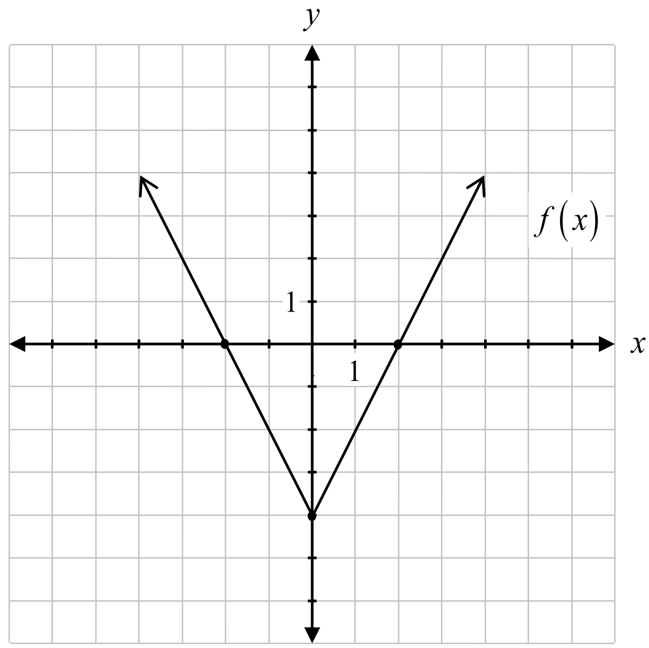
Question 32 R1

Given the graph of $f(x)$, state the domain of $\frac{1}{f(x)}$.

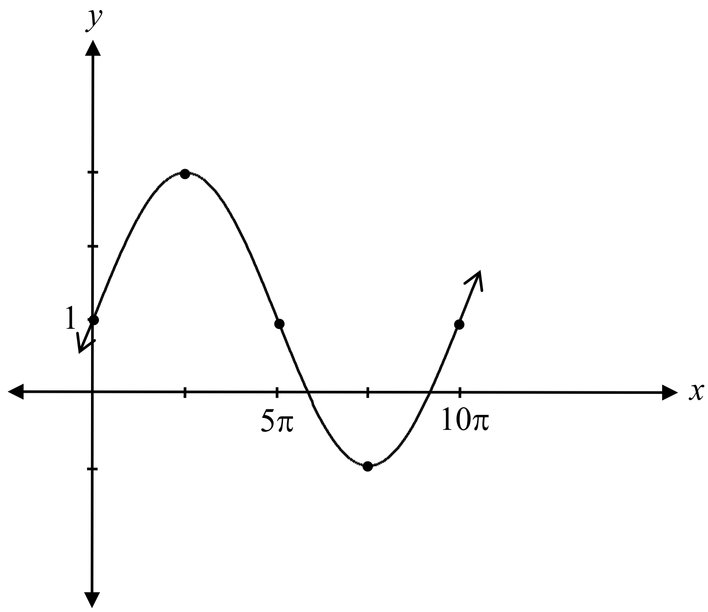


Solution

Domain: $\{x \in \mathbb{R},\ x \neq \pm 2\}$ 1 mark (½ mark for $x \neq 2$, ½ mark for $x \neq -2$)

1 mark

Determine the values of A, B, and D of the sinusoidal function in the form $y = \mathrm{A}\sin(\mathrm{B}x) + \mathrm{D}$.



Solution

A = $\underline{\quad 2 \quad}$ 1 mark for A

B = $\underline{\quad \frac{1}{5} \quad}$ 1 mark for B

D = $\underline{\quad 1 \quad}$ 1 mark for D

3 marks

Question 34 T2

Determine if the point $\left(-\frac{\sqrt{7}}{5}, \frac{2}{5}\right)$ is on the unit circle.

Justify your answer.

Solution

$$x^2 + y^2 = 1$$

$$\text{Left-hand side} = \left(-\frac{\sqrt{7}}{5}\right)^2 + \left(\frac{2}{5}\right)^2$$

$$= \frac{7}{25} + \frac{4}{25}$$

$$= \frac{11}{25}$$

$$\frac{11}{25} \neq 1$$

$\therefore$ not on the unit circle

1 mark for justification

1 mark

Solve, algebraically.

$$\frac{{}_nC_5}{{}_nC_4} = 6$$

Solution

$$\frac{\dfrac{n!}{(n-5)!5!}}{\dfrac{n!}{(n-4)!4!}} = 6$$

½ mark for substitution into equation

$$\frac{n!(n-4)!4!}{n!(n-5)!5!} = 6$$

$$\frac{\cancel{n!}\,(n-4)\,\cancel{(n-5)!}\,\cancel{4!}}{\cancel{n!}\,\cancel{(n-5)!}\,5 \bullet \cancel{4!}} = 6$$

1 mark for factorial expansion
(½ mark for numerical factors; ½ mark for factors with variables)
1 mark for simplification of factorials
(½ mark for numerical factors; ½ mark for factors with variables)

$$\frac{n-4}{5} = 6$$

$$n - 4 = 30$$

$$n = 34$$

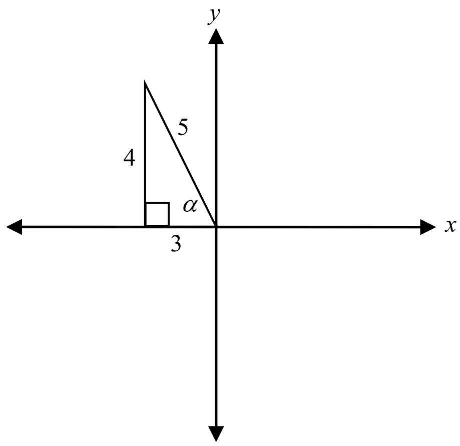
½ mark for solving for n

3 marks

Question 36 T6

Given $\sin\alpha = \frac{4}{5}$, where α is in quadrant II, determine the exact value of $\sin 2\alpha$.

Solution



$$x^2 + y^2 = r^2$$
$$x^2 + 16 = 25$$
$$x^2 = 9$$
$$x = \pm 3$$
$$x = -3$$

$\cos\alpha = -\frac{3}{5}$

½ mark for value of x
½ mark for $\cos\alpha$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$
$$= 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right)$$
$$= -\frac{24}{25}$$

1 mark for substitution into correct identity

2 marks

Note(s):

- Accept any of the following values for x: $x = \pm 3$; $x = -3$; or $x = 3$.

Given the functions $f(x) = x + 1$ and $g(x) = \sqrt{x}$,

a) determine the equation of $g\left(f(x)\right)$.

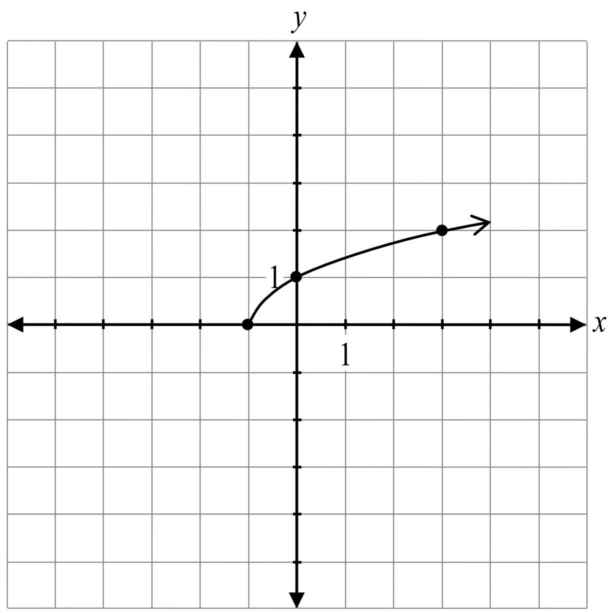
b) sketch the graph of $g\left(f(x)\right)$.

Solution

a) $g\left(f(x)\right) = \sqrt{x+1}$

1 mark

b)



1 mark for domain of $g\left(f(x)\right)$

1 mark for shape consistent with $g\left(f(x)\right)$

2 marks

Question 38 T4

Steve is asked to determine an equation with a larger period than the period of the graph of $y = \cos(2x)$.

Justify why Steve's answer of $y = \cos(6x)$ is incorrect.

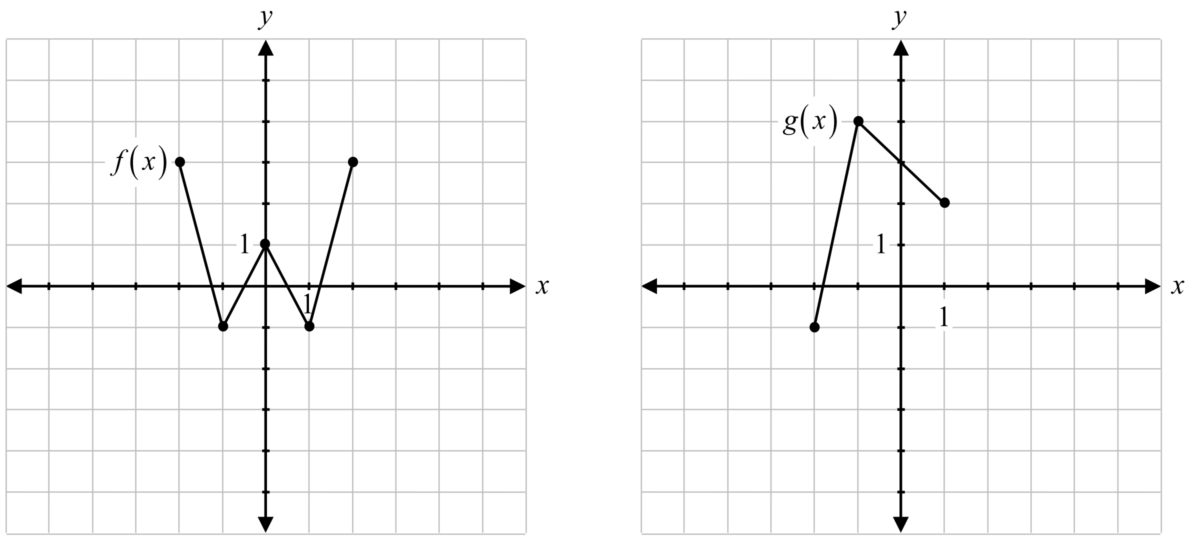
Solution

Steve's equation needs to have a value of $|b|$ less than 2. **1 mark**

or

Steve's graph would have a period of $\frac{2\pi}{6} = \frac{\pi}{3}$, which is smaller than $\frac{2\pi}{2} = \pi$, the period of the given graph.

Given the graphs of $f(x)$ and $g(x)$,



a) determine the value of $(f \bullet g)(-1)$.

b) determine the value of $g(f(0))$.

Solution

a) $(f \bullet g)(-1) = (-1)(4)$
$= -4$

1 mark for value of $(f \bullet g)(-1)$

1 mark

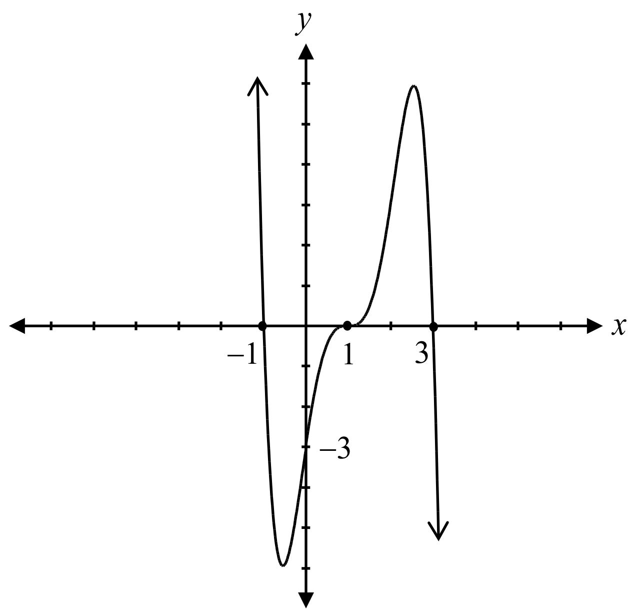
b) $f(0) = 1$
$g(f(0)) = 2$

½ mark for $f(0)$
½ mark for $g(f(0))$ consistent with $f(0)$ value

1 mark

Sketch the graph of $P(x) = -(x-1)^3(x-3)(x+1)$.

Solution



1 mark for x-intercepts
½ mark for y-intercept
1 mark for multiplicity (degree 3 at $x = 1$)
½ mark for end behaviour

3 marks

Question 41 T2, T3

The point $\left(-\sqrt{3},1\right)$ is on the terminal arm of an angle $\theta,$ in standard position.

a) Determine $\tan\theta$.

b) Determine a possible value of $\theta,$ in radians.

Solution

a) $\tan\theta = -\dfrac{1}{\sqrt{3}}$

1 mark

b) $\theta = \dfrac{5\pi}{6}$

1 mark

Describe the transformation used to obtain the graph of $y = \log_5 x$ given the graph of $y = 5^x$.

Solution

The graph of $y = \log_5 x$ is obtained by reflecting the graph of $y = 5^x$ over the line $y = x$.

1 mark

or

The graph of $y = \log_5 x$ is the inverse of $y = 5^x$.

Question 43 T5

Solve $\sin\theta = -\frac{\sqrt{3}}{2}$, where $\theta \in \mathbb{R}$.

Solution

$\theta = \frac{4\pi}{3}, \frac{5\pi}{3}$ — 1 mark for values of θ (½ mark for each value)

$$\theta = \begin{Bmatrix} \frac{4\pi}{3} + 2k\pi,\ k \in \mathbb{Z} \\ \frac{5\pi}{3} + 2k\pi,\ k \in \mathbb{Z} \end{Bmatrix}$$

1 mark for general solution

2 marks

or

$\theta = 240°, 300°$

$$\theta = \begin{Bmatrix} 240° + 360°k,\ k \in \mathbb{Z} \\ 300° + 360°k,\ k \in \mathbb{Z} \end{Bmatrix}$$

Given that the point (a, b) is on the graph of $f(x)$, describe how you would determine the corresponding point on the graph of $y = \sqrt{f(x)}$.

Solution

The value of *a* stays the same, square root the value of *b*.

1 mark

Question 45 T6

Evaluate.

$$\cos\left(\frac{\pi}{20}\right)\cos\left(\frac{\pi}{5}\right)-\sin\left(\frac{\pi}{20}\right)\sin\left(\frac{\pi}{5}\right)$$

Solution

$\cos\left(\frac{\pi}{20}+\frac{\pi}{5}\right)$ ½ mark for substitution of an appropriate identity

$\cos\left(\frac{\pi}{20}+\frac{4\pi}{20}\right)$

$\cos\left(\frac{5\pi}{20}\right)$

$\cos\left(\frac{\pi}{4}\right)$

$\frac{\sqrt{2}}{2}$ ½ mark for exact value

1 mark

Question 46 R2, R5

Describe the transformations used to obtain the graph of the function $y = f(-x + 6) - 8$ from the graph of $y = f(x)$.

Solution

Reflect the graph of $y = f(x)$ over the y-axis and then translate 6 units right and 8 units down.

1 mark for horizontal reflection
1 mark for horizontal translation
1 mark for vertical translation

3 marks

Note(s):

- Deduct 1 mark if correct transformations are given in the wrong order.

Question 47 R14

State the equations of all the asymptotes of the function, $y = \dfrac{1}{3x+1}$.

Solution

$y = 0$ 1 mark for horizontal asymptote

$x = -\dfrac{1}{3}$ 1 mark for vertical asymptote

2 marks

Determine the zeros of the polynomial function $P(x) = 2x^3 + 5x^2 - 4x - 3$.

Solution

$P(1) = 2(1)^3 + 5(1)^2 - 4(1) - 3$ 1 mark for identifying one possible value of *x*

$P(1) = 0$

$(x-1)$ is a factor

$$\begin{array}{r|rrrr} 1 & 2 & 5 & -4 & -3 \\ & \downarrow & 2 & 7 & 3 \\ \hline & 2 & 7 & 3 & 0 \end{array}$$

1 mark for synthetic division (or for any equivalent strategy)

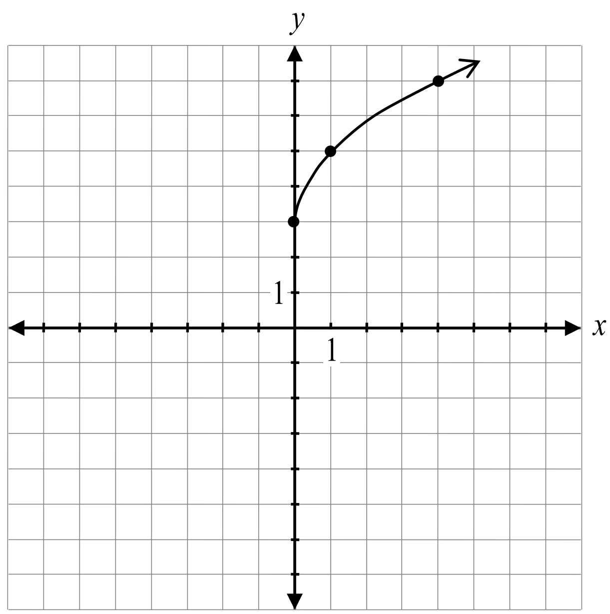
$P(x) = (x-1)\left(2x^2 + 7x + 3\right)$ ½ mark for consistent factors

$0 = (x-1)(2x+1)(x+3)$

$x = 1 \quad x = -\frac{1}{2} \quad x = -3$ ½ mark for all zeros

3 marks

Determine the equation of the radical function represented by the graph.



Solution

$y = 2\sqrt{x} + 3$ 1 mark for vertical stretch
1 mark for vertical translation

or

$y = \sqrt{4x} + 3$ 1 mark for horizontal stretch
1 mark for vertical translation

2 marks