 Date: Answer key

OUTCOME R11 – Review

1. Use the Remainder Theorem to determine the **remainder** resulting from each division.

a) $(3x^3 - 4x^2 + 6x - 9) \div (x + 1)$

$P(-1) = 3(-1)^3 - 4(-1)^2 + 6(-1) - 9$

$P(-1) = -3 - 4 - 6 - 9$

$P(-1) = -22$

The remainder is -22

b) $(3x^2 - 8x + 4) \div (x - 2)$

$P(2) = 3(2)^2 - 8(2) + 4$

$P(2) = 12 - 16 + 4$

$P(2) = 0$

The remainder is 0

c) $(6x^3 - 5x^2 - 7x + 9) \div (x + 5)$

$P(-5) = 6(-5)^3 - 5(-5)^2 - 7(-5) + 9$

$P(-5) = -750 - 125 + 35 + 9$

$P(-5) = -831$

The remainder is -831

2. **Divide** each of the following

a) $(x^4 - 3x^3 + 2x^2 + 55x - 11)$ by $(x + 3)$

-3	1	-3	2	55	-11
+	↓	-3	18	-60	15
×	1	-6	20	-5	4

$$\frac{x^4 - 3x^3 + 2x^2 + 55x - 11}{x+3} = x^3 - 6x^2 + 20x - 5 + \frac{4}{x+3}$$

b) $\frac{2x^3 - 10x^2 - 15x - 20}{x+5}$

-5	2	-10	-15	-20
+	↓	-10	100	-425
×	2	-20	85	-445

$$\frac{2x^3 - 10x^2 - 15x - 20}{x+5} = 2x^2 - 20x + 85 - \frac{445}{x+5}$$

c) $(7x^5 + 5x^4 + 23x^2 + 8)$ by $(x + 2)$

-2	7	5	0	23	0	8
+	↓	-14	18	-36	26	-52
x	7	-9	18	-13	26	44

$$\frac{7x^5+5x^4+23x^2+8}{x+2} = 7x^4-9x^3+18x^2-13x+26-\frac{44}{x+2}$$

d) $(8x^3 - 1) \div (x - 2)$

2	8	0	0	-1
+	↓	16	32	64
x	8	16	32	63

$$\frac{8x^3-1}{x-2} = 8x^2+16x+32+\frac{63}{x-2}$$

3. For $2x^3 + 5x^2 - kx + 9 \div (x + 3)$, determine the value of k if the remainder if 6.

$P(-3) = 0$

$2(-3)^3 + 5(-3)^2 - k(-3) + 9 = 0$

$2(-27) + 5(9) + 3k + 9 = 0$

$-54 + 45 + 3k + 9 = 0$

$\frac{3k}{3} = \frac{0}{3}$

$k = 0.$

4. When the polynomial $x^3 - x^2 + ax + b$ is divided by $x - 1$ the remainder is 0, but when divided by $x + 2$ the remainder is -18.

Determine the value of a and b under these circumstances.

$P(1) = 0$

$\therefore (1)^3 - (1)^2 + a(1) + b = 0$

$1 - 1 + a + b = 0$

$a + b = 0$

$b = -a$

$\therefore x^3 - x^2 + ax + b = x^3 - x^2 + ax - a$

$P(-2) = -18$

$\therefore (-2)^3 - (-2)^2 + a(-2) - a = -18$

$-8 - 4 - 2a - a = -18$

$\frac{-3a}{-3} = \frac{-6}{-3}$

$a = 2$

$b = -a$

$b = -(2)$

$\therefore a = 2,\ b = -2$

5. **Explain** how you know $(x + 2)$ will not be a factor of the polynomial $P(x) = -3x^3 + 2x^2 + 10x + 5$

↳ Intergal zero possibilities are $\pm 1, \pm 5$

-2 is not a factor of the constant term, so $(x+2)$ will not be a factor of $P(x)$.

6. State the possible **integral zeros** of each polynomial.

a) $P(n) = n^3 - 2n^2 - 5n + 12$ $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

b) $P(p) = p^4 + 4p^3 + 3p^3 + 8p - 25$ $\pm 1, \pm 5, \pm 25$

c) $y = x^4 - 11x^3 - 2x^2 + 10$ $\pm 1, \pm 2, \pm 5, \pm 10$

7. State whether each polynomial has $(x+2)$ as a **factor.**

a) $5x^2+6x-8$

$P(-2) = (5)(-2)^2 + 6(-2) - 8$

$= 20 - 12 - 8$

$= 0$

Since $P(-2) = 0$, yes $(x+2)$ is a factor

b) $2x^4 - 3x^3 - 5x^2$

$P(-2) = 2(-2)^4 - 3(-2)^3 - 5(-2)^2$

$= 32 + 24 - 20$

$= 36$

Since $P(-2) \neq 0$, $(x+2)$ is not a factor

c) $3x^3 + x^2 - 12x - 4$

$P(-2) = 3(-2)^3 + (-2)^2 - 12(-2) - 4$

$= -24 + 4 + 24 - 4$

$= 0$

Since $P(-2) = 0$, yes $(x+2)$ is a factor

9. Determine the value(s) of k so that the binomial $(x+k)$ is a factor of the polynomial $x^2 - 8x - 20$

$P(-k) = 0$

$(-k)^2 - 8(-k) - 20 = 0$

$k^2 + 8k - 20 = 0$

$(k+10)(k-2) = 0$

$k = -10, \; k = 2$

8. **Factor** completely.

a) $x^3 + x^2 - 16x - 16$

-1	1	1	-16	-16
$+$	$\downarrow$	-1	0	16
$\times$	1	0	-16	0

$\therefore x^3+x^2-16x-16 = (x+1)(x^2-16)$

$= (x+1)(x-4)(x+4)$

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b) $x^3 - 2x^2 - 6x - 8$

4	1	-2	-6	-8
$+$	$\downarrow$	4	8	8
$\times$	1	2	2	0

$\therefore x^3-2x^2-6x-8 = (x-4)(x^2+2x+2)$

c) $2x^3 + 5x^2 - 7$

1	2	5	0	-7
+	↓	2	7	7
x	2	7	7	0

$\therefore 2x^3+5x^2-7 = (x-1)(2x^2+7x+7)$

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d) $x^4 + 4x^3 - 7x^2 - 34x - 24$

-1	1	4	-7	-34	-24
+	↓	-1	-3	10	24
x	1	3	-10	-24	0

-2	1	3	-10	-24
+	↓	-2	-2	24
x	1	1	-12	0

$\therefore x^4+4x^3-7x^2-34x-24 = (x+1)(x^3+3x^2-10x-24)$

$= (x+1)(x+2)(x^2+x-12)$

$= (x+1)(x+2)(x+4)(x-3)$

Date: ______________

10. **Solve** each of the following polynomial equations.

a) $x^3 + 9x^2 + 23x + 15 = 0$

$$\begin{array}{c|cccc} -1 & 1 & 9 & 23 & 15 \\ + & \downarrow & -1 & -8 & -15 \\ \hline \times & 1 & 8 & 15 & 0 \end{array}$$

$$x^3+9x^2+23x+15 = (x+1)(x^2+8x+15)$$
$$= (x+1)(x+5)(x+3)$$

$$\therefore (x+1)(x+5)(x+3) = 0$$

$x=-1,\ x=-5,\ x=-3$

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b) $10x^3 - 21x^2 - x + 6 = 0$

$$\begin{array}{c|cccc} 2 & 10 & -21 & -1 & 6 \\ + & \downarrow & 20 & -2 & -6 \\ \hline \times & 10 & -1 & -3 & 0 \end{array}$$

$$10x^3-21x^2-x+6 = (x-2)(10x^2-x-3)$$
$$= (x-2)(2x+1)(5x-3)$$

$$\therefore (x-2)(2x+1)(5x-3) = 0$$

$x = 2,\ x = -1/2,\ x = 3/5$

MPC40S Date: Answer key

OUTCOME R12 – Review

1. For each polynomial function, state the **degree**, the **leading coefficient**, and the **y - intercept** of each polynomial function. Also, describe the **end behaviour** of the graph.

a) $y = -x^3 + 2x + 3$

Degree 3 Leading Coefficient -1 y – intercept 3

End Behaviour Extends up in Q2 and down in Q4

b) $y = 5 + 9x^4$

Degree 4 Leading Coefficient +9 y – intercept 5

End Behaviour Extends up in Q2 and up in Q1

c) $y = 3x^4 + 3x^2 - 2x$

Degree 4 Leading Coefficient +3 y – intercept 0

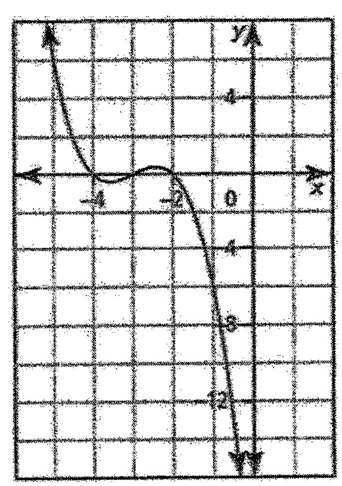
End Behaviour Extends up in Q2 and up in Q1

d) $y = -2(x+1)^2(x-2)(x-3)^2$

Degree 5 Leading Coefficient -2 y – intercept 36

End Behaviour Extends down in Q4 and up in Q2

2. For the graph of the following polynomial function, determine the following:



a) The least possible **degree**.

3

b) The sign of the **leading coefficient**.

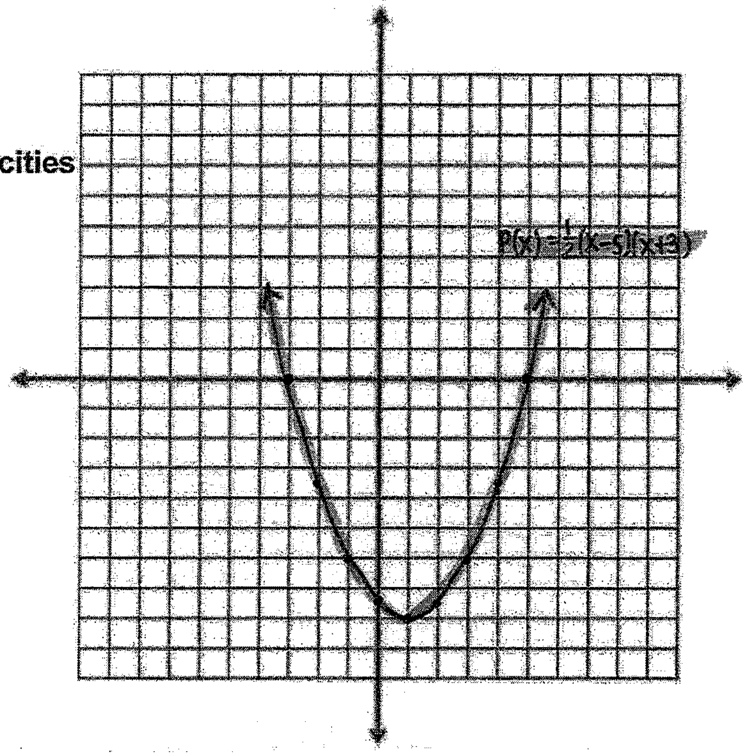
Negative

c) The **zeros** and their **multiplicities**.

X = -4 , multiplicity of 1
X = -3 , multiplicity of 1
X = -2 , multiplicity of 1

3. For each polynomial function provided, **sketch** a graph of the function and answer the following questions.

a) $P(x) = \frac{1}{2}(x-5)(x+3)$



i) Determine the **zeros** and their **multiplicities**

X = 5 , M1

X = -3 , M1

ii) Determine the **y – intercept**

$P(0) = \frac{1}{2}(0-5)(0+3)$

$P(0) = \frac{1}{2}(-15)$

$P(0) = -\frac{15}{2}$ @ $(0, -15/2)$

iii) Describe the **end behaviour**

up in Q2 , up in Q1

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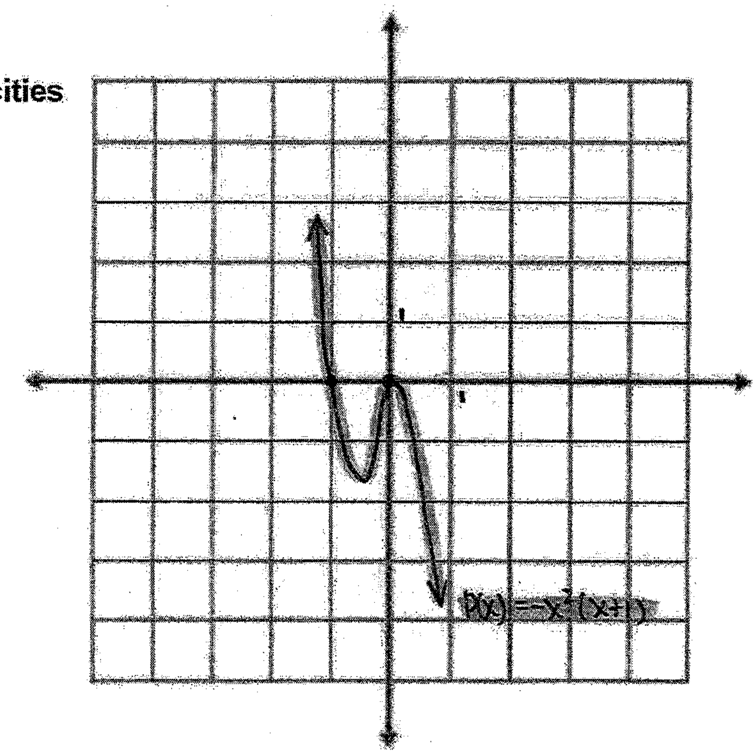
b) $P(x) = -x^2(x+1)$



i) Determine the **zeros** and their **multiplicities**

X = 0 , M2

X = -1 , M1

ii) Determine the **y – intercept**

$P(0) = -(0)^2(0+1)$

$P(0) = -(0)(1)$

$P(0) = 0$ @ $(0, 0)$

iii) Describe the **end behaviour**

up in Q2 , up in Q4

c) $P(x) = (x-1)^2(x+2)^2$

i) Determine the **zeros** and their **multiplicities**

X=1 , M2

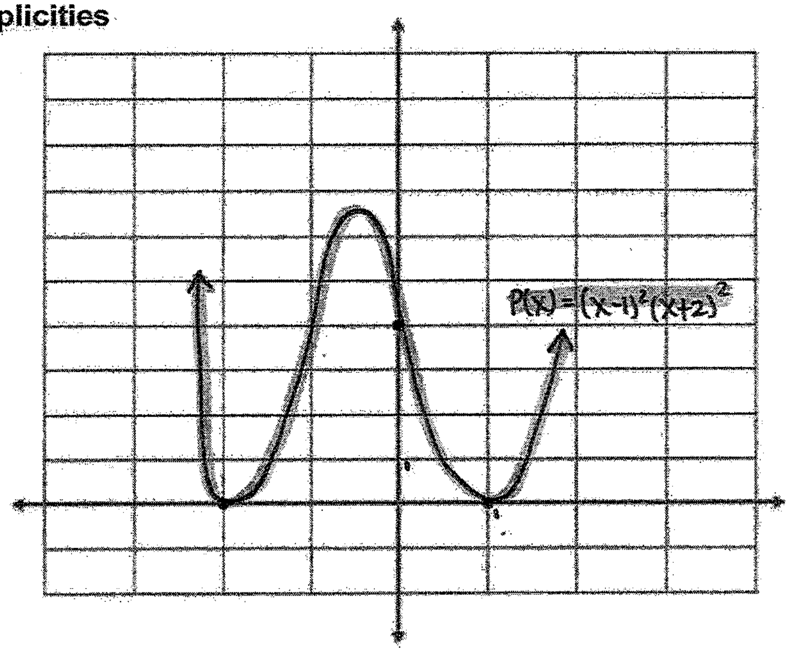
X=-2, M2

ii) Determine the **y – intercept**

$P(0) = (0-1)^2(0+2)^2$

$P(0) = (1)(4)$

$P(0) = 4$ @ (0,4)



iii) Describe the **end behaviour**

up in Q2 , up in Q1

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d) $P(x) = x(x+1)^3(x-2)^2$

i) Determine the **zeros** and their **multiplicities**

X=0 , M1

X=-1, M3

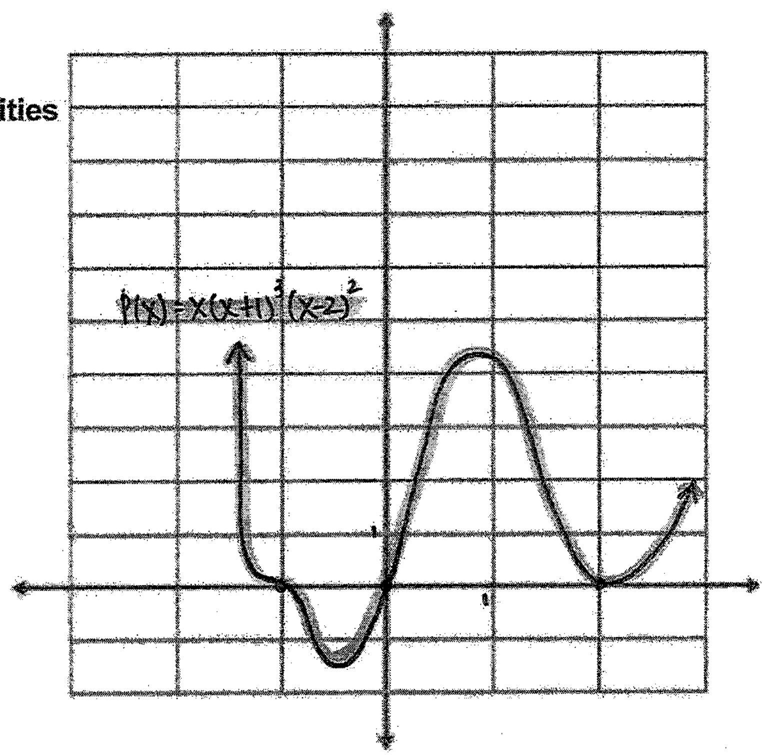
X=2, M2

ii) Determine the **y – intercept**

$P(0) = (0)(0+1)^3(0-2)^2$

$P(0) = (0)(1)(4)$

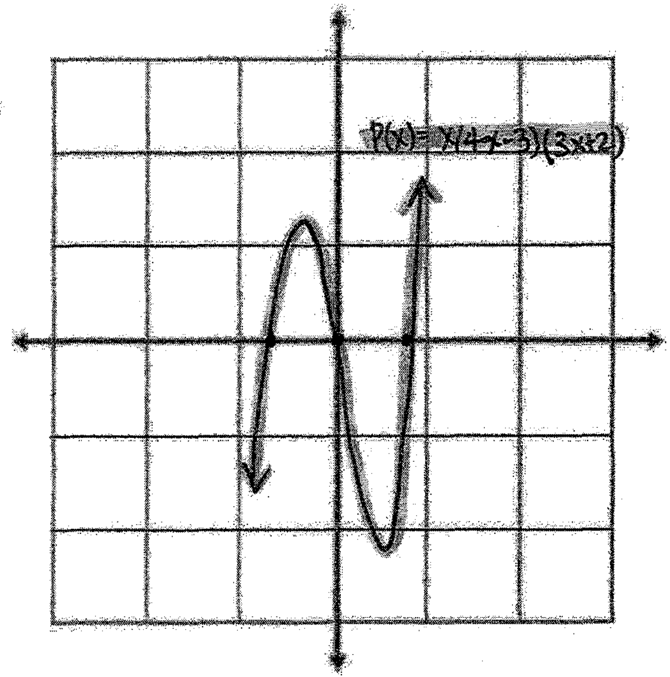
$P(0) = 0$ @ (0,0)



iii) Describe the **end behaviour**

up in Q2 and up in Q1

e) $P(x) = x(4x-3)(3x+2)$



i) Determine the **zeros** and their **multiplicities**

X=0, M1
X=3/4, M1
X=-2/3, M1

ii) Determine the **y – intercept**

$P(0) = (0)(4(0)-3)(3(0)+2)$

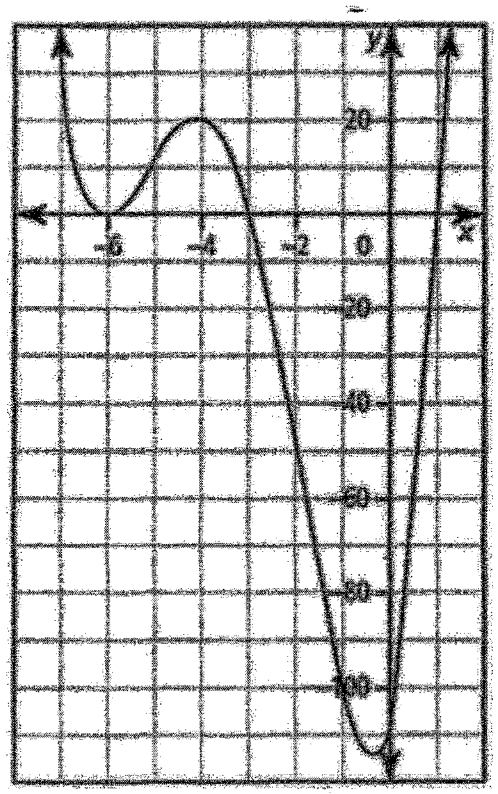
$P(0) = 0(-3)(2)$

$P(0) = 0 \quad @ (0,0)$

iii) Describe the **end behaviour**

down in Q3, up in Q1

4. For the graph of the following polynomial function, determine the following:



a) The least possible **degree**

4

b) The sign of the **leading coefficient**

positive

c) The **zeros** and their **multiplicities**.

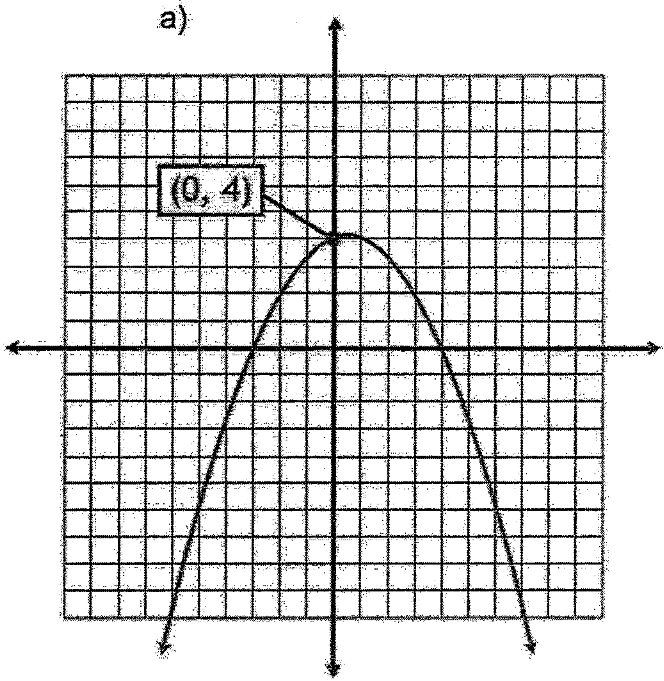
X = -6 , multiplicity of 2

X = -3, multiplicity of 1

X = 1 , multiplicity of 1

5. Determine the **polynomial function** corresponding to each graph. Leave your answer in factored form.

a)



$y = a(x+3)(x-4)$

$4 = a(0+3)(0-4)$

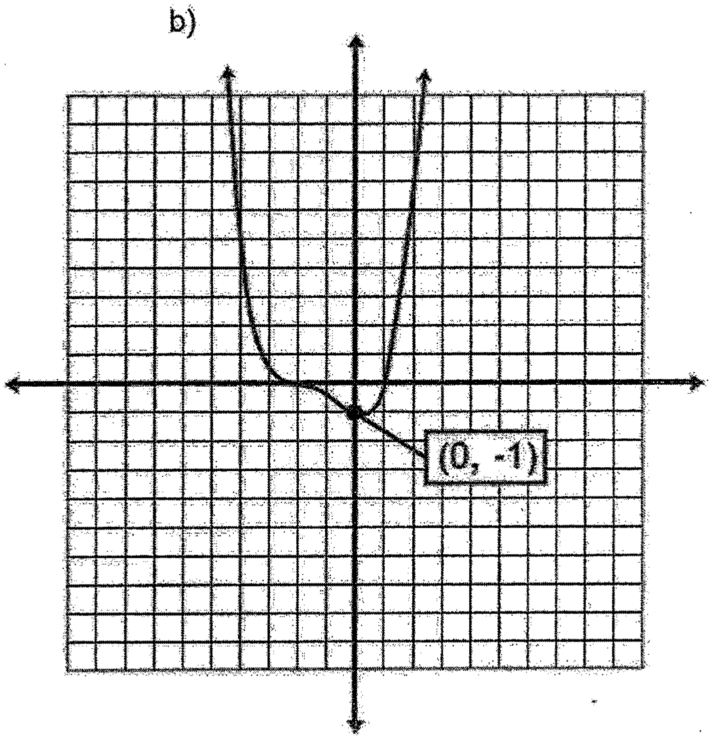
$4 = a(3)(-4)$

$\frac{4}{-12} = \frac{a(-12)}{-12}$

$-\frac{1}{3} = a$

$y = -\frac{1}{3}(x+3)(x-4)$

b)



$y = a(x+2)^3(x-1)$

$-1 = a(0+2)^3(0-1)$

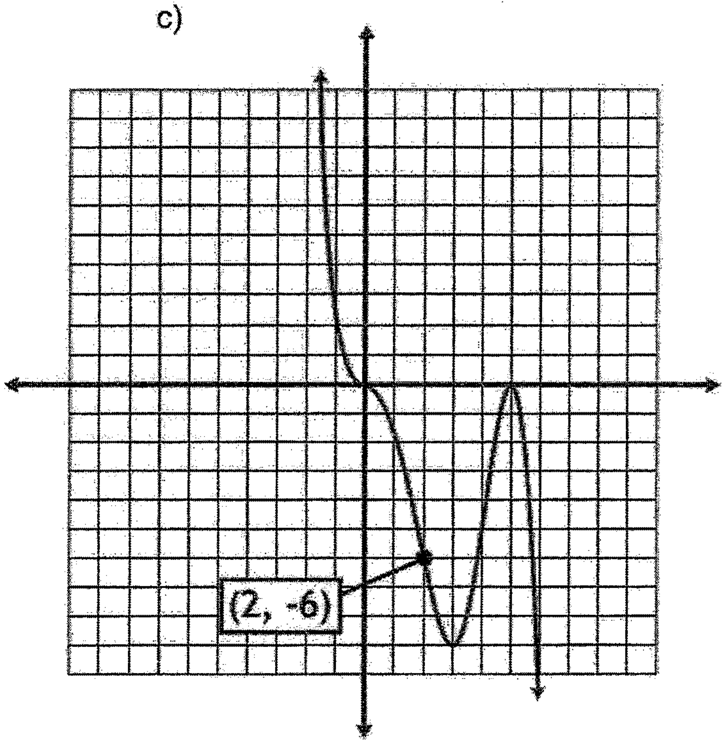
$-1 = a(8)(-1)$

$\frac{-1}{-8} = \frac{a(-8)}{-8}$

$\frac{1}{8} = a$

$y = \frac{1}{8}(x+2)^3(x-1)$

c)



$y = a(x)^3(x-5)^2$

$-6 = a(2)^3(2-5)^2$

$-6 = a(8)(9)$

$-6 = a(72)$

$\frac{-6}{72} = \frac{a(72)}{72}$

$-\frac{1}{12} = a$

$y = -\frac{1}{12}(x)^3(x-5)^2$

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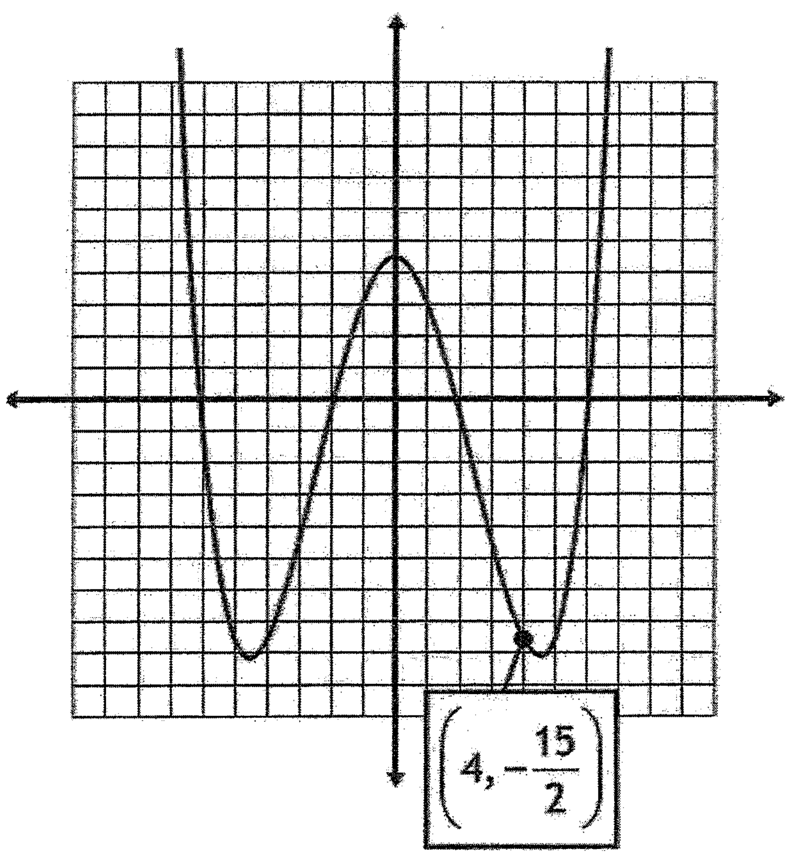
d)



$y = a(x+6)(x+2)(x-2)(x-6)$

$-\frac{15}{2} = a(4+6)(4+2)(4-2)(4-6)$

$-\frac{15}{2} = a(10)(6)(2)(-2)$

$\frac{-\frac{15}{2}}{-240} = \frac{a(-240)}{-240}$

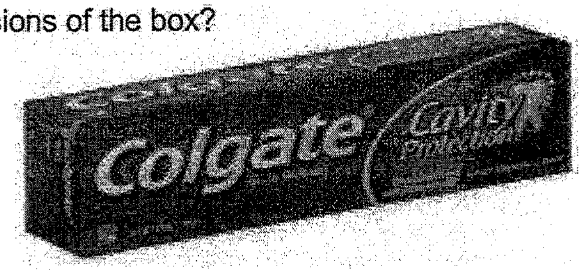
$\frac{-15}{-480} = a$

$\frac{1}{32} = a$

$y = \frac{1}{32}(x+6)(x+2)(x-2)(x-6)$

6. A toothpaste box has square ends. The length of the box is 12 cm greater than the width.

The volume of the box is 135 cm^3. What are the dimensions of the box?



Volume = Length x width x height

$$V(x) = (12 + x)(x)(x)$$

$$135 = (12+x)(x^2)$$

$$135 = 12x^2 + x^3$$

$$0 = x^3 + 12x^2 - 135$$

$$0 = (x-3)(x^2+15x+45)$$

3	1	12	0	-135
+	↓	3	45	135
x	1	15	45	0

$x = 3$

The only solution is $x = 3$

$$x = \frac{-15 \pm \sqrt{45}}{2}$$

~~x = -4.146~~
~~x = -10.854~~
} cannot have negative dimensions.

∴ Length : 12 + 3 = 15 cm

width : 3 cm

height : 3 cm

The dimensions of the box are 15 cm by 3 cm by 3 cm