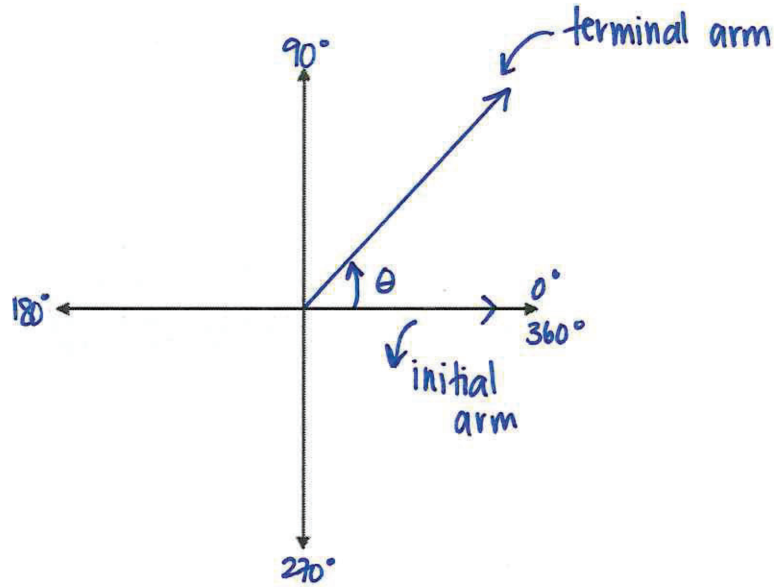
Chapter 4: TRIGONOMETRY AND THE UNIT CIRCLE

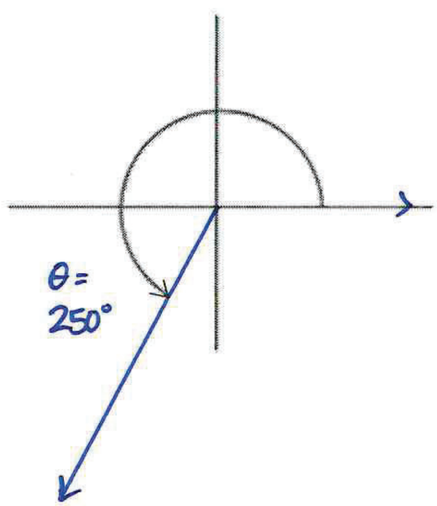
4.1 – Angles and Angle Measure

An angle in standard position has its centre at the origin and its initial arm along the positive x-axis

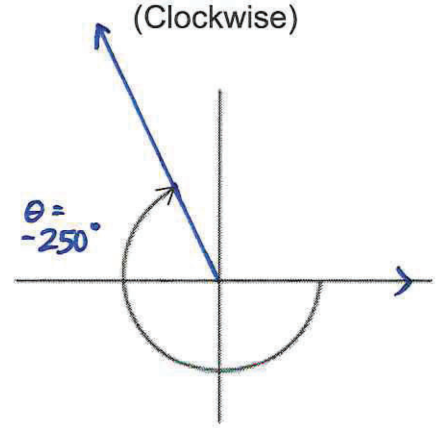
There are positive and negative angles.



Positive Angles
(Counter-clockwise)



Negative Angles
(Clockwise)



Example #1

In which **quadrant** is the terminal arm of each angle located?

a) 400° ___I___

b) 700° ___IV___

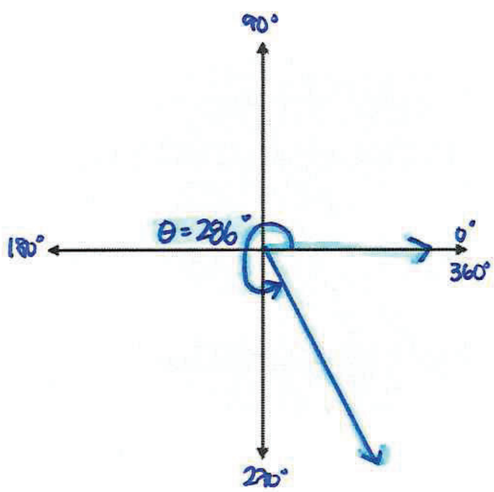
c) – 65° ___IV___

d) – 150° ___III___

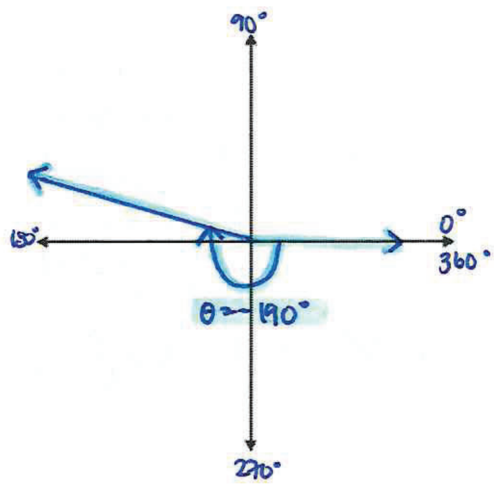
Example #2

Sketch each angle in **standard position**.

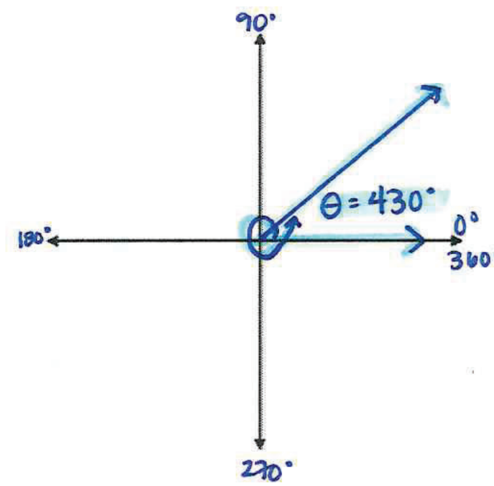
a) 286°



b) −190°



c) 430°

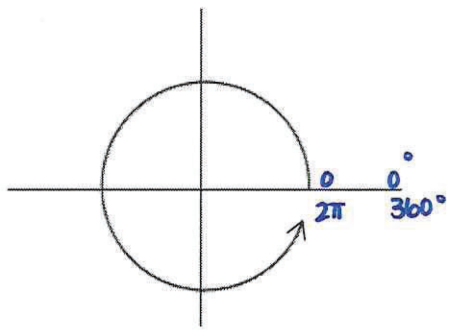


Radian Measure of an Angle

- The formula for the **circumference** of a circle is $C = 2\pi r$
- The **unit circle** has a radius = 1
- Therefore, the circumference of the unit circle is $C = 2\pi$

$2\pi = 6.283185\ldots$

This means that the distance traveled from the initial arm all around the circle and back again is 6.283185…



Revolutions	Degrees	Radian Measure	
1 revolution	$360°$	2π radians	6.283185… radians
$\frac{1}{2}$ revolution	$180°$	π radians	3.141592… radians
$\frac{1}{4}$ revolution	$90°$	$\frac{\pi}{2}$ radians	1.570796… radians
$\frac{3}{4}$ revolution	$270°$	$\frac{3\pi}{2}$ radians	4.712388… radians
$\frac{1}{360}$ revolution	$1°$	$\frac{\pi}{180}$ radians	0.017453… radians

Note that 1 radian = $\left(\frac{180°}{\pi}\right) \approx 57.3°$

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Converting Degrees to Radians: Multiply by $\left(\frac{\pi}{180^\circ}\right)$

Example #3

Express the following angle measures in **radians**.

a) 30°

$30^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{30^\circ\pi}{180^\circ}$

$= \frac{\pi}{6}$

b) 225°

$225^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{225^\circ\pi}{180^\circ}$

$= \frac{5\pi}{4}$

c) 720°

$720^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{720^\circ\pi}{180^\circ}$

$= 4\pi$

Converting Radians to Degrees: Multiply by $\left(\frac{180^\circ}{\pi}\right)$

Example #4

Express the following angle measures in **degrees**

a) $\frac{2\pi}{3}$

$\frac{2\pi}{3}\left(\frac{180^\circ}{\pi}\right) = \frac{360^\circ\pi}{3\pi}$

$= 120^\circ$

b) 1.6

$1.6\left(\frac{180^\circ}{\pi}\right) = \frac{288^\circ}{\pi}$

$= 91.7^\circ$

c) $\frac{5\pi}{6}$

$\frac{5\pi}{6}\left(\frac{180^\circ}{\pi}\right) = \frac{900^\circ\pi}{6\pi}$

$= 150^\circ$

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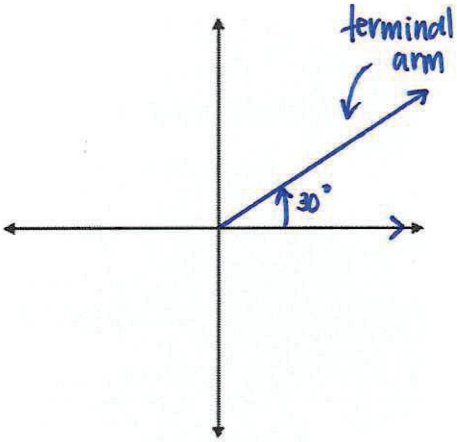
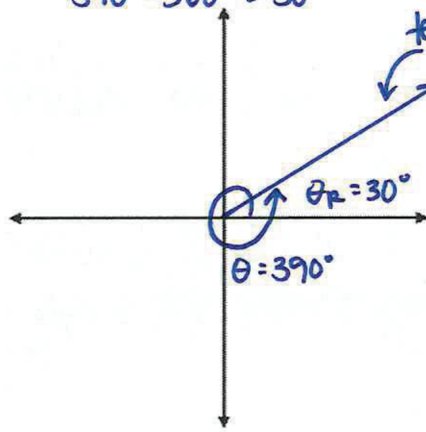
Coterminal Angles

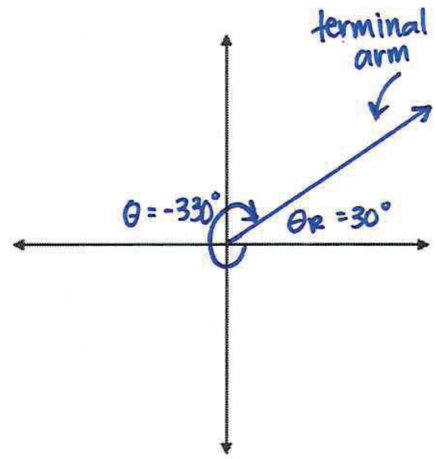
Coterminal Angles are angles in standard position that share the same terminal arm.

Example: Sketch $\theta = 30°$ as an angle in standard position, and show that $\theta = 390°$ and $\theta = -330°$ are **coterminal angles**.



$390° - 360° = 30°$

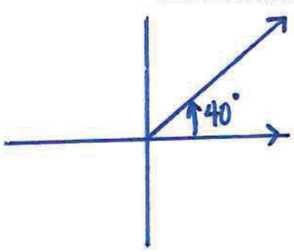




The coterminal angle can be found by **adding** or **subtracting** revolutions; either $\pm 360°$ when given degree measure or $\pm 2\pi$ when given radian measure. There are an infinite number of coterminal angles.

Example #5

Determine 3 **coterminal angles** for $40°$.



$40° + 360° = 400°$

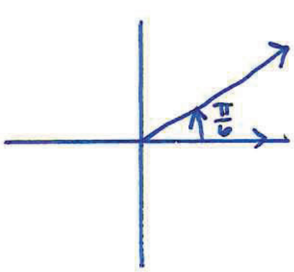
$40° - 360° = -320°$

$40° + 360° + 360° = 760°$

$400°$, $-320°$ and $760°$ are all coterminal with $40°$

Example #6

Determine 3 **coterminal angles** for $\frac{\pi}{6}$



$\frac{\pi}{6} + 2\pi = \frac{13\pi}{6}$

$\frac{\pi}{6} - 2\pi = -\frac{11\pi}{6}$

$\frac{\pi}{6} + 2\pi + 2\pi = \frac{25\pi}{6}$

$\frac{13\pi}{6}, -\frac{11\pi}{6}, \frac{25\pi}{6}$ are all coterminal with $\frac{\pi}{6}$

General Form of Coterminal Angles

Degrees: $\theta \pm 360^\circ n$, $n \in N$

Radians: $\theta \pm 2\pi n$, $n \in N$

Represents all the possible coterminal angles.

↳ n is a Natural # (1, 2, 3, ...)

Example #7

Express the angles **coterminal** with 50° in general form.

$50^\circ \pm 360^\circ(n)$, $n \in N$

Example #8

Express a general form for all **coterminal angles** of $\frac{5\pi}{3}$

$\therefore \frac{5\pi}{3} \pm 2\pi(n)$, $n \in N$

Example #9

Determine a **coterminal angle** to 740° over the interval $\underbrace{-360^\circ < \theta < 0^\circ}_{\text{Restriction!}}$ only angles between are acceptable.

$740^\circ - 360^\circ(1) = 380^\circ$ ×

$740^\circ - 360^\circ(2) = 20^\circ$ ×

$740^\circ - 360^\circ(3) = -340^\circ$ ✓

$740^\circ - 360^\circ(4) = -700^\circ$ ×

$\therefore$ -340° is the only coterminal angle over this interval.

Example #10

Determine all **coterminal angles** to $\frac{5\pi}{3}$ over the interval $[-4\pi, 2\pi]$ → Think with denominator of 3

$\left[-\frac{12\pi}{3}, \frac{6\pi}{3}\right]$

$\frac{5\pi}{3} + 2\pi(1) = \frac{11\pi}{3}$ ×

$\frac{5\pi}{3} - 2\pi(1) = -\frac{\pi}{3}$ ✓

$\frac{5\pi}{3} - 2\pi(2) = -\frac{7\pi}{3}$ ✓

$\frac{5\pi}{3} - 2\pi(3) = -\frac{13\pi}{3}$ ×

$\therefore -\frac{\pi}{3}$ & $-\frac{7\pi}{3}$ are the coterminal angles over this interval

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Arc Length

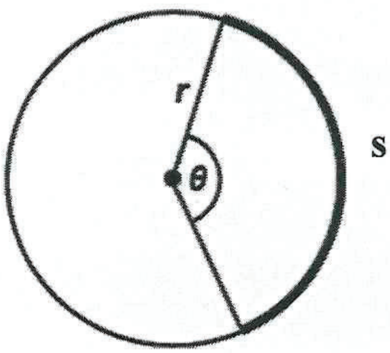
The central angle is the relationship between the length of the arc and the radius of the circle.

The equation that represents this relationship is:

$S = \theta r$ where: $S =$ arc length, $r =$ radius — must be the same units

$\theta =$ central angle

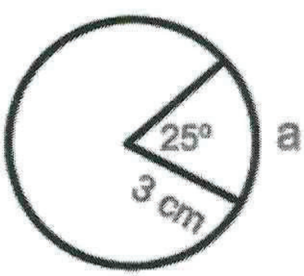
↳ Must use radians



Note: If there is no unit attached to the angle measure (ex: $\theta = 2.5$) it is assumed to be in **radians**.

Example #11

Determine the **arc length**.



$\theta = 25°$

↳ Must be radians

$25°\left(\frac{\pi}{180°}\right)$

$= \frac{5\pi}{36}$

$S = \theta r$

$S = \left(\frac{5\pi}{36}\right)(3)$

$S = 1.309$ cm

Example #12

A bicycle tire has a radius of 0.5 m and travels a distance of 1.5 m. Determine the rotated angle, in degrees.

$S = \theta r$

$\frac{1.5}{0.5} = \frac{(\theta)(0.5)}{0.5}$

$3 = \theta$

↳ radians

$3\left(\frac{180°}{\pi}\right) = 171.887°$

Example #13

Given the following information determine the missing value.

a) $r = 8.7$ cm, $\theta = 75°$ determine arc length

$S = \theta r$

$75°\left(\frac{\pi}{180°}\right) = \frac{5\pi}{12}$

$S = \left(\frac{5\pi}{12}\right)(8.7)$

$S = 11.388$ cm

b) $\theta = 1.8$, $S = 4.7$ mm, determine the radius

$S = \theta r$

$\frac{4.7}{1.8} = \frac{(1.8)r}{1.8}$

2.611 mm $= r$

c) $r = 5$ m, $S = 13$ m, determine the measure of the central angle

$S = \theta r$

$\frac{13}{5} = \frac{(\theta)(5)}{5}$

$2.6 = \theta$

↳ radians, so no units