

3.4

Graphing Polynomials

Chapter 3: POLYNOMIAL FUNCTIONS
3.4 – Equations and Graphs of Polynomial Functions

Example #1

a) Determine the **zeroes** of the following cubic function

$$f(x) = x^3 - x^2 - 4x + 4$$

$$0 = x^3 - x^2 - 4x + 4$$

$$0 = (x-1)(x-2)(x+2)$$

xints: 1, 2, -2

b) Determine the **y – intercept** of the function

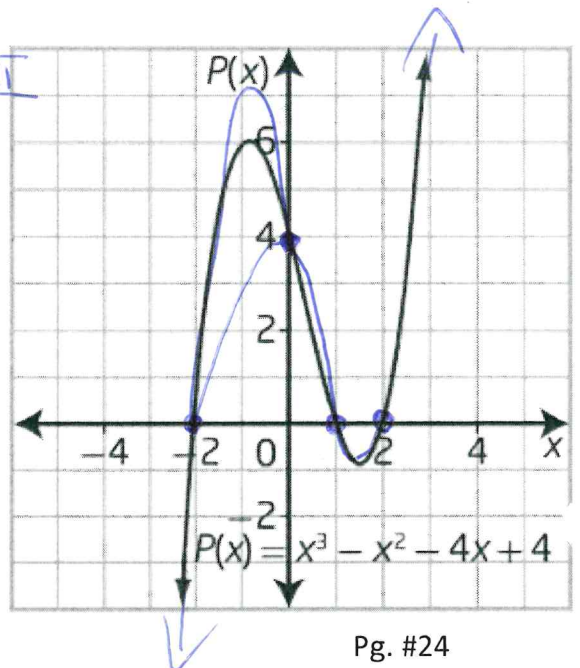
set $x = 0$

$$f(0) = 0^3 - 0^2 - 4(0) + 4 = 4$$

c) Summarize what we know about this function

Degree	3
Leading Coefficient	1
End Behaviour	up into Q I down into Q III
Zeroes	1, 2, -2
y – Intercept	4
Intervals (sign Diagram)	

d) Look at the sketch of the polynomial function.



Multiplicity of a Zero

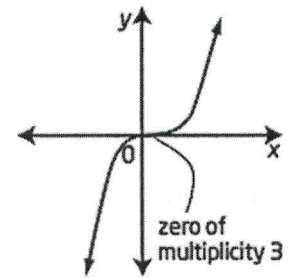
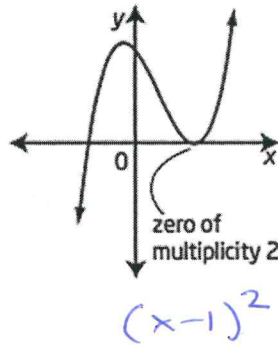
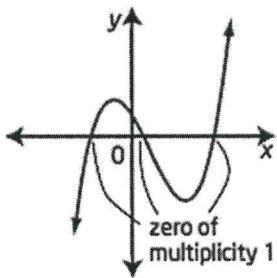
If $P(x)$ has a factor $(x - a)$ that is repeated n times, we say that $x = a$ is a zero of multiplicity n .

For example:

$$y = (x + 1)^2(x - 2)(x - 4)^3 \quad \left\{ \begin{array}{l} x = -1 \text{ is a zero of multiplicity } \underline{2} \\ x = 2 \text{ is a zero of multiplicity } \underline{1} \\ x = 4 \text{ is a zero of multiplicity } \underline{3} \end{array} \right.$$

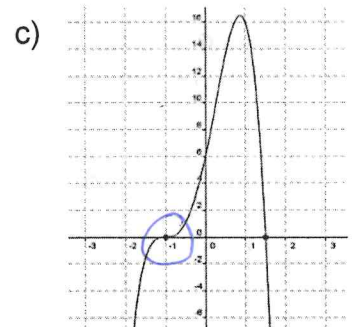
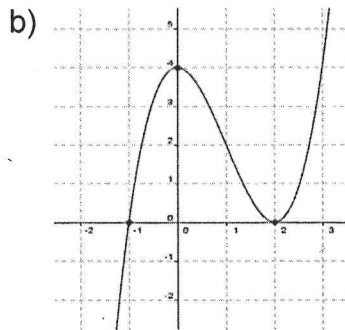
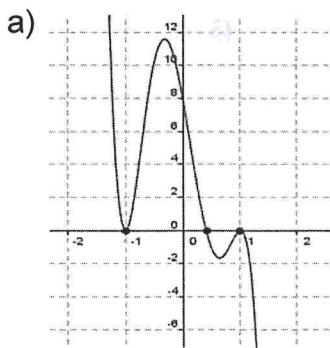
Multiplicity is the number of times the zero of a polynomial occurs. (The number of times a factor is repeated)

The shape of the graph of a function close to a zero (x - intercept) depends on its multiplicity.



Example #2

Determine the **zeroes** of each polynomial function and their **multiplicities** from the given graphs.



Zero	Multiplicity
-1	2
0.5	1
1	2

Zero	Multiplicity
-1	1
2	2

Zero	Multiplicity
-1	3

Example #3

Sketch the following graphs:

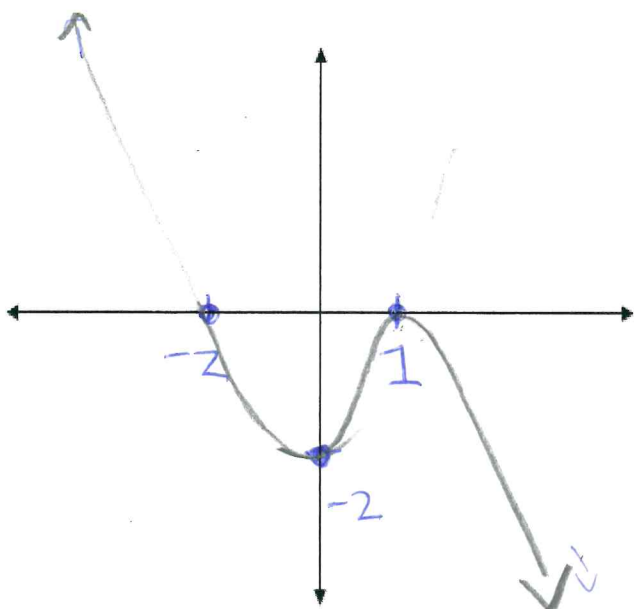
a) $y = -(x - 1)^2(x + 2)$

$y = -(x-1)(x-1)(x+2)$

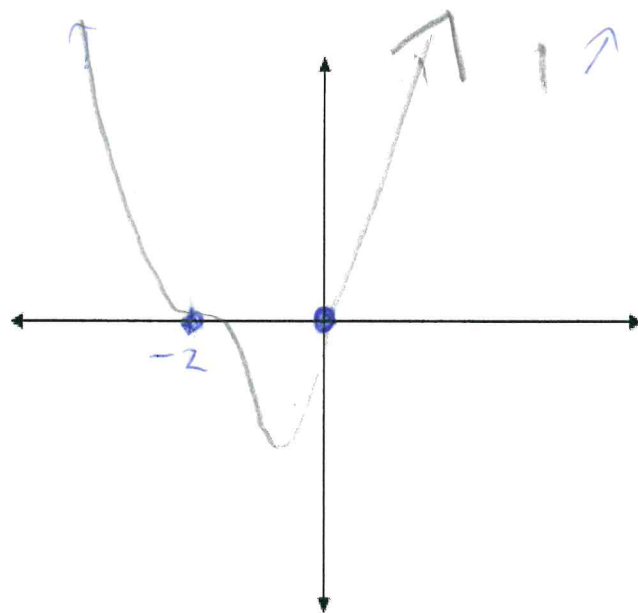
Degree	3
Leading Coefficient	← <u>negative</u>
End Behaviour	up into Q II down into Q IV
Zeroes	1 and -2
y - intercept	
Intervals (Sign Diagram)	

b) $y = (x)(x + 2)^3$

Degree	4
Leading Coefficient	<u>positive</u>
End Behaviour	up into Q I and Q II
Zeroes	-2 and 0
y - intercept	
Intervals (Sign Diagram)	



y int:
 $y = -(0-1)^2(0+2)$
 $y = -(1)(2)$
 $y = -2$

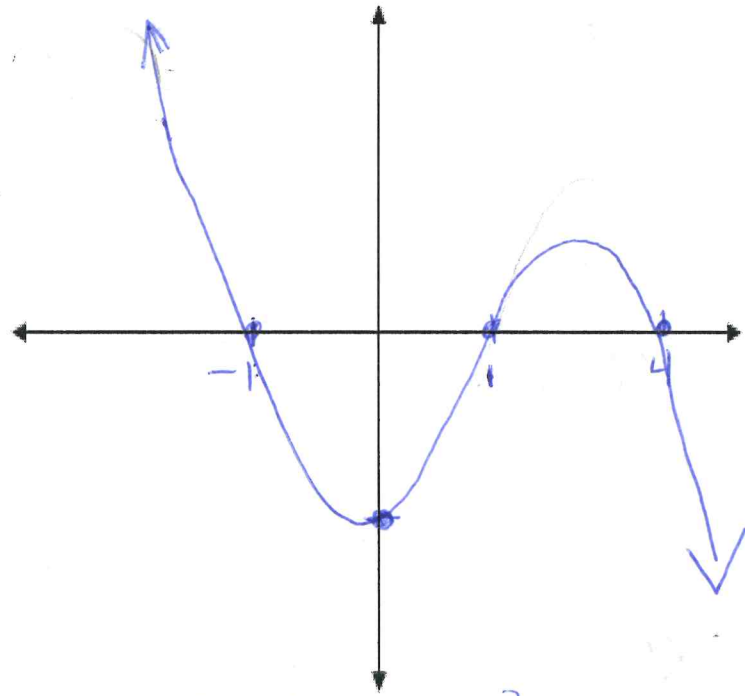


$y = x(x+2)^3$
 $y = 0(0+2)^3$
 $y = 0$

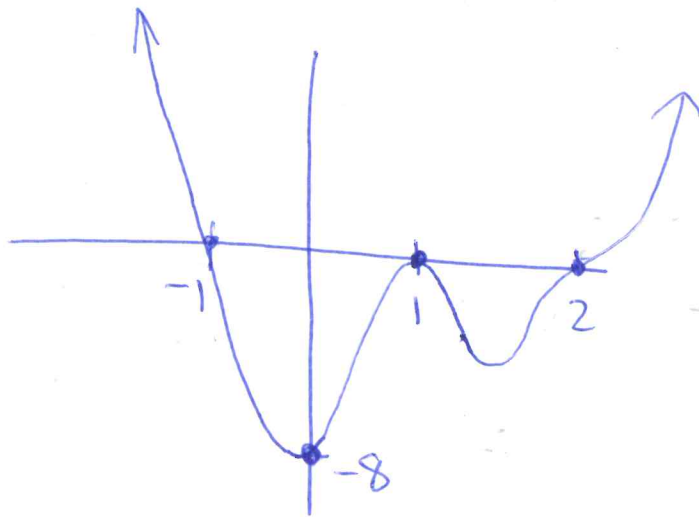
Example #4Sketch the graph of $P(x) = -x^3 + 4x^2 + x - 4$

$$P(0) = -1(-4)(-1)(1)$$

$$a) \quad P(x) = -(x-4)(x-1)(x+1)$$



$$b) \quad P(x) = (x-1)^2(x+1)(x-2)^3$$

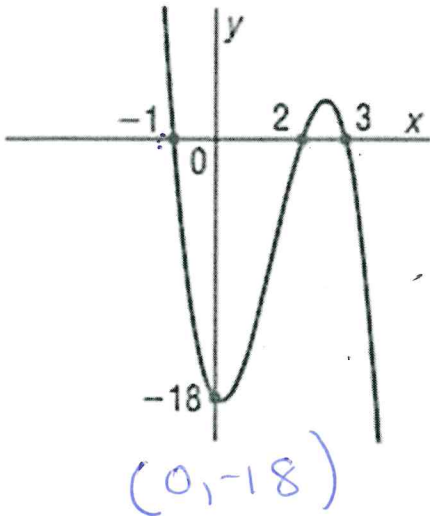


$$P(0) = (-1)^2(1)(-2)^3$$

$$= 1(1)(-8)$$

Example #5

Determine the **equation** for the following polynomial function given the graph below.



$$y = a(x+1)(x-2)(x-3)$$

$$-18 = a(0+1)(0-2)(0-3)$$

$$-18 = a(1)(-2)(-3)$$

$$-18 = a(6)$$

$$-3 = a$$

$$y = -3(x+1)(x-2)(x-3)$$

Example #6

The zeroes of a quartic function are at -2, -1, and 3, with multiplicities of 1, 1, and 2 respectively.

Determine the **equation** of the function that satisfies this condition and passes through the point (1, 24).

$$y = a(x+2)^1(x+1)^1(x-3)^2$$

$$24 = a(1+2)(1+1)(1-3)^2$$

$$24 = a(3)(2)(4)$$

$$24 = 24a$$

$$1 = a$$

$$y = (x+2)(x+1)(x-3)^2$$