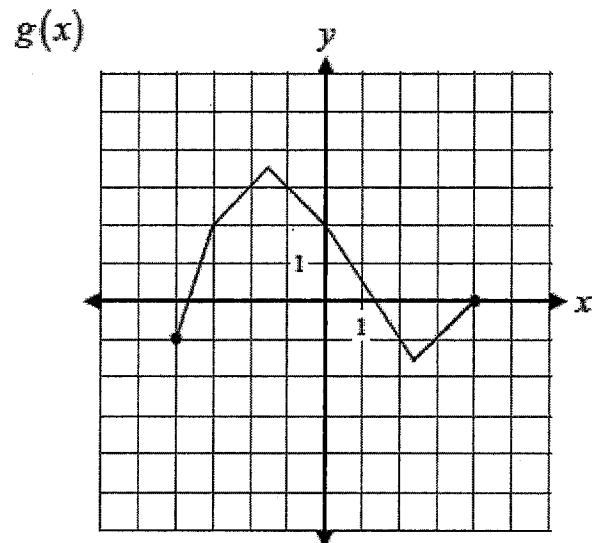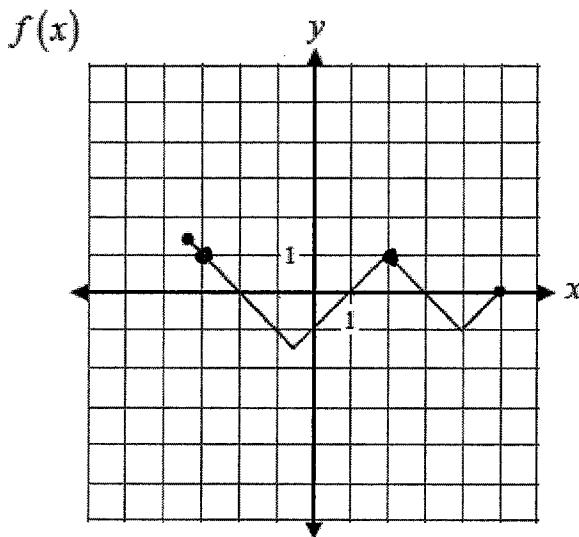
a) Determine the value of $[f \cdot g](0)$.

$$f(0) \cdot g(0)$$

not open circle!

$$= (-1)(2)$$

$$= -2$$

b) Determine the value of $g(f(4))$.

$$f(4) = -1$$

$$g(-1) = 3$$

$$\therefore g(f(4)) = 3$$

c) Determine a value for k where $f(k) = 1$.

$$k = -3 \quad \underline{\underline{\text{or}}} \quad k = 2$$