P. 12 SOLUTIONS Continuity

Monday, February 15, 2021 11:23 AM

(1) $f(x) = \begin{cases} x+7, & x<2 \\ 9, & x=2 \\ 3x+3, & x>0 \end{cases}$

(I) $f(2) = 9$

(II) $\lim\limits_{x\to 2^+} f(x) = 9 = \lim\limits_{x\to 2^-} f(x)$ $\therefore$ $f(x)$ is continuous @ $x=2$

(III) $\lim\limits_{x\to 2} f(x) = f(2)$

(2) $g(x) = \begin{cases} 4x^2-2x, & x<3 \\ 10x-1, & x=3 \\ 30, & x>3 \end{cases}$

(I) $f(3) = 30$

II $\lim\limits_{x\to 3^+} f(x) = 30$ $\lim\limits_{x\to 3^-} f(x) = 29$

$\lim\limits_{x\to 3} f(x)$ DNE $\therefore$ $f(x)$ is not continuous at $x=3$

(3) $f(x) = \begin{cases} 5x+7, & x<3 \\ 7x+1, & x>3 \end{cases}$

$f(3)$ is not defined

$\therefore$ $f(x)$ is discontinuous @ $x=3$

(4) $f(x) = \sec x$

$f(x) = \frac{1}{}$ $\cos x \neq 0$

$\cos x \qquad x \neq -\frac{\pi}{2}, \frac{\pi}{2}$

$\therefore f(x) = \sec x$ is discontinuous @ $x = \frac{\pi}{2} + k\pi \quad k \in I$

(5) $f(x) = \sec x$ is not defined at $x = \pm\frac{\pi}{2}$

(6) $f(x) = \csc x \qquad \sin x \neq 0$

$f(x) = \frac{1}{\sin x} \qquad x \neq 0$

$\therefore f(x)$ is discontinous @ $x = 0$

(7) $f(x) = \begin{cases} 3x^2 - 11x - 4, & x \leq 4 \\ ax^2 - 2x - 1, & x > 4 \end{cases}$

$f(4) = 3(4)^2 - 11(4) - 4$
$= 0$

$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x)$

$0 = a(4)^2 - 2(4) - 1$

$0 = 16a - 9$

$\frac{9}{16} = a$

8) $f(x) = \begin{cases} -6x - 12, & x < -3 \\ a^2 - 5a, & x = -3 \\ 6, & x > -3 \end{cases}$

$f(-3) = a^2 - 5a$

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = 6$

$\therefore \lim_{x \to 3} f(x) = 6$

Since $\lim_{x \to 3} f(x) = f(-3)$

It follows that

$$6 = a^2 - 5a$$
$$0 = a^2 - 5a - 6$$
$$0 = (a-6)(a+1)$$
$$a = 6 \qquad a = -1$$

(9) $f(x) = \dfrac{x^2+5x-24}{x^2-x-6}$

$= \dfrac{(x+8)(x-3)}{(x-3)(x+2)}$

$x = 3$
removable discontinuity
(A.K.A: Point discontinuity)

$x = -2$
infinite discontinuity
(aka: Asymptotic)

(10) $f(x) = \dfrac{1}{|x-2|}$

discontinuous @ $x = 2$
$f(2)$ is not defined.

(11) $f(x) = \dfrac{x^2-1}{x+1}$

$f(x) = \dfrac{(x-1)\cancel{(x+1)}}{\cancel{x+1}}$

Point discontinuity at $x = -1$

(12) $f(x) = |x+2|$

<u>continuous</u>

(but not differentiable @ $x = -2$)

(13) $f(x) = \{ \; 1-x \;, \; x < 1$

Both $y = 1-x$ and $y = x$

(13) $f(x) = \begin{cases} 1-x, & x<1 \\ x, & x \geq 1 \end{cases}$

Both $y = 1-x$ and $y=x$ are continuous. $\therefore$ the only discontinuity could occur at $x=1$

$f(1) = 1$

$\lim_{x\to 1^+} f(x) = 1 \qquad \lim_{x\to 1^-} f(x) = 0$

$\therefore \lim_{x\to 1} f(x)$ DNE and $f(x)$ is discontinuous @ $x=1$ (Jump!)

(14) $f(x) = e^x \sin x$

Both $y=e^x$ and $y=\sin x$ are continuous functions. $\therefore$ the product of these continuous functions also results in a continuous function.

(15) $f(x) = \ln x$

This function is continuous on its domain $(0, \infty)$

(16) $f(x) = \begin{cases} bx^2-1, & x<1 \\ x, & x \geq 1 \end{cases}$

$f(1) = 1$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x)$

$1 = b-1$

$2 = b$

Note

$\lim_{x\to 1} f(x) = f(1)$

(17) $f(x) = \begin{cases} -2x+b, & x<1 \\ \ln x, & x \geq 1 \end{cases}$

$f(1) = \ln 1$
$= 0$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x)$

$0 = -2+b$

$2 = b$

Note:

$\lim_{x \to 1} f(x) = f(1)$

(18) $f(x) = \begin{cases} 2x^2 - x - b, & x<1 \\ 2e^{x-1}, & x \geq 1 \end{cases}$

$f(1) = 2e^{1-1}$
$= 2$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x)$

$2 = 2-1-b$

$2 = 1-b$

$-1 = b$

note:

$\lim_{x \to 1} f(x) = f(1)$