LAFERTY

Grade 12
Pre-Calculus Mathematics
[MPC40S]

Chapter 2

Radical Functions

Outcomes

R13

12P.R.13. Graph and analyze radical functions (limited to functions involving one radical)

Snd chp 2 exercises

MPC40S Date: ____________ **R13**

Chapter 2: RADICAL FUNCTIONS

2.1 – Radical Functions and Transformations

Radical Function: A function containing a radical

____

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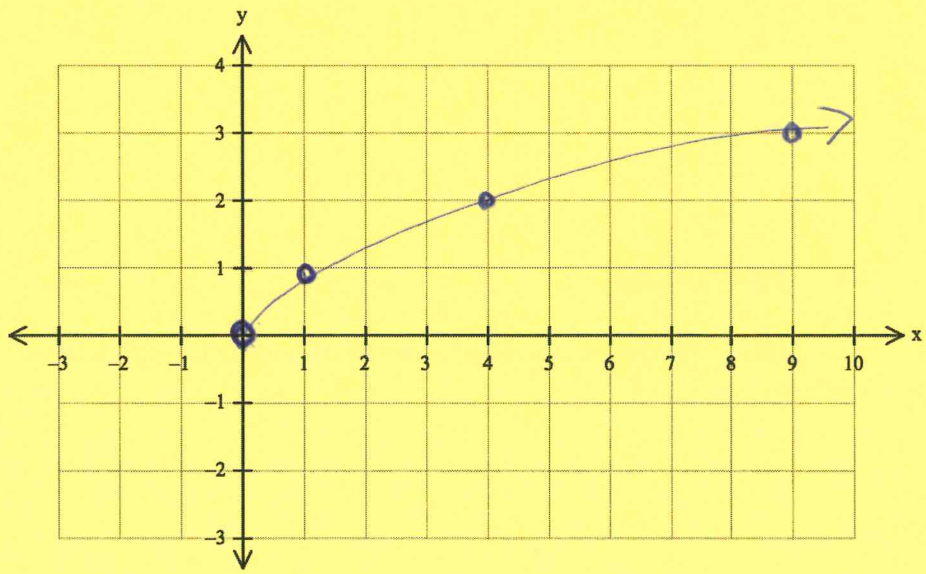
Example #1

Sketch the graph of the function $y = \sqrt{x}$.

State the **domain** and **range** of the radical function.

x	y
0	0
1	1
4	2
9	3



Domain: $x \geq 0$ $[0, \infty)$

Range: $y \geq 0$ $[0, \infty)$

General Transformation Model for Radical Functions

You can graph a radical function of the form $y = a\sqrt{b(x-h)} + k$ by transforming the graph of $y = \sqrt{x}$ based on the values of a, b, h, and k. The effects of changing parameters in radical functions are the same as the effects of changing parameters in other types of functions.

Example #2

we need at least 2 points and correct general shape.

Sketch the graph of the following radical functions.

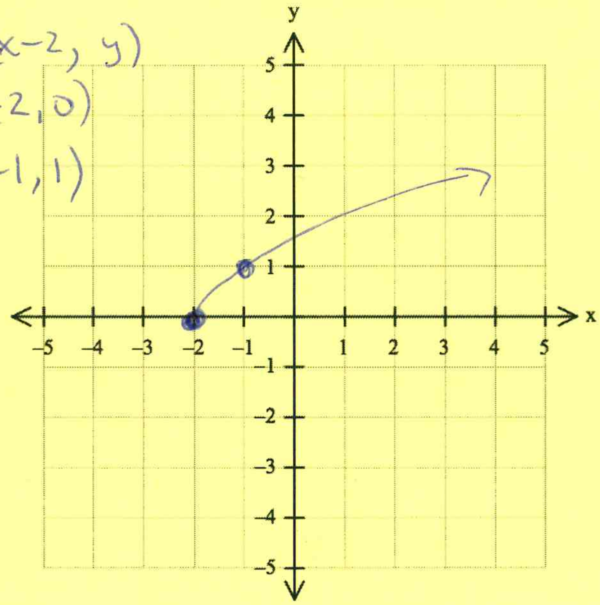
a) $y = \sqrt{x+2}$ left 2 units

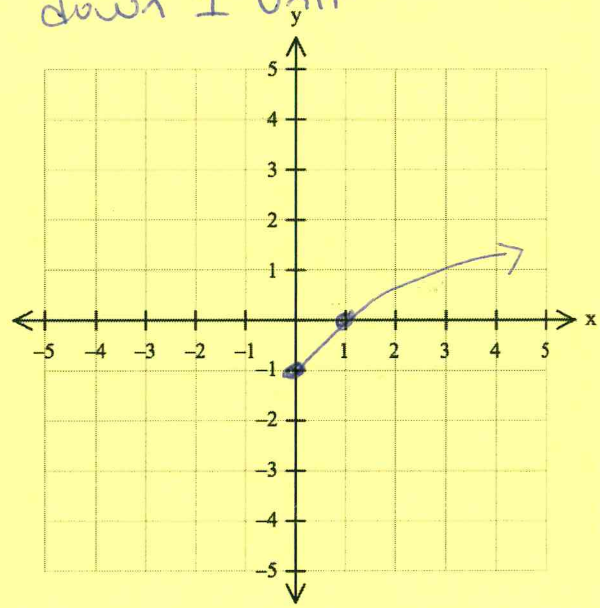
b) $y = \sqrt{x} - 1$ down 1 unit

$(x, y) \to (x-2, y)$

$(0,0) \to (-2, 0)$

$(1,1) \to (-1, 1)$





Example #3

Using the given equations, **explain** in words the transformations needed to transform the graph of $y = \sqrt{x}$.

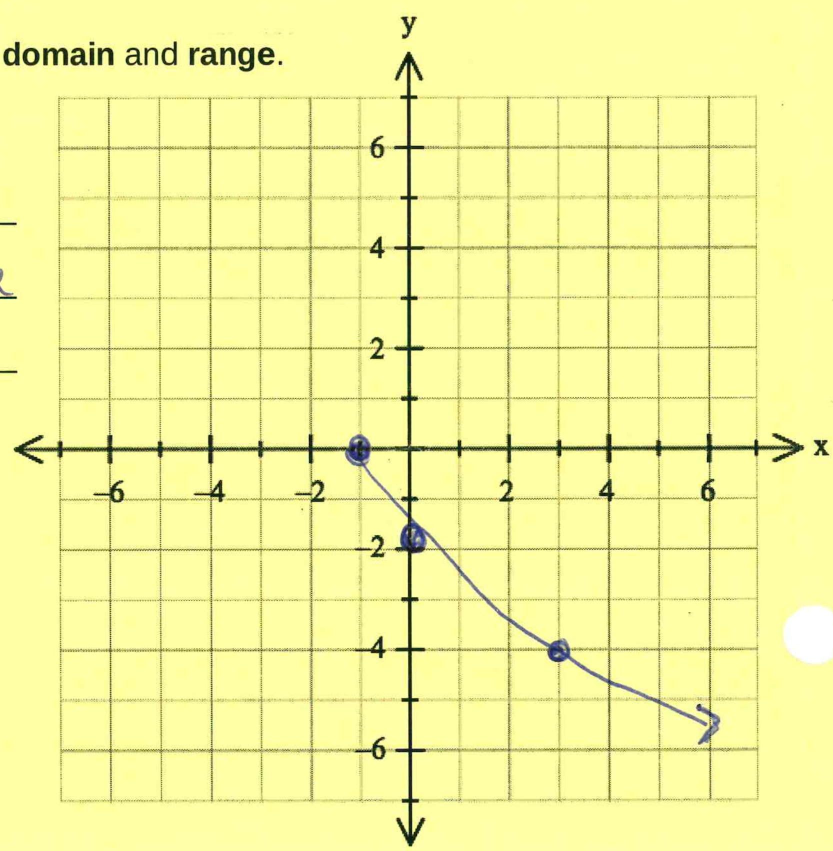
Then **sketch** each graph and state the **domain** and **range**.

a) $y = -2\sqrt{x+1}$ $+0$

→ reflect over x-axis

→ vertical stretch by a factor of 2.

→ shift left 1



Domain: $x \geq -1$

Range: $y \leq 0$

when $x = 3$

$y = -2\sqrt{4}$ $(3, -4)$

$y = -4$

b) $y = \sqrt{2x - 4}$

$y = \sqrt{2(x-2)}$

horizontal compression by a factor of 2

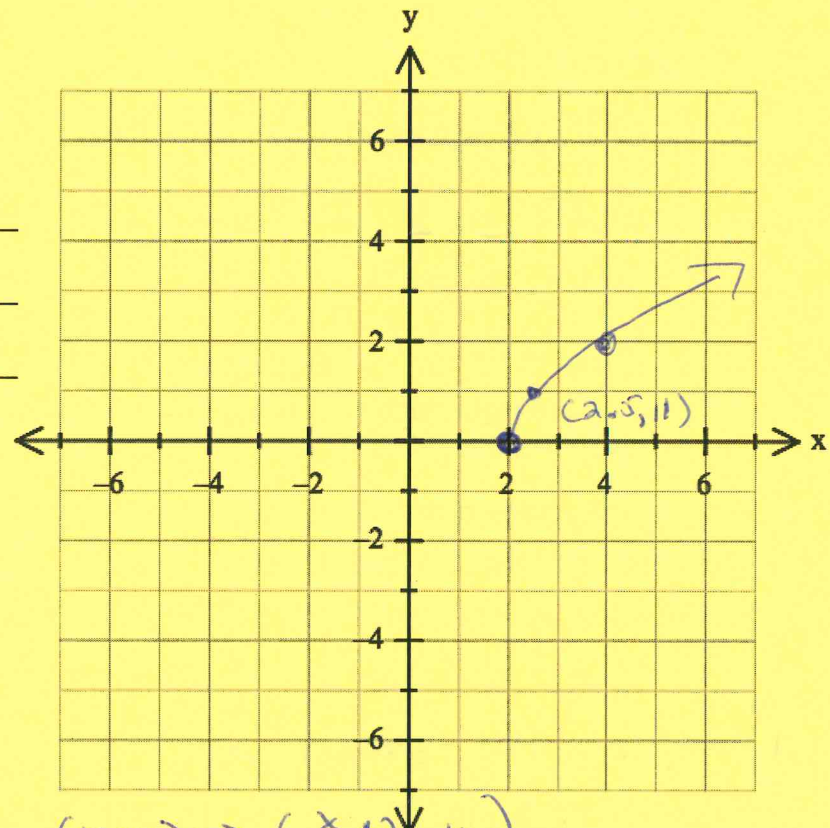
shift right 2

Domain: $[2, \infty)$

Range: $[0, \infty)$

when $x = 4$

$y = \sqrt{2(4-2)}$
$= 2$



$(x, y) \to (\frac{x}{2} + 2, y)$
$(0,0) \to (2,0)$
$(1,1) \to (2.5, 1)$

c) $y + 1 = \sqrt{-x + 3}$

$y = \sqrt{-(x-3)} - 1$

reflection over y-axis.

shift right 3

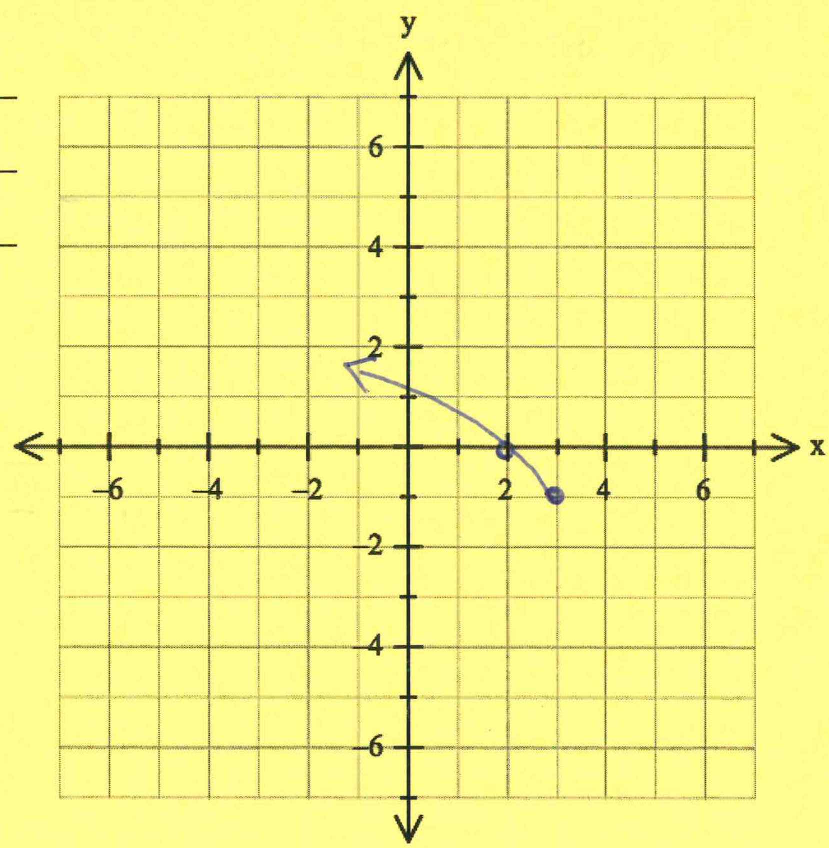
shift down 1

Domain: $x \leq 3$

Range: $y \geq -1$

when $x = 2$

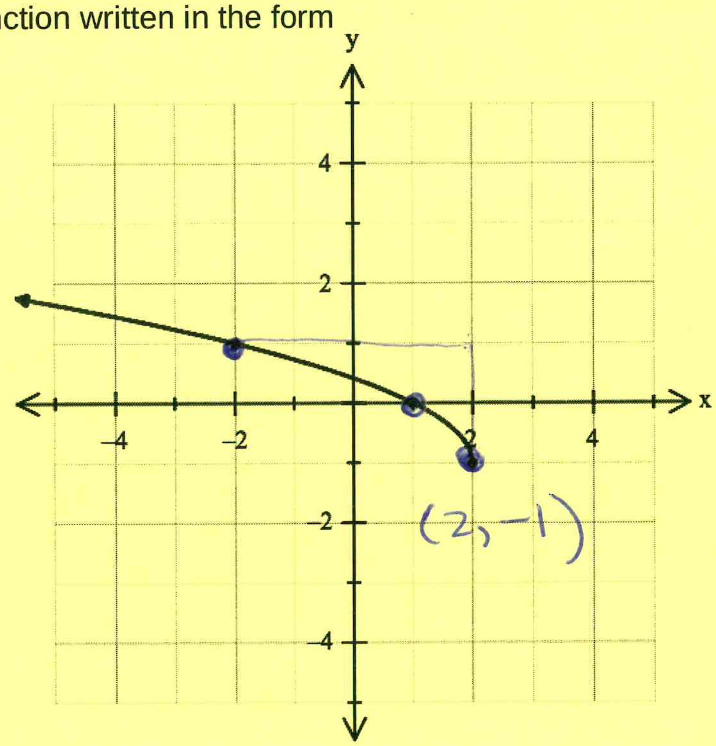
$y = \sqrt{-(-1)} - 1$
$y = 0$



Example #4

Determine the **equation** of the following function written in the form

$y = a\sqrt{b(x-h)} + k$



The point (0,0) on the graph of $y = \sqrt{x}$ is **not** affected by any stretches or any reflections.

Use the **image point** of ____________ to find the value of _______ and _______

① determine START point (h, k)

② Do we have reflections?

③ Are we stretching/compression.

Answer: ______________________________

$$\boxed{y = \_\_\sqrt{\_\_(x-2)} - 1}$$

$$\boxed{y = 2\sqrt{-\tfrac{1}{4}(x-2)} - 1}$$

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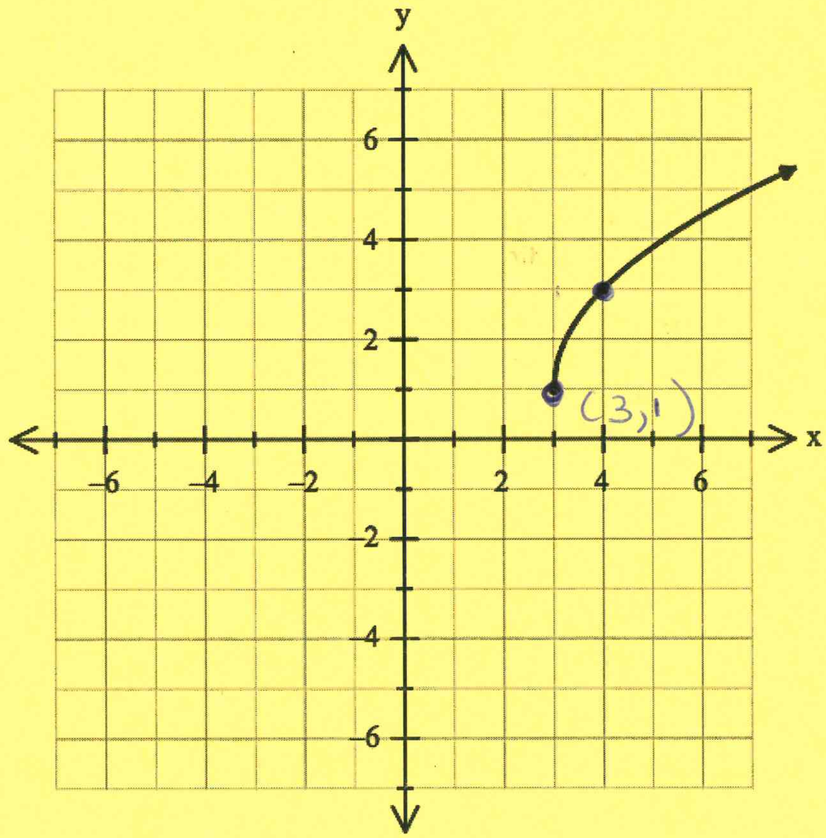
Example #5

Determine the **equation** of the following function written in the form

$y = a\sqrt{b(x-h)} + k$



Answer: ________________________

$y = 2\sqrt{(x-3)} + 1$

Chapter 2: RADICAL FUNCTIONS
2.2 – Square Root of a Function

Example #1

Sketch the following graphs on the same Cartesian plane.

$y = x + 2$ and $y = \sqrt{x+2}$

$f(x) = x+2$ $y = \sqrt{f(x)}$

$(x, y) \longrightarrow (x, \sqrt{y})$

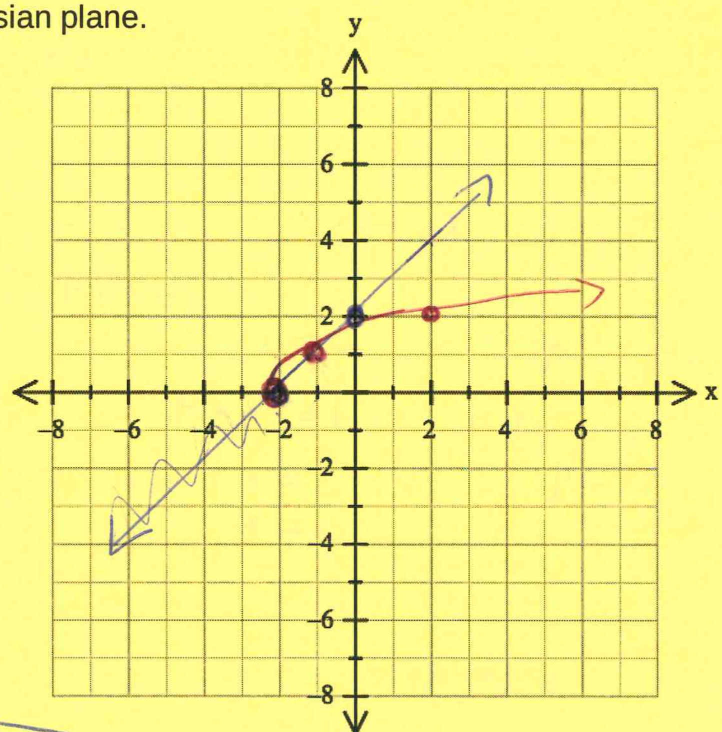
* no negative y-values!

* points with y values that are 0 and 1 remain invariant.

$(-2, 0) \longrightarrow (-2, 0)$

$(-1, 1) \longrightarrow (-1, 1)$

$(2, 4) \longrightarrow (2, 2)$



note: ~~when the~~ the points between $0 < y < 1$ the graph of $\sqrt{f(x)}$ is above the graph of $f(x)$

	$y = x + 2$	$y = \sqrt{x+2}$
x – intercept	-2	-2
y – intercept	2	$\sqrt{2}$
Domain	$x \in \mathbb{R}$	$x \geq -2$
Invariant Points	$(-2, 0)$	$(-1, 1)$

Steps to sketching $y = \sqrt{f(x)}$ given the graph of $y = f(x)$

1. Cross out the part of the graph where the y values are negative.

2. Place a point on the graph where $y = 0$.

3. Place a point on the graph where $y = 1$.

4. To find other points on the graph, keep the x values the same and take the **square root** of the y values.

5. Connect all points in a smooth curve. Note: In <u>between</u> the invariant points, the square root graph should be above the original graph. Where the y values are greater than 1, the square root graph should be below the original graph.

Example #2

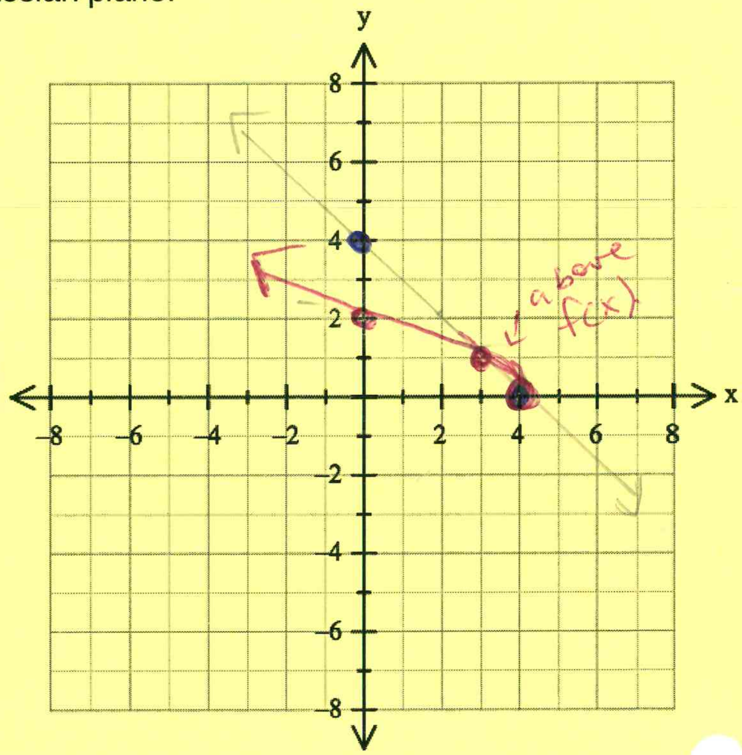
Sketch the following graphs on the same Cartesian plane.

State the **domain** and any **invariant points**.

$y = -x + 4$ and $y = \sqrt{-x + 4}$

x int: $0 = -x+4$
$x = 4$

$(x, y) \to (x, \sqrt{y})$



Domain	$x \in \mathbb{R}$	$x \leq 4$

$(-\infty, 4]$

when $y=0$
$0 = -x+4$
$x = 4$

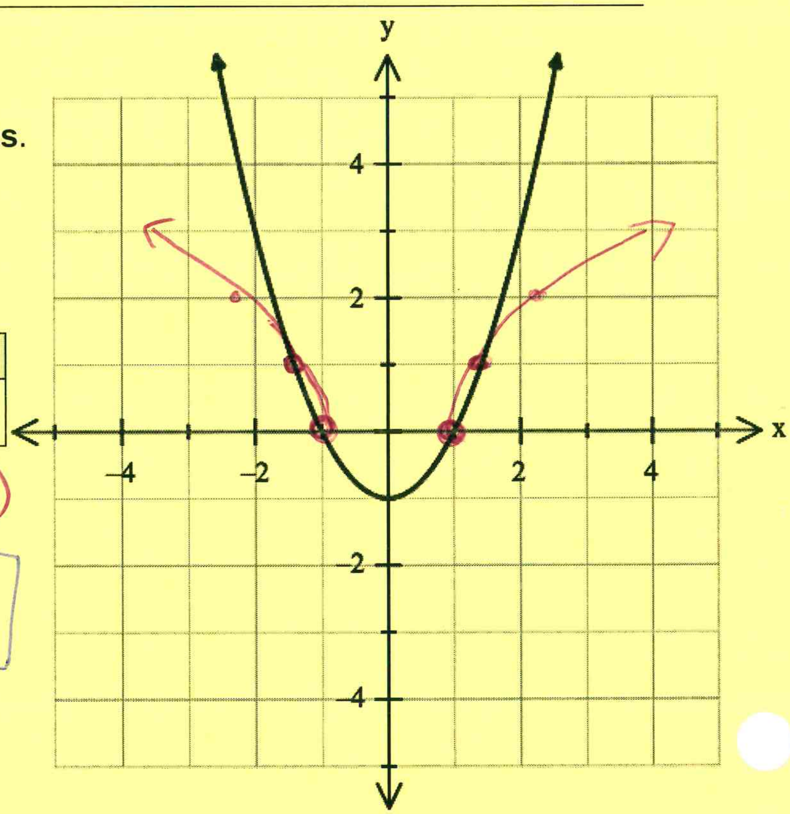
$y = 1$
$1 = -x+4$
$-3 = -x$
$3 = x$

Example #3

Sketch the following functions.

State the **domain** and any **invariant points**.

a) $y = x^2 - 1$ and $y = \sqrt{x^2 - 1}$



Domain	$x \in \mathbb{R}$.	$x \leq -1,\ x \geq 1$

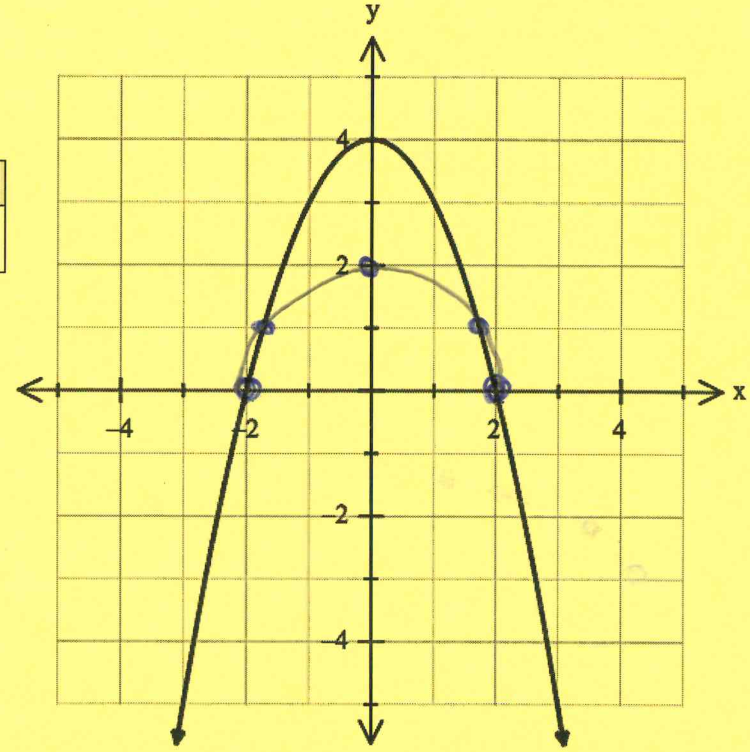
$(-\infty, -1] \cup [1, \infty)$

Invariant points: $\boxed{(-1,0), (1,0)}$

when $y = 1$
$1 = x^2 - 1$
$2 = x^2$
$\boxed{\pm\sqrt{2} = x}$

b) $y = 4 - x^2$ and y= $\sqrt{4 - x^2}$

Domain	$x \in \mathbb{R}$	$[-2, 2]$



$1 = 4 - x^2$

$-3 = -x^2$

$3 = x^2$

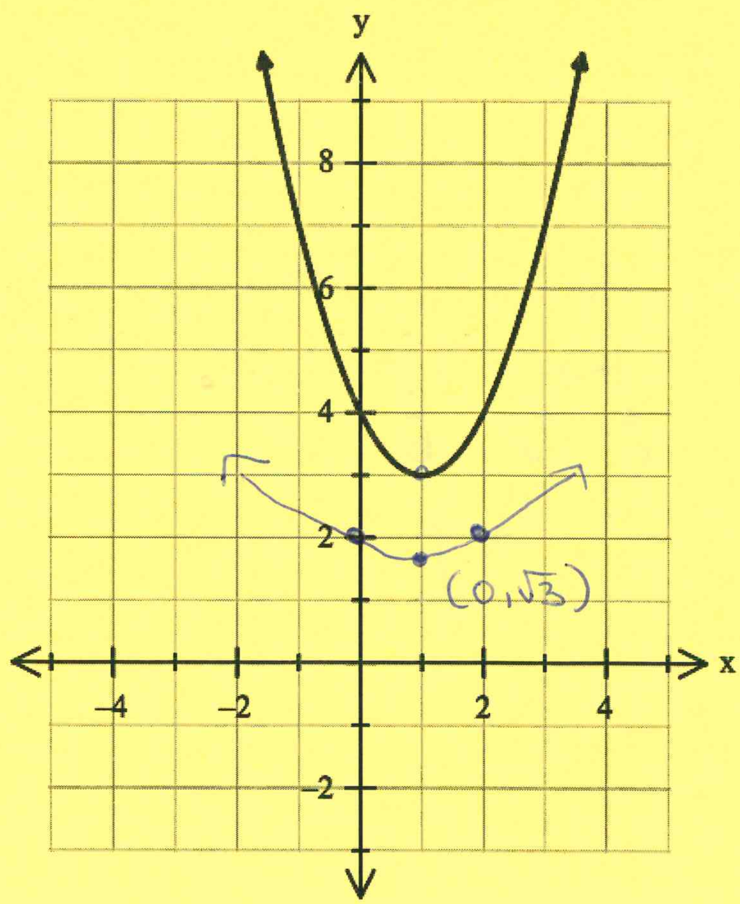
$\pm\sqrt{3} = x$

Invariant points

$(\pm 2, 0)$ $(\pm\sqrt{3}, 1)$

c) $f(x) = (x - 1)^2 + 3$ and $y = \sqrt{f(x)}$

Domain	$x \in \mathbb{R}$	$x \in \mathbb{R}$.



$(x, y) \rightarrow (x, \sqrt{y})$

$(0, 3) \rightarrow (0, \sqrt{3})$

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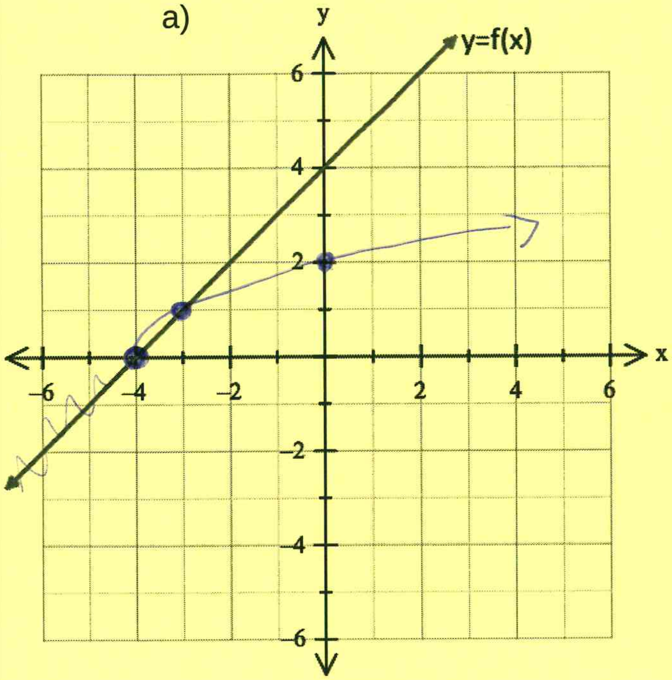
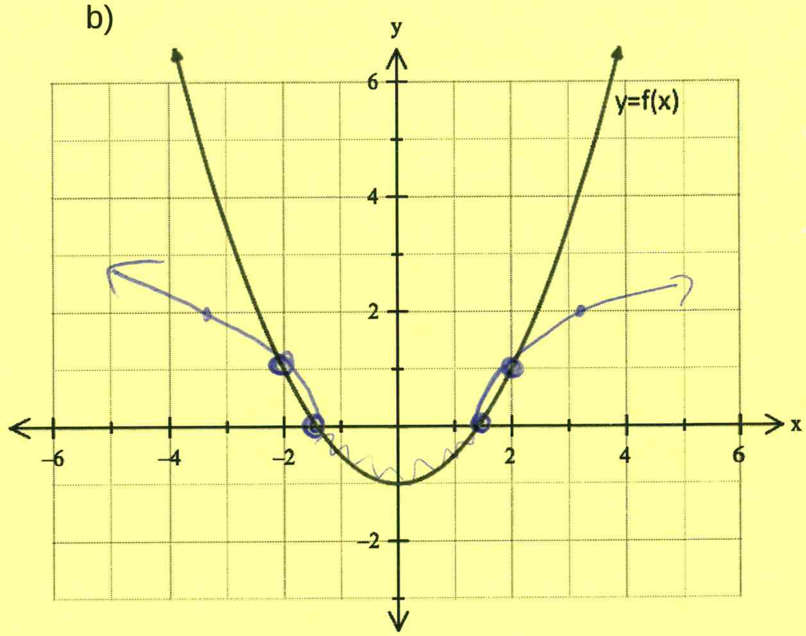
Example #4

Using the graph of $y = f(x)$, **sketch** the graphs of $y = \sqrt{f(x)}$.

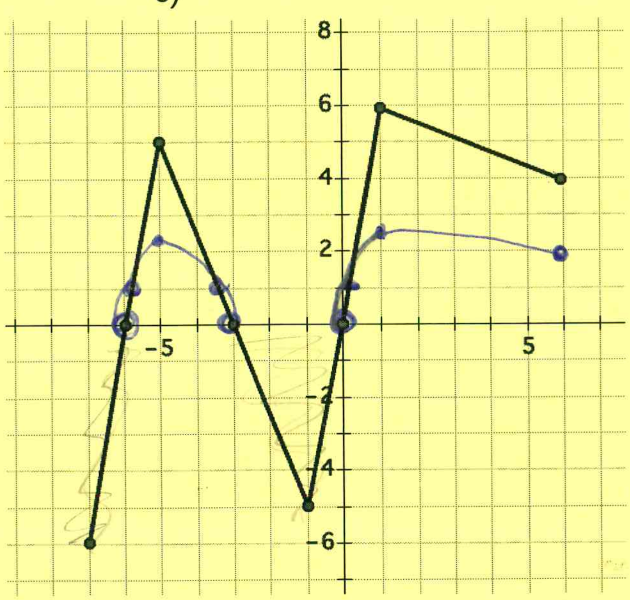
a)



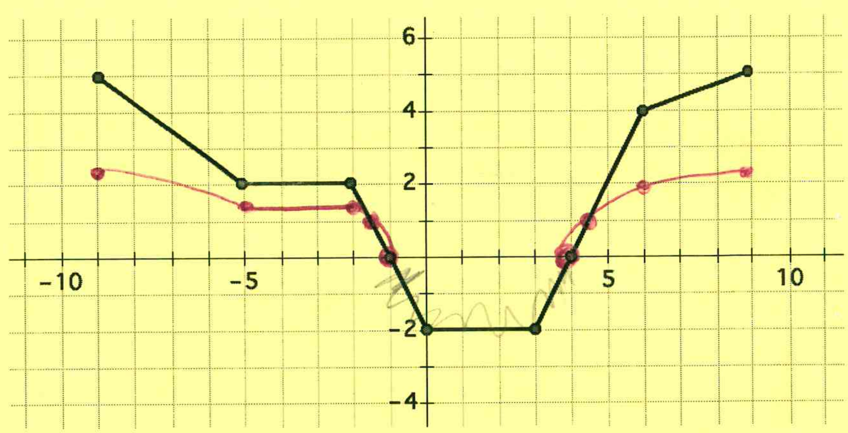
b)



c)



d)



Chapter 2: RADICAL FUNCTIONS
2.3 – Solving Radical Equations

Example #1

a) **Solve** the following equation **algebraically**: $0 = \sqrt{x+4} - 3$

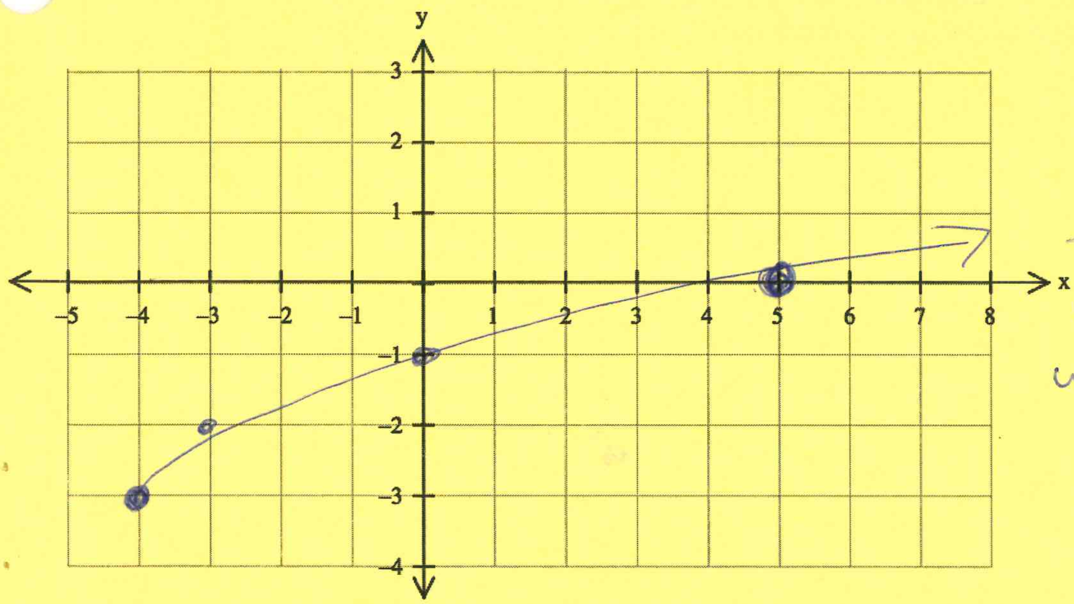
$$(3)^2 = (\sqrt{x+4})^2$$

$$9 = x + 4$$

$$5 = x$$

$y = \sqrt{x}$

b) Using the graph provided, determine the **x – intercept** of the graph of: $y = \sqrt{x+4} - 3$



$(x, y) \to (x-4, y-3)$

$(0,0) \to (-4,-3)$

$(1,1) \to (-3,-2)$

$(4,2) \to (0,-1)$

$0 = \sqrt{x+4} - 3$.

we did this!

$x = 5$.

Note: The zero., root, and x intercept of a function all represent the same thing, the plane where the function crosses the x – axis. For this example, this also represents the solution to the equation.

Example #2

a) **Solve** the following equation **algebraically**.

$$\left(\sqrt{x+5}\right)^2 = (x-1)^2$$

$$x+5 = (x-1)(x-1)$$

$$x+5 = x^2 - 2x + 1$$

$$0 = x^2 - 3x - 4$$

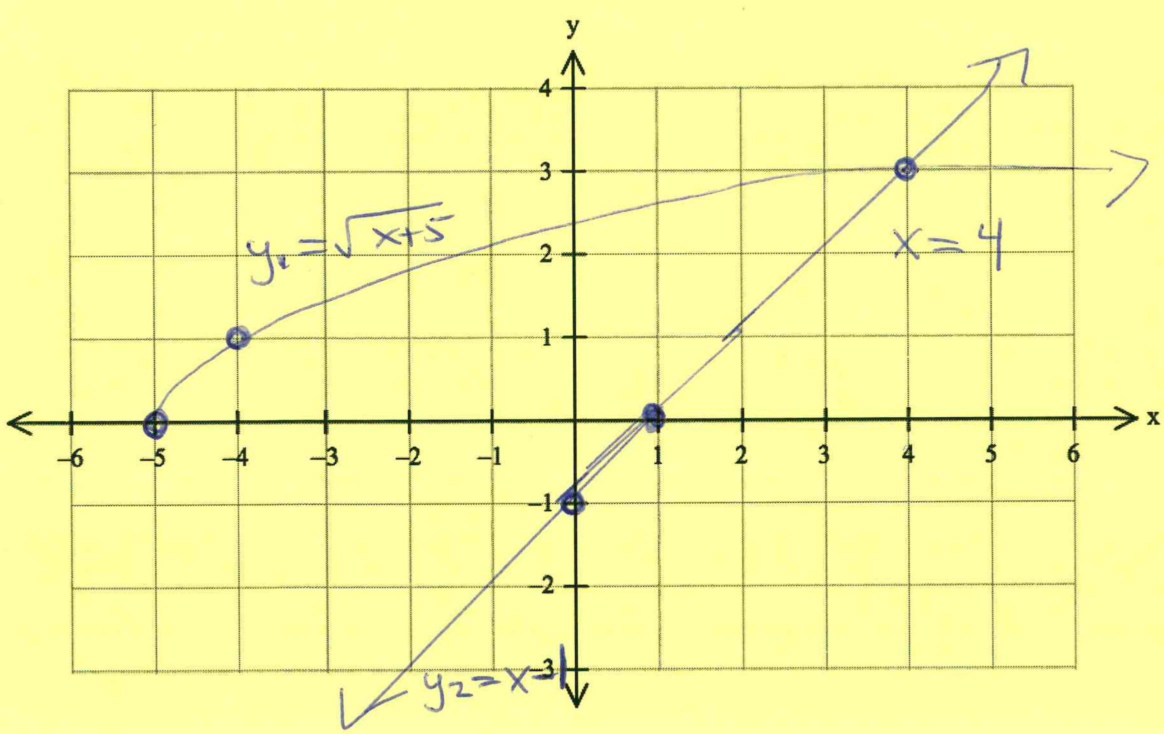
$$0 = (x-4)(x+1)$$

$x = 4$ ✓ $\quad$ $x = \cancel{-1}$ extraneous

b) **Solve** the same equation **graphically**.

To do this, let's separate the equation into two individual functions and graph each. The solution will be the x value of the point of intersection between the two graphs.

$y_1 = \sqrt{x+5}$ $\quad\quad$ $y_2 = x - 1$



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Example #3

Solve each of the following equations **algebraically** and **graphically**.

a) $\sqrt{x+5} - 3 = 0$

$$\boxed{y = \sqrt{x+5} - 3}$$

or

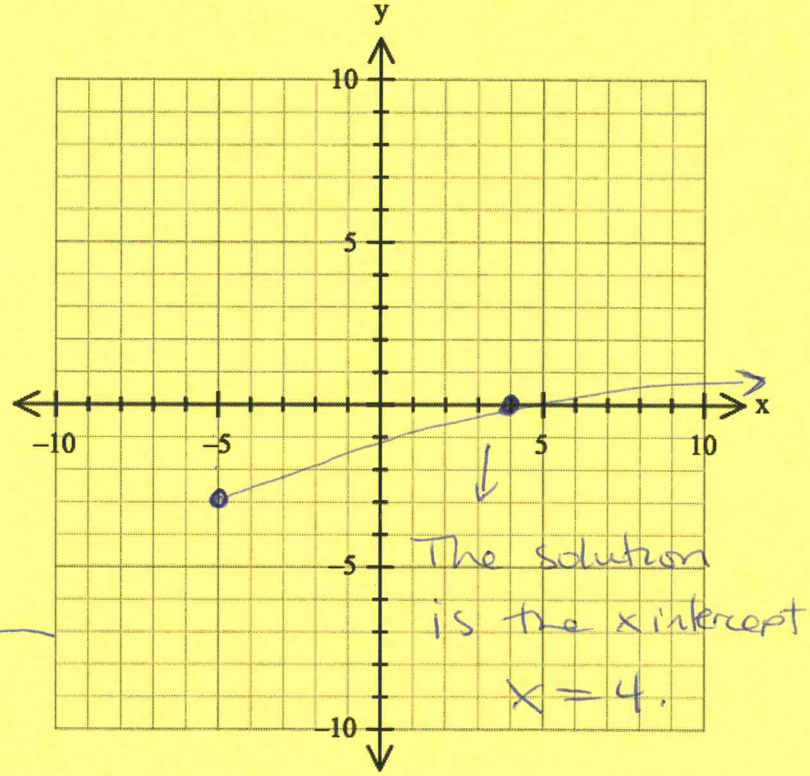
$$\sqrt{x+5} = 3$$

$$y_1 = \sqrt{x+5} \qquad y_2 = 3$$

$$(\sqrt{x+5})^2 = (3)^2$$

$$x + 5 = 9$$

$$x = 4$$



b) $(\sqrt{6-x})^2 = (4-x)^2$

$$y_1 = \sqrt{6-x} \qquad y_2 = 4 - x$$

$$6 - x = (4-x)(4-x)$$

$$6 - x = 16 - 8x + x^2$$

$$0 = x^2 - 7x + 10$$

$$0 = (x-5)(x-2)$$

$$x = 2 \checkmark \qquad \cancel{x = 5}$$

$$y_1 = \sqrt{-x+6}$$

$$y_1 = \sqrt{-(x-6)}$$