Grade 12
Pre-Calculus Mathematics
Achievement Test

Marking Guide

January 2018

Manitoba

A group of 7 friends decide to go to a movie.

Determine how many ways the friends can sit in a row if two of the friends refuse to sit next to each other.

Solution

$7! - 6!\,2! = 3600$ ways

½ mark for 7!
1 mark for product of 6!2!
(½ mark for 6!, ½ mark for 2!)
½ mark for subtraction

2 marks

Gabrielle listens to her radio at a sound level of 80 dB. She attended a music concert that had a sound level of 115 dB. Determine how many times more intense the music concert was than the radio.

You may use the formula:

$$\beta = 10\log\left(\frac{I}{I_0}\right)$$

where β is the intensity level of sound, measured in dB

I is the intensity of sound

I_0 is the standard minimum intensity that a person can hear

Solution

Radio:

$$80 = 10\log\left(\frac{I}{I_0}\right)$$

$$8 = \log\left(\frac{I}{I_0}\right)$$

$$10^8 = \frac{I}{I_0}$$ ½ mark for exponential form

$$10^8 I_0 = I$$

Music concert:

$$115 = 10\log\left(\frac{I}{I_0}\right)$$

$$11.5 = \log\left(\frac{I}{I_0}\right)$$

$$10^{11.5} = \frac{I}{I_0}$$ ½ mark for exponential form

$$10^{11.5} I_0 = I$$

$$\frac{\text{intensity of music concert}}{\text{intensity of radio}} = \frac{10^{11.5} I_0}{10^8 I_0}$$

$$= 10^{3.5}$$

$$= 3162.27766$$

$$= 3162.278$$

1 mark for comparison

2 marks

Solve, algebraically.

$$2(7)^x = 3^{2x-3}$$

Solution

$\log\left(2\left(7^x\right)\right) = \log 3^{2x-3}$	½ mark for applying logarithms
$\log 2 + x\log 7 = (2x-3)\log 3$	1 mark for product law 1 mark for power law
$\log 2 + x\log 7 = 2x\log 3 - 3\log 3$	
$\log 2 + 3\log 3 = 2x\log 3 - x\log 7$	½ mark for collecting terms with x
$\log 2 + 3\log 3 = x(2\log 3 - \log 7)$	
$\dfrac{\log 2 + 3\log 3}{2\log 3 - \log 7} = x$	½ mark for isolating x
$15.872\ 483 = x$	
$15.872 = x$	½ mark for evaluating quotient of logarithms

4 marks

Solve for θ, algebraically, over the interval $[0, 2\pi]$.

$$\csc^2\theta + 2\csc\theta - 8 = 0$$

Solution

$(\csc\theta + 4)(\csc\theta - 2) = 0$

$\csc\theta = -4 \qquad \csc\theta = 2$ — 1 mark for solving for $\csc\theta$

$\sin\theta = -\frac{1}{4} \qquad \sin\theta = \frac{1}{2}$ — 1 mark for reciprocal

$\theta_r = 0.252\ 680$

$\theta = 3.394 \qquad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$ — 2 marks for solving for θ (½ mark for each value)

$\theta = 6.031$

4 marks

or

$\theta = 3.394 \qquad \theta = 0.524$

$\theta = 6.031 \qquad \theta = 2.618$

Question 5 P1

You have forgotten the code to unlock your cell phone. You know the code is made up of four numbers from 0 to 9.

Determine the number of possible codes, if repetition is allowed.

Solution

$\underline{10} \bullet \underline{10} \bullet \underline{10} \bullet \underline{10} = 10\ 000$

1 mark

In the binomial expansion of $\left(\frac{7}{x^3} - 3x^7\right)^n$, the 5th term contains x^7.

Determine the value of n.

Solution

$$x^7 = \left(\frac{1}{x^3}\right)^{n-4}\left(x^7\right)^4$$

1 mark for $k = 4$
½ mark for substitution

$$x^7 = \left(x^{-3}\right)^{n-4}\left(x^7\right)^4$$

$$x^7 = x^{-3n+12+28}$$

$$7 = -3n + 40$$

$$-33 = -3n$$

$$11 = n$$

½ mark for solving for n

2 marks

Question 7 R6

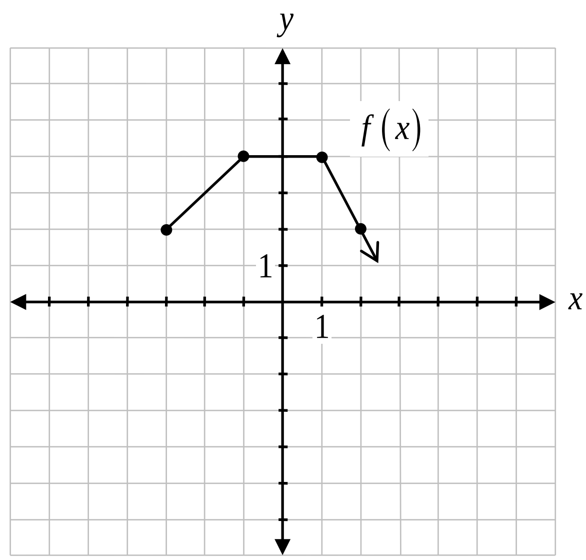
Given the domain of $f(x)$ is $\{-6,1,3,4\}$ and the range of $f(x)$ is $\{-4,7,10,15\}$, state the domain of $f^{-1}(x)$.

Solution

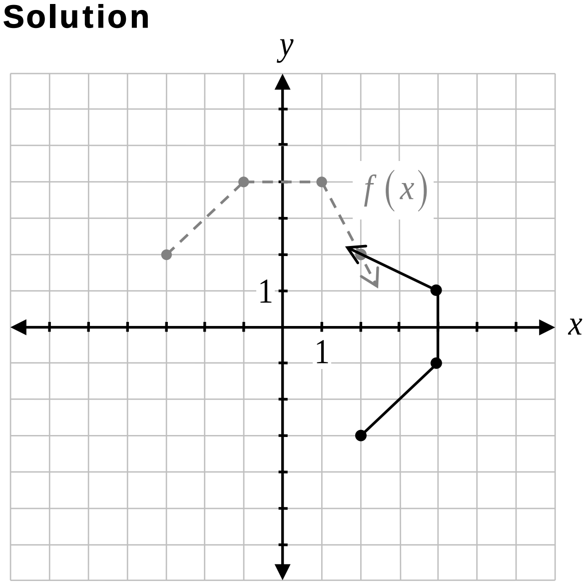
$\{-4,7,10,15\}$

1 mark

Given the graph of $y = f(x)$, sketch the graph of its inverse.



Solution



1 mark

Question 9 T6

Prove the following identity for all permissible values of θ.

$$\frac{1+\cos\theta}{1-\sin^2\theta} = \sec\theta + \tan^2\theta + 1$$

Solution

Method 1

Left-Hand Side	Right-Hand Side
$\dfrac{1+\cos\theta}{1-\sin^2\theta}$	$\sec\theta + \tan^2\theta + 1$
$\dfrac{1+\cos\theta}{\cos^2\theta}$	
$\dfrac{1}{\cos^2\theta} + \dfrac{1}{\cos\theta}$	
$\sec^2\theta + \sec\theta$	
$\tan^2\theta + 1 + \sec\theta$	

1 mark for algebraic strategies

1 mark for logical process to prove the identity

1 mark for correct substitution of appropriate identities

3 marks

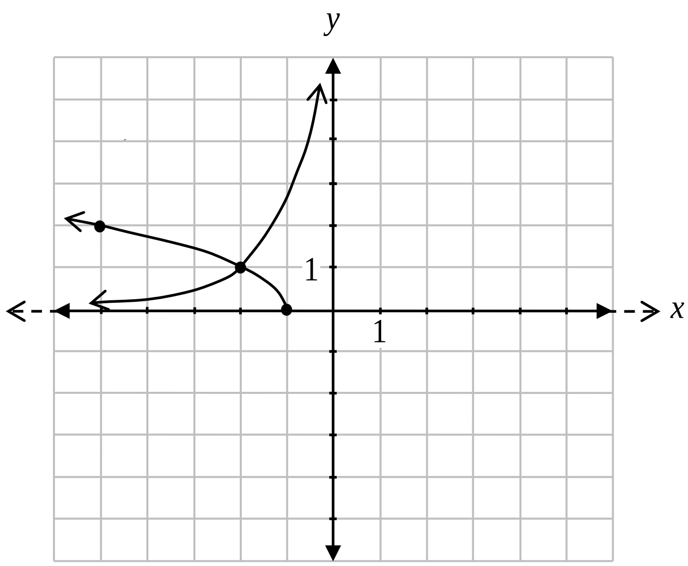
Solution

Method 2

Left-Hand Side	Right-Hand Side	
$\dfrac{1+\cos\theta}{\cos^2\theta}$	$\dfrac{1}{\cos\theta}+\sec^2\theta$	
	$\dfrac{1}{\cos\theta}+\dfrac{1}{\cos^2\theta}$	1 mark for algebraic strategies
	$\dfrac{\cos\theta+1}{\cos^2\theta}$	1 mark for logical process to prove the identity
		1 mark for correct substitution of appropriate identities

3 marks

Thomas used graphs to solve the equation $e^{x+2} = \sqrt{-(x+1)}$.



He incorrectly states the solution as $(-2, 1)$.

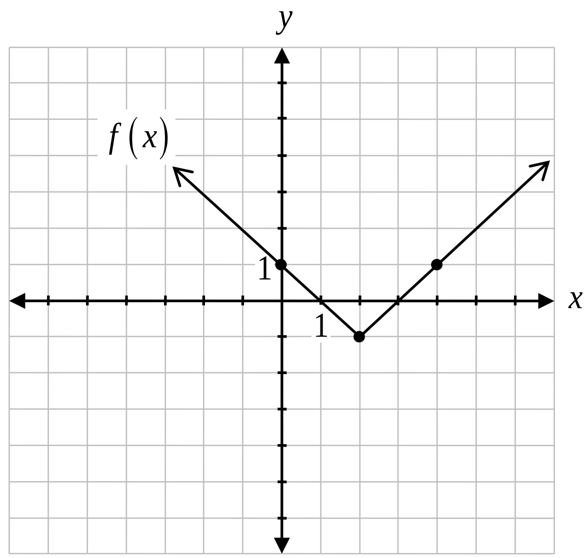
Describe how Thomas should have stated the solution.

Solution

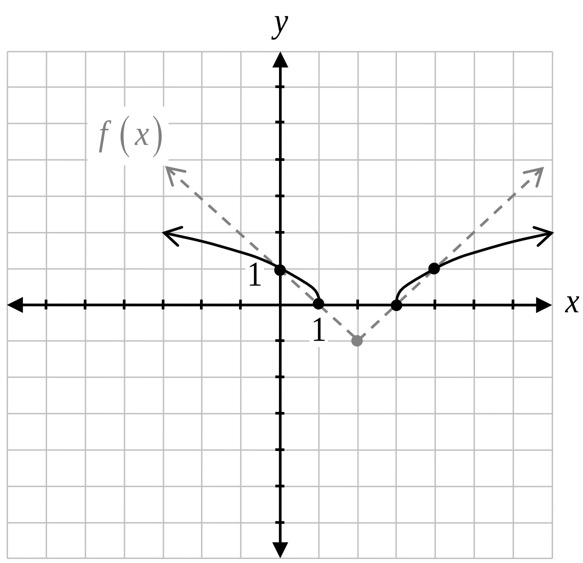
He stated his solution as a coordinate point; his solution should have only been the value of *x*.

1 mark

Given the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$.



Solution



1 mark for restricting domain
½ mark for shape between both pairs of invariant points
½ mark for shape above both pairs of invariant points

2 marks

Question 12 R11

When a polynomial, $P(x)$, is divided by $(x-2)$ the resulting equation is

$$\frac{P(x)}{x-2} = x^2 - x + 1 + \frac{3}{x-2}.$$

a) Explain why $x-2$ is not a factor of $P(x)$.

b) Determine the equation for the polynomial function $P(x)$.

Solution

a) There is a remainder when $P(x)$ is divided by $x-2$. **1 mark**

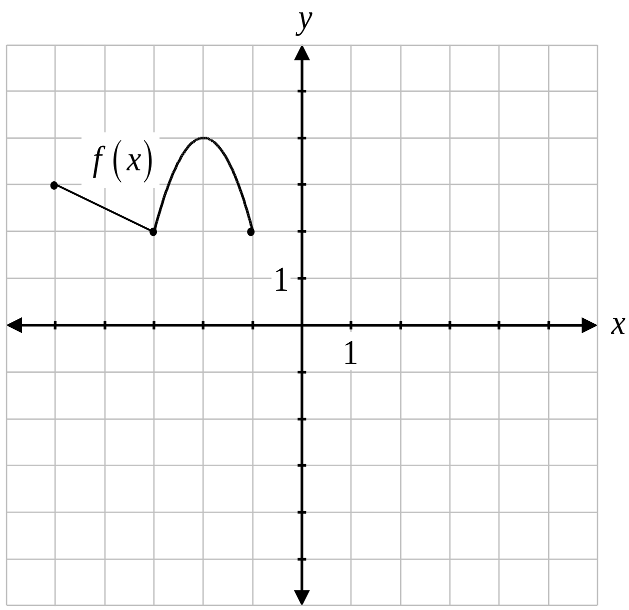
b) $P(x) = \underline{(x-2)(x^2 - x + 1) + 3}$ **1 mark**

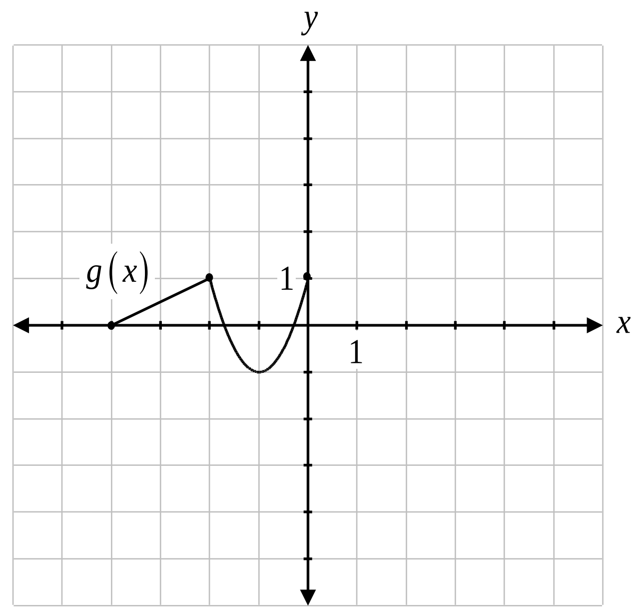
or

$P(x) = \underline{x^3 - 3x^2 + 3x + 1}$

Question 13 R2, R5

Determine the equation for $g(x)$ in terms of $f(x)$.





Solution

$g(x) = \underline{-f(x-1)+3}$

1 mark for vertical reflection
1 mark for horizontal translation
1 mark for vertical translation

3 marks

Question 14 P4

Explain why the binomial expansion of $(2x+y)^9$ does not have a middle term.

Solution

The expansion contains $n+1$ terms. Since n equals 9, there are 10 terms, which would not allow for a middle term.

1 mark

Question 15 R8

Using the laws of logarithms, completely expand the expression $\log\left(\frac{5\sqrt{a}}{b^3}\right)$.

Solution

$\log 5 + \frac{1}{2}\log a - 3\log b$	1 mark for product law 1 mark for power law (½ mark for each) 1 mark for quotient law

3 marks

Answer Key for Selected Response Questions

Question	Answer	Learning Outcome
16	D	T1
17	B	R12
18	A	R7
19	C	T1
20	D	P2
21	B	R14
22	B	R3
23	C	T6
24	A	R9

Question 25 T3

Evaluate the following expression.

$$\tan\left(\frac{2\pi}{3}\right)\csc\left(\frac{-2\pi}{3}\right)+\cos(3\pi)$$

Solution

$$\left(-\sqrt{3}\right)\left(-\frac{2}{\sqrt{3}}\right)+(-1)$$
$$2-1$$
$$1$$

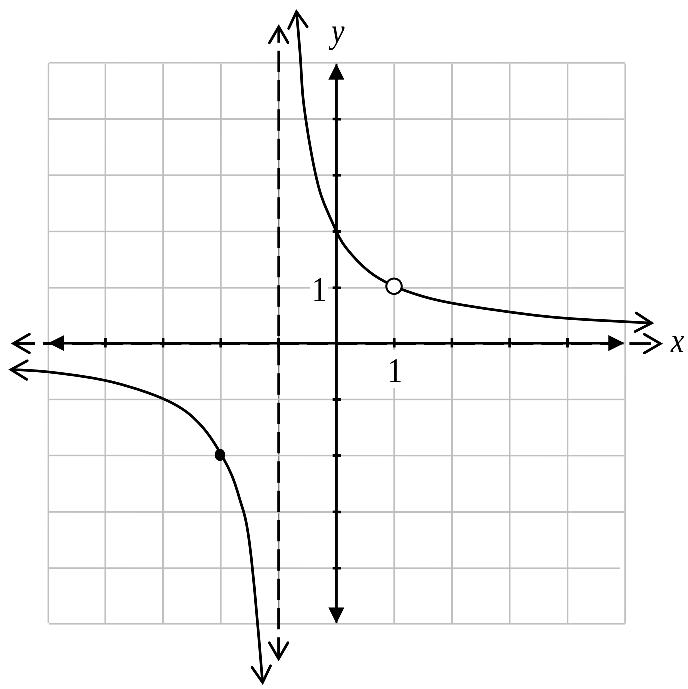
1 mark for $\tan\left(\frac{2\pi}{3}\right)$ (½ mark for quadrant, ½ mark for value)

1 mark for $\csc\left(-\frac{2\pi}{3}\right)$ (½ mark for quadrant, ½ mark for value)

1 mark for $\cos(3\pi)$

3 marks

State the range of the graph below.



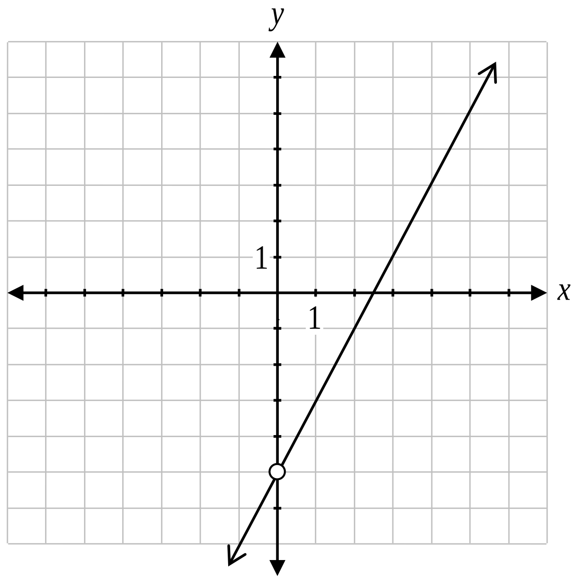
Solution

Range: $\{y \in \mathbb{R},\ y \neq 0 \text{ and } y \neq 1\}$ 1 mark (½ mark for $y \neq 0$, ½ mark for $y \neq 1$)

1 mark

Sketch the graph of the function $f(x) = \dfrac{2x^2 - 5x}{x}$.

Solution



1 mark for point of discontinuity (hole) at $(0, -5)$
(½ mark for $x = 0$, ½ mark for $y = -5$)
1 mark for shape of a linear function

2 marks

Question 28 R12

State a possible value of n if the polynomial function $P(x) = (x-1)^2(x+2)^n$ has a range of $[0, \infty)$.

Solution

$n = 2$ **1 mark**

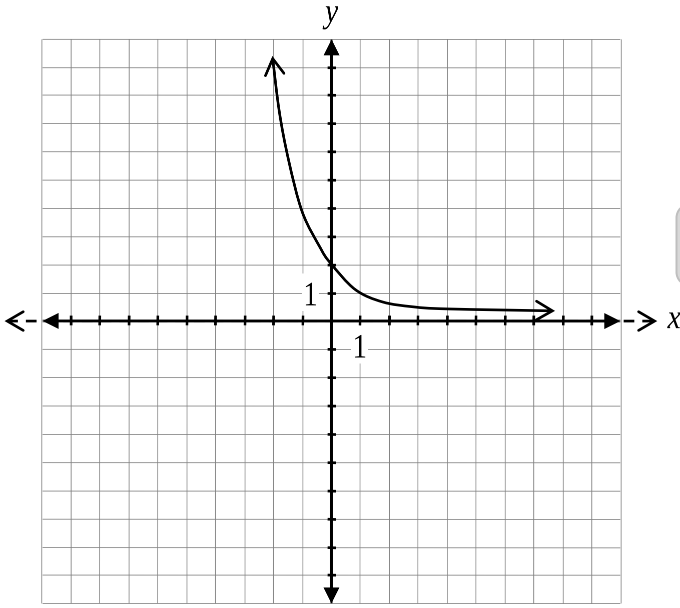
Note(s):

- Accept any even positive value of n, including zero.

Question 29 R9

Sketch the graph of $y = \left(\frac{1}{2}\right)^{x-1}$.

Solution



1 mark for decreasing exponential function
1 mark for horizontal translation

2 marks

Question 30 R10

Solve.

$$\log_x 27 = 3$$

Solution

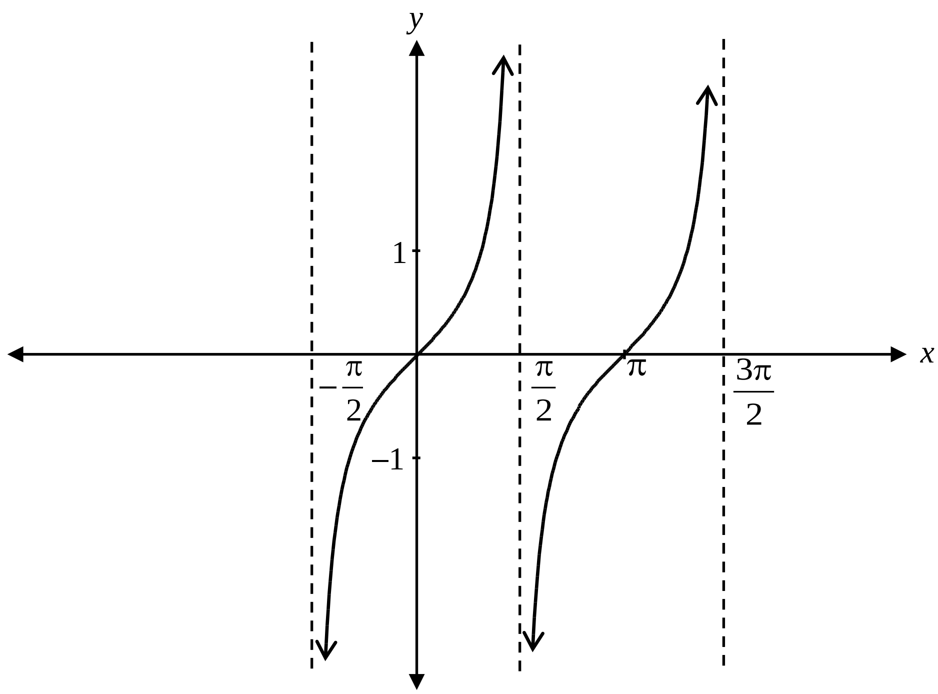
$x^3 = 27$ 1 mark for exponential form

$x = 3$

1 mark

Sketch at least two periods of the graph $y = \tan x$.

Solution



1 mark for increasing trigonometric function

1 mark for asymptotic behaviour approaching $x = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$

2 marks