MPC40S Date: Answer Key

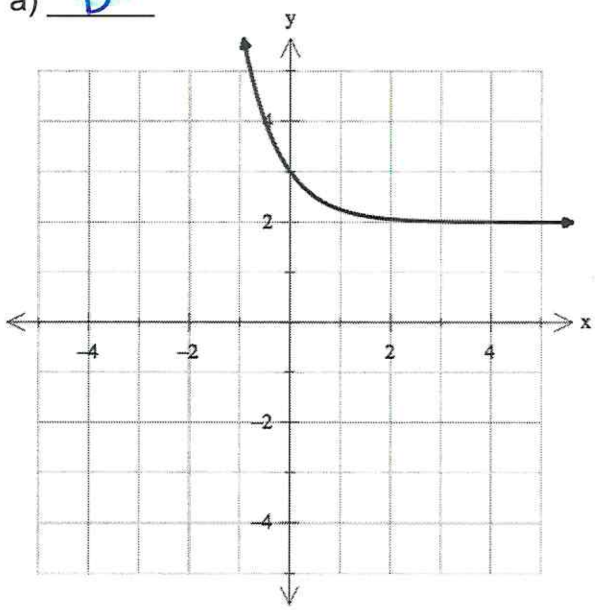
Review – CHAPTER 7/8 Questions

7.1/7.2 – Transformations of Exponential Functions

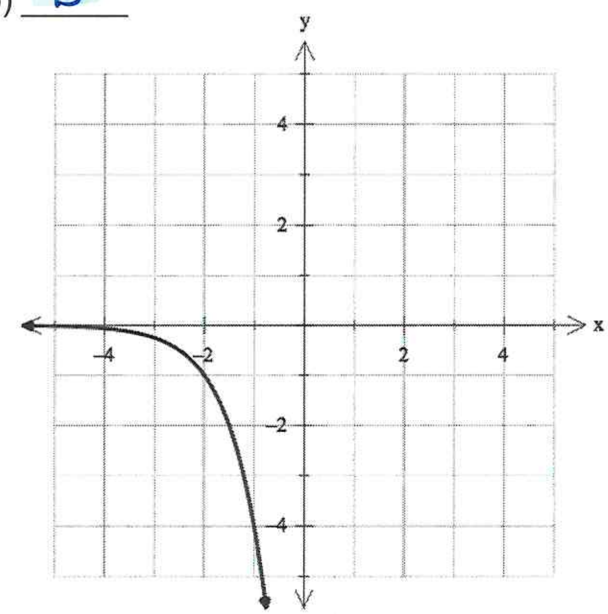
1. Match the following exponential equation with the corresponding graph.

A) $y = 4^{-(x+2)}$ B) $y = -4^{x+2}$ C) $y = -4^{x} + 2$ D) $y = 4^{-x} + 2$

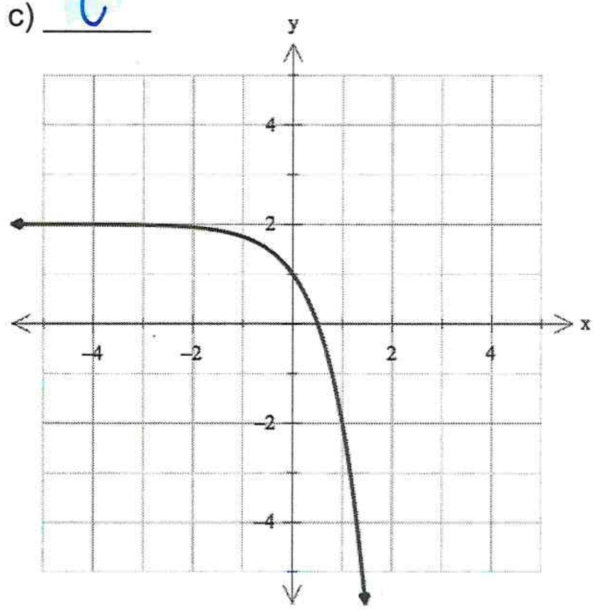
a) D



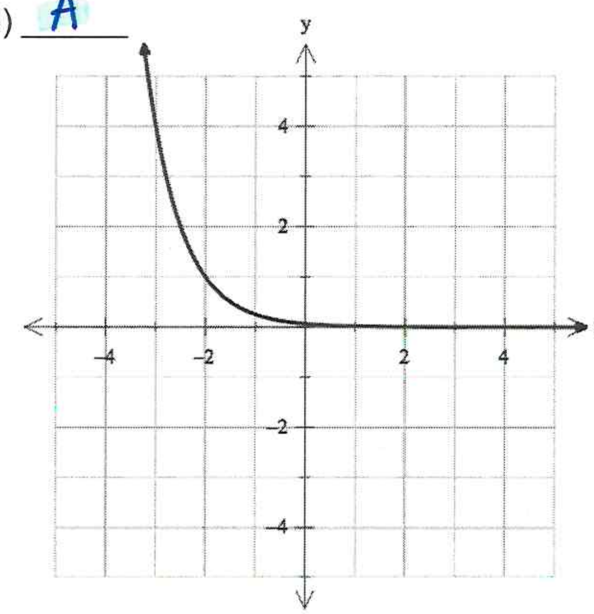
b) B



c) C



d) A



2. For each of the following exponential functions, determine each of the following characteristics.

a) $f(x) = -6(3^x) + 2$

i) Domain: $x \in \mathbb{R}$ ii) Range: $y < 2$

iii) Horizontal Asymptote: $y = 2$

iv) x – intercept

$0 = -6(3^x) + 2$

$-2 = -6(3)^x$

$\frac{1}{3} = 3^x$

$3^{-1} = 3^x$

$\therefore -1 = x$

@ $(-1, 0)$

v) y – intercept

$y = -6(3^0) + 2$

$y = -6(1) + 2$

$y = -6 + 2$

$y = -4$

@ $(0, -4)$

b) $f(x) = 2\left(\frac{1}{3}\right)^x$

i) Domain: $(-\infty, \infty)$ ii) Range: $(0, \infty)$

iii) Horizontal Asymptote: $y = 0$

iv) x – intercept

$0 = 2\left(\frac{1}{3}\right)^x$

$0 = \left(\frac{1}{3}\right)^x$

No solution

No x-intercept

v) y – intercept

$y = 2\left(\frac{1}{3}\right)^0$

$y = 2(1)$

$y = 2$

@ $(0, 2)$

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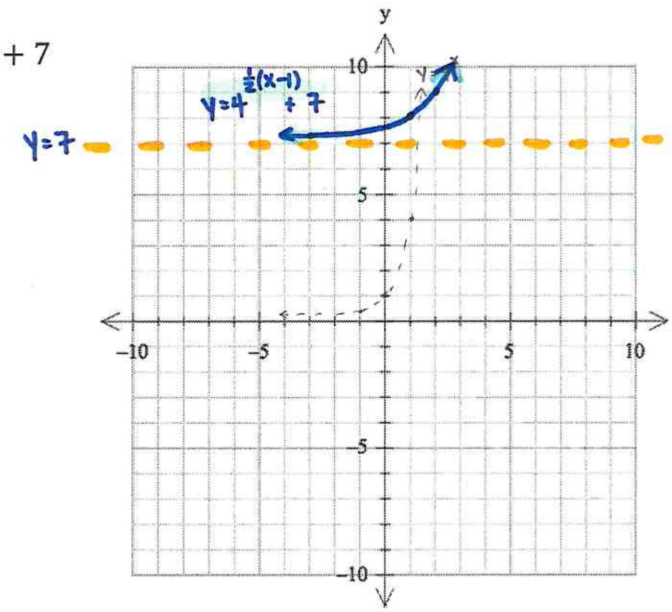
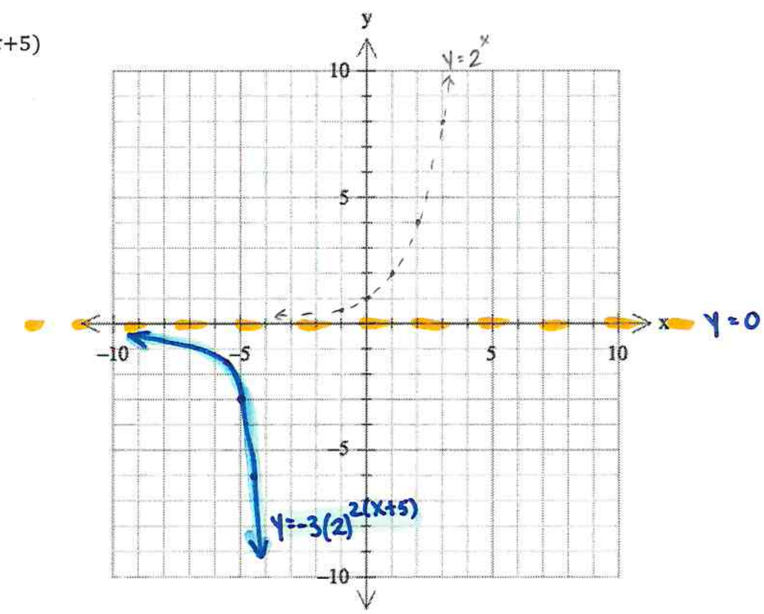
3. **Sketch** the graph of the following functions.

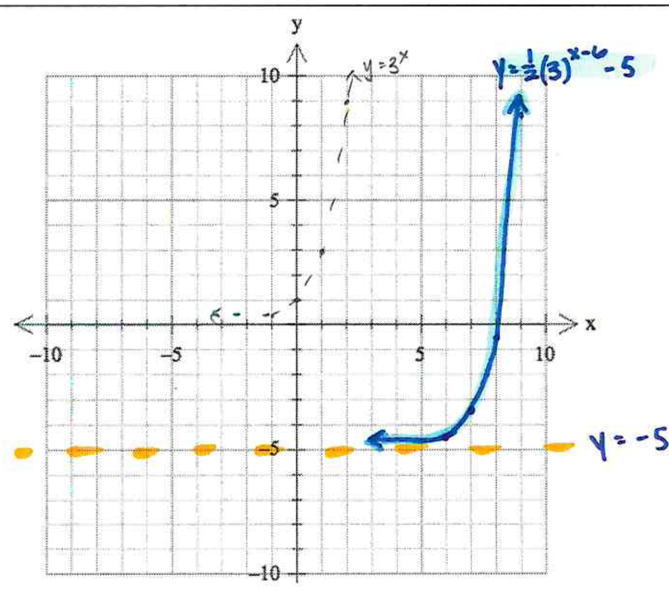
a) $y = 4^{\frac{1}{2}(x-1)} + 7$



b) $y = -3(2)^{2(x+5)}$



c) $y = \frac{1}{2}(3)^{x-6} - 5$



4. Using the information below, write the **equation** for each transformation in the form of $y = a(c)^{b(x-h)} + k$

a) $f(x) = \left(\frac{1}{2}\right)^x$ is stretched vertically by a factor of 3, reflected over the y – axis, translated 4 units left and 3 units down.

$$y = 3\left(\tfrac{1}{2}\right)^{-(x+4)} - 3$$

b) $g(x) = 3^x$ is horizontally stretched by a factor of $\frac{1}{2}$, reflected in the x – axis and translated 7 units up.

$$y = -(3)^{2x} + 7$$

c) $j(x) = 2^x$ and $y = -4j(2(x-3)) + 5$

$$y = -4(2)^{x-3} + 5$$

7.3 – Solving Exponential Equations

1. **Solve** the following equations.

a) $2^{x^2} = 16$

$2^{x^2} = 2^4$

$\therefore x^2 = 4$

$x = \pm\sqrt{4}$

$x = \pm 2$

b) $8^{2x+1} = 64$

$2^{3(2x+1)} = 2^6$

$\therefore 3(2x+1) = 6$

$6x + 3 = 6$

$\frac{6x}{6} = \frac{3}{6}$

$x = 1/2$

c) $\frac{1}{4^{x-2}} = 64$

$4^{-1(x-2)} = 64$

$2^{-2(x-2)} = 2^6$

$\therefore -2(x-2) = 6$

$-2x + 4 = 6$

$\frac{-2x}{-2} = \frac{2}{-2}$

$x = -1$

d) $\frac{2(5)^{2x-9}}{2} = \frac{250}{2}$

$5^{2x-9} = 125$

$5^{2x-9} = 5^3$

$\therefore 2x - 9 = 3$

$\frac{2x}{2} = \frac{12}{2}$

$x = 6$

e) $27^{x+2} = \left(\frac{1}{3}\right)^{3-6x}$

$3^{3(x+2)} = 3^{-1(3-6x)}$

$\therefore 3(x+2) = -1(3-6x)$

$3x + 6 = -3 + 6x$

$\frac{9}{3} = \frac{3x}{3}$

$3 = x$

f) $\left(\frac{1}{6}\right)^{3x-2} = 36^{x+4}$

$6^{-1(3x-2)} = 6^{2(x+4)}$

$\therefore -1(3x-2) = 2(x+4)$

$-3x + 2 = 2x + 8$

$\frac{-5x}{-5} = \frac{6}{-5}$

$x = -6/5$

h) $3^{4x}(3) = 27^{2x}$

$3^{4x}(3^1) = 3^{3(2x)}$

$3^{4x+1} = 3^{6x}$

$\therefore 4x+1 = 6x$

$\frac{1}{2} = \frac{2x}{2}$

$1/2 = x$

i) $2^{x-1} = (128^x)(2^x)$

$2^{x-1} = (2^{7x})(2^x)$

$2^{x-1} = 2^{7x+x}$

$\therefore x-1 = 7x+x$

$\frac{-1}{7} = \frac{7x}{7}$

$-1/7 = x$

j) $2^{x-2} + 3 = 5$

$2^{x-2} = 2$

$\therefore x-2 = 1$

$x = 3$

k) $\left(\frac{1}{5}\right)^{x-3} - 4 = -3$

$5^{-(x-3)} = 1$

$5^{-(x-3)} = 5^0$

$\therefore -(x-3) = 0$

$-x+3 = 0$

$3 = x$

2. A city has 67 000 people. The population decreases by 5% each year. The situation can be represented by the equation $P = 67\,000(0.95)^n$, where P is the population of the city and n is the number of years. Determine the population of the city in 5 years and 3 months.

$P = ?$

$n = 5.25$ years

$P = (67000)(0.95)^{5.25}$

$P = 51\,182.763$

The population is approximately 51 183 people.

8.1 – Understanding Logarithms

1. **Evaluate** each of the following

a) $\log_8 64$ let $y = \log_8 64$ $8^y = 64$ $8^y = 8^2$ $\therefore y = 2$	b) $\log 1000$ let $y = \log_{10} 1000$ $10^y = 1000$ $10^y = 10^3$ $\therefore y = 3$
c) $\log_3 81$ let $y = \log_3 81$ $3^y = 81$ $3^y = 3^4$ $\therefore y = 4$	d) $\log_7 1$ let $y = \log_7 1$ $7^y = 1$ $y = 0$
e) $\log_4 2$ let $y = \log_4 2$ $4^y = 2$ $2^{2y} = 2^1$ $\frac{2y}{2} = \frac{1}{2}$ $y = 1/2$	f) $\log 0.01$ let $y = \log_{10} 0.01$ $10^y = 0.01$ $10^y = 10^{-2}$ $\therefore y = -2$

2. Rewrite each expression in **logarithmic form**.

a) $3^5 = 243$ $\log_3(243) = 5$	b) $16^{\frac{1}{4}} = 2$ $\log_{16}(2) = \frac{1}{4}$	c) $2^{-2} = 0.25$ $\log_2(0.25) = -2$	d) $5^{2m} = n + 4$ $\log_5(n+4) = 2m$

3. Rewrite each expression in **exponential form**.

a) $\log_4 64 = 3$ $4^3 = 64$	b) $\log_4 8 = \frac{3}{2}$ $4^{3/2} = 8$	c) $\log 10\,000 = 4$ $10^4 = 10\,000$	d) $\log_6(x-2) = y$ $6^y = x - 2$

4. **Estimate** each of the following. Round your final answer to the nearest tenth.

a) $\log_3 30$

let $y = \log_3 30$

$3^y = 30$

we know

$3^3 = 27$

$3^4 = 64$

Estimate $\log_3 30 = 3.1$

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b) $\log_5 80$

let $y = \log_5 80$

$5^y = 80$

we know

$5^2 = 25$

$5^3 = 125$

Estimate $\log_5 80 = 2.7$

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c) $\log 25$

let $y = \log 25$

$10^y = 25$

we know

$10^1 = 10$

$10^2 = 100$

Estimate $\log 25 = 1.4$

8.2 – Transformations of Logarithmic Functions

1. For each of the following logarithmic functions, answer each of the following questions.

a) $y = \log_3 x$

i) Complete the following table of values.

x	3	9	27
y	1	2	3

ii) Domain: $x > 0$ Range: $y \in \mathbb{R}$

b) $y = \log_5(x - 2)$

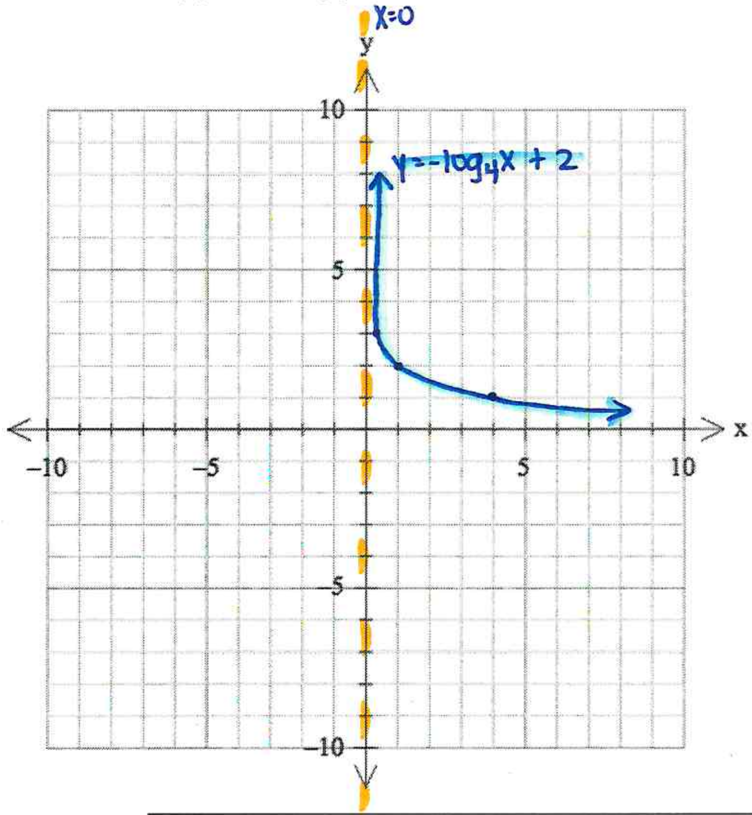
i) Complete the following table of values.

x	7	27	127
y	1	2	3

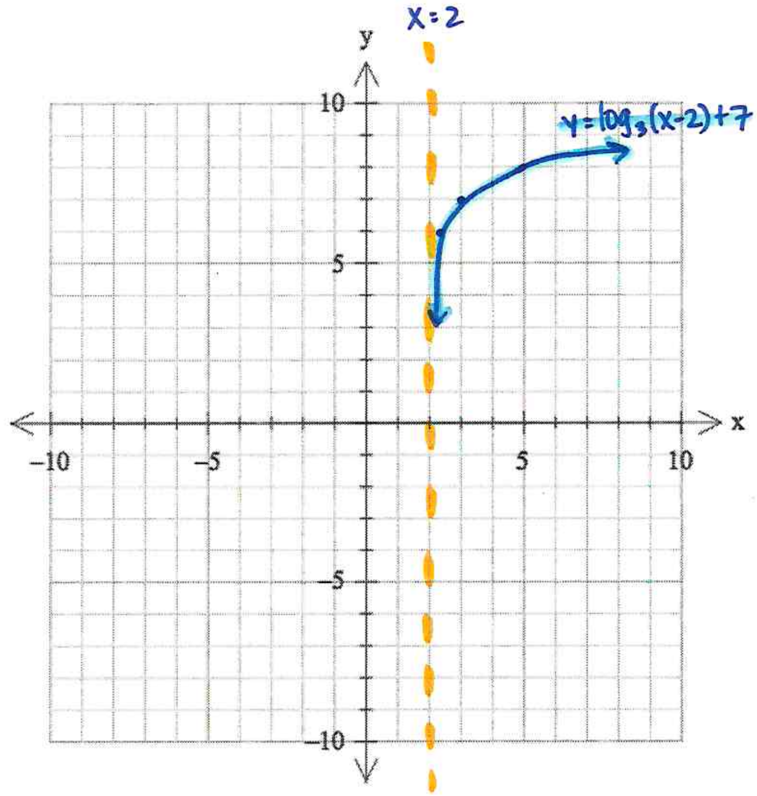
ii) Domain: $(2, \infty)$ Range: $(-\infty, \infty)$

2. **Sketch** the graph of the following logarithmic functions.

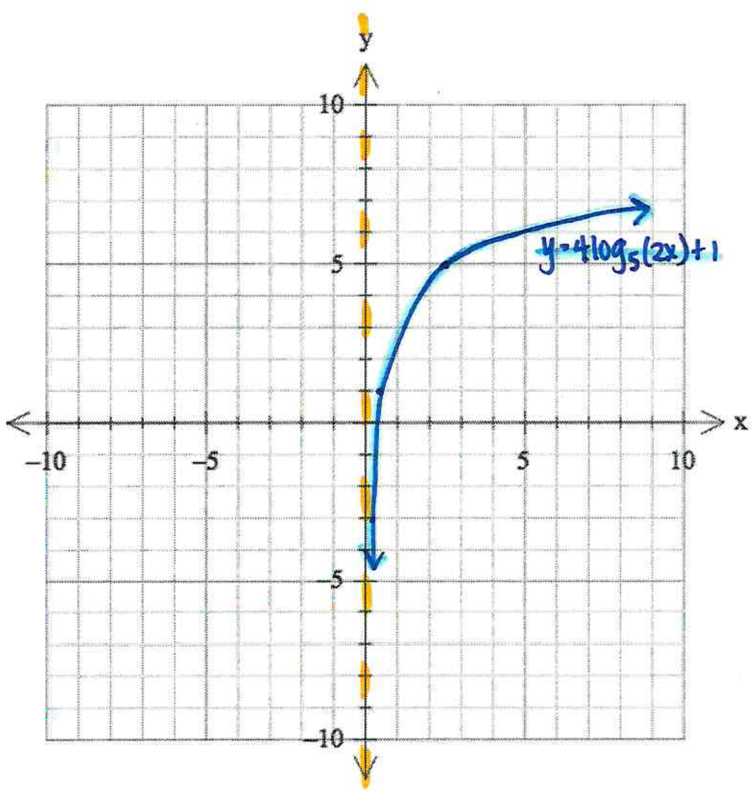
a) $y = -\log_4 x + 2$



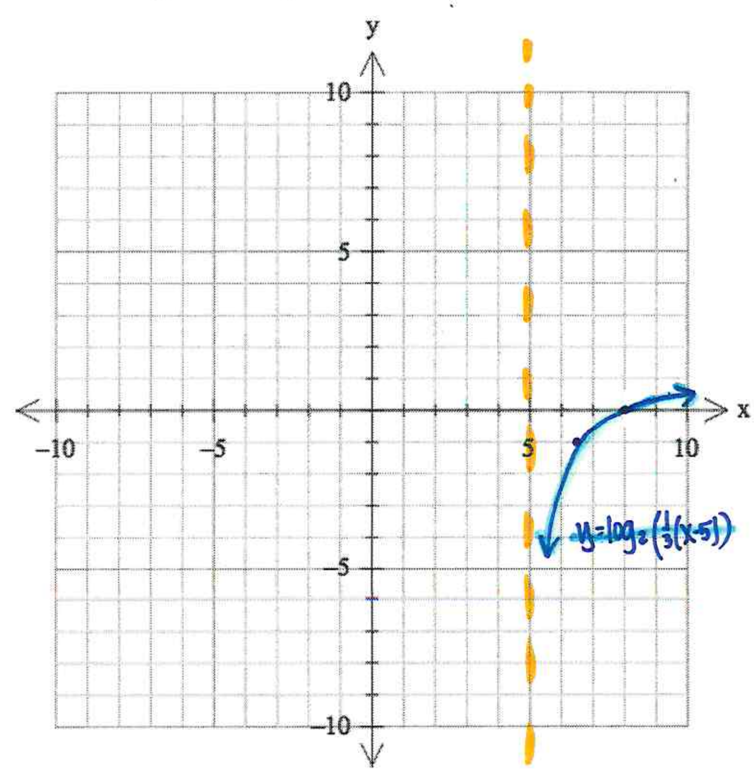
b) $y = \log_3(x-2) + 7$



c) $y = 4\log_5(2x) + 1$



d) $y = \log_2\left(\frac{1}{3}(x-5)\right)$



3. For each of the following logarithmic functions, determine each of the following characteristics.

a) $y = 3\log_2(2(x-4))$

i) Domain: $x > 4$ ii) Range: $y \in \mathbb{R}$

iii) Vertical Asymptote: $x = 4$

iv) x – intercept

$\frac{0}{3} = \frac{3\log_2(2(x-4))}{3}$

$0 = \log_2(2(x-4))$

$2^0 = 2(x-4)$

$1 = 2(x-4)$

$\frac{1}{2} = x-4$

$\frac{9}{2} = x$ @ $(9/2, 0)$

v) y – intercept

$y = 3\log_2(2(0-4))$

$y = 3\log_2(-8)$

not possible

$\therefore$ no y-int.

b) $y = \log\left(\frac{1}{2}(x-10)\right)$

i) Domain: $x > 10$ ii) Range: $y \in \mathbb{R}$

iii) Vertical Asymptote: $x = 10$

iv) x – intercept

$0 = \log(\frac{1}{2}(x-10))$

$10^0 = \frac{1}{2}(x-10)$

$1 = \frac{1}{2}(x-10)$

$2 = x-10$

$12 = x$ @ $(12, 0)$

v) y – intercept

$y = \log(\frac{1}{2}(0-10))$

$y = \log(\frac{1}{2}(-10))$

$y = \log(-5)$

not possible

$\therefore$ no y-int.

c) $y = \log_4(x+4)$

i) Domain: $x > -4$ ii) Range: $y \in \mathbb{R}$

iii) Vertical Asymptote: $x = -4$

iv) x – intercept

$0 = \log_4(x+4)$

$4^0 = x+4$

$1 = x+4$

$-3 = x$

@ $(-3,0)$

v) y – intercept

$y = \log_4(0+4)$

$y = \log_4 4$

$y = 1$

@ $(0,1)$

d) $y = \log_6\big(-(x-1)\big) + 2$

i) Domain: $x < 1$ ii) Range: $y \in \mathbb{R}$

iii) Vertical Asymptote: $x = 1$

iv) x – intercept

$0 = \log_6(-(x-1)) + 2$

$-2 = \log_6(-(x-1))$

$6^{-2} = -(x-1)$

$\frac{1}{36} = -x+1$

$x = 1 - \frac{1}{36}$

$x = \frac{35}{36}$ @ $(\frac{35}{36}, 0)$

v) y – intercept

$y = \log_6(-(0-1)) + 2$

$y = \underbrace{\log_6(1)} + 2$

let $x = \log_6(1)$

$6^x = 1$

$x = 0$

$y = 0 + 2$

$y = 2$ @ $(0,2)$

8.3 – Laws of Logarithms

1. **Expand** each logarithmic expression.

a) $\log_7\left(\frac{x^2y}{z}\right)$ $= \log_7 x^2 + \log_7 y - \log_7 z$

$= 2\log_7 x + \log_7 y - \log_7 z$

b) $\log_3(x\sqrt{yz})$ $= \log_3 x + \log(yz)^{\frac{1}{2}}$

$= \log_3 x + \frac{1}{2}(\log y + \log z)$

$= \log_3 x + \frac{1}{2}\log y + \frac{1}{2}\log z$

c) $\log_5(xyz)^3$ $= 3\log_5(xyz)$

$= 3(\log_5 x + \log_5 y + \log_5 z)$

$= 3\log_5 x + 3\log_5 y + 3\log_5 z$

d) $\log_2(xy\sqrt[3]{z})$ $= \log_2 x + \log_2 y + \log_2 \sqrt[3]{z}$

$= \log_2 x + \log_2 y + \frac{1}{3}\log_2 z$

2. Using the laws of logarithms, **simplify** the expressions below, <u>then</u> **evaluate** each expression.

a) $3\log_8 4 + \log_8 4 + \log_8 2$

$= \log_8(4^3 \cdot 4 \cdot 2)$

$= \log_8 512$

$= 3$

b) $2\log_2 4 + \log_2 5 - \log_2 10$

$= \log_2\left(\frac{4^2 \cdot 5}{10}\right)$

$= \log_2 8$

$= 3$

c) $\log_5(25\sqrt{5})$

$= \log_5 25 + \frac{1}{2}\log_5 5$

$= 2 + \frac{1}{2}(1)$

$= \frac{5}{2}$

d) $\frac{1}{2}\log 9 - \log 3$

$= \log\left(\frac{\sqrt{9}}{3}\right)$

$= \log 1$

$= 0$

8.4 – Logarithmic and Exponential Equations

1. **Solve** the following equations.

a) $\log_2(3-2x) - \log_2(2-x) = \log_2 3$

$\log_2\left(\frac{3-2x}{2-x}\right) = \log_2 3$

$\frac{3-2x}{2-x} = 3$

$3-2x = 3(2-x)$

$3-2x = 6-3x$

$\cancel{x = 3}$ extraneous

$\therefore$ no solution

b) $\log_4(x^2+1) - \log_4 6 = \log_4 5$

$\log_4\left(\frac{x^2+1}{6}\right) = \log_4 5$

$\frac{x^2+1}{6} = 5$

$x^2+1 = 30$

$x^2 = 29$

$x = \pm\sqrt{29}$

c) $\log_2 x + \log_2(x-2) = \log_2(9-2x)$

$\log_2(x(x-2)) = \log_2(9-2x)$

$x(x-2) = 9-2x$

$x^2-2x = 9-2x$

$x^2-9 = 0$

$(x+3)(x-3) = 0$

$\cancel{x = -3}$, $x = 3$

extraneous

d) $2\log(3-x) = \log 4 + \log(6-x)$

$\log(3-x)^2 = \log(4(6-x))$

$(3-x)^2 = 4(6-x)$

$9-6x+x^2 = 24-4x$

$x^2-2x-15 = 0$

$(x-5)(x+3) = 0$

$\cancel{x = 5}$, $x = -3$

extraneous

2. **Solve** the following equations.

a) $\log_2 x + \log_2(x-7) = 3$

$\log_2(x(x-7)) = 3$

$2^3 = x(x-7)$

$8 = x^2 - 7x$

$0 = x^2 - 7x - 8$

$0 = (x-8)(x+1)$

$x = 8$, $\cancel{x = -1}$ extraneous

b) $\log_2 x = 3 - \log_2(x+2)$

$\log_2 x + \log_2(x+2) = 3$

$\log_2(x(x+2)) = 3$

$2^3 = x(x+2)$

$8 = x^2 + 2x$

$0 = x^2 + 2x - 8$

$0 = (x+4)(x-2)$

$\cancel{x = -4}$ extraneous, $x = 2$

c) $\log_2(2-2x) + \log_2(1-x) = 5$

$\log_2((2-2x)(1-x)) = 5$

$2^5 = (2-2x)(1-x)$

$32 = 2 - 2x - 2x + 2x^2$

$0 = 2x^2 - 4x - 30$

$0 = 2(x^2 - 2x - 15)$

$0 = 2(x-5)(x+3)$

$\cancel{x = 5}$ extraneous, $x = -3$

d) $\log_2(x-6) = 3 - \log_2(x-4)$

$\log_2(x-6) + \log_2(x-4) = 3$

$\log_2((x-6)(x-4)) = 3$

$2^3 = (x-6)(x-4)$

$8 = x^2 - 10x + 24$

$0 = x^2 - 10x + 16$

$0 = (x-8)(x-2)$

$x = 8$, $\cancel{x = 2}$ extraneous

3. **Solve** the following equations.

a) $9^x = 51$

$\log 9^x = \log 51$

$x \log 9 = \log 51$

$x = \frac{\log 51}{\log 9}$

$x = 1.789$

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b) $4^{x+3} = 260$

$\log 4^{x+3} = \log 260$

$(x+3)\log 4 = \log 260$

$x\log 4 + 3\log 4 = \log 260$

$x \log 4 = \log 260 - 3\log 4$

$x = \frac{\log 260 - 3\log 4}{\log 4}$ $x = 1.011$

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c) $4^{x-3} = 11^{4-x}$

$\log 4^{x-3} = \log 11^{4-x}$

$(x-3)\log 4 = (4-x)\log 11$

$x\log 4 - 3\log 4 = 4\log 11 - x\log 11$

$x\log 4 + x\log 11 = 4\log 11 + 3\log 4$

$x(\log 4 + \log 11) = 4\log 11 + 3\log 4$

$x = \frac{4\log 11 + 3\log 4}{\log 4 + \log 11}$

$x = 3.634$

d) $2^x = 5^{x+1}$

$\log 2^x = \log 5^{x+1}$

$(x)\log 2 = (x+1)\log 5$

$x\log 2 = x\log 5 + \log 5$

$x\log 2 - x\log 5 = \log 5$

$x(\log 2 - \log 5) = \log 5$

$x = \dfrac{\log 5}{\log 2 - \log 5}$

$x = -1.756$

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e) $2(5^x) = 4^{x-1}$

$\log(2 \cdot 5^x) = \log 4^{x-1}$

$\log 2 + x\log 5 = (x-1)\log 4$

$\log 2 + x\log 5 = x\log 4 - \log 4$

$x\log 5 - x\log 4 = -\log 4 - \log 2$

$x(\log 5 - \log 4) = -\log 4 - \log 2$

$x = \dfrac{-\log 4 - \log 2}{\log 5 - \log 4}$

$x = -9.319$

4. The formula $A = A_0 e^{-0.2t}$ represents the rate at which a radioactive substance deteriorates where A_0 is the initial value, A is the final value and t is time, in years.

a) If the initial value of the substance is 80 grams, determine how much substance remains after 3 years.

$A_0 = 80$
$t = 3$
$A = ?$

$$A = A_0 e^{-0.2t}$$
$$A = (80)(e^{-0.2(3)})$$
$$A = 80e^{-0.6}$$
$$A = 43.905$$

There will be 43.905 grams.

b) How much time is needed for the substance to reduce to half of its original size?

$t = ?$
$A_0 = 80$
$A = 40$

$$A = A_0 e^{-0.2t}$$
$$40 = 80e^{-0.2t}$$
$$\frac{1}{2} = e^{-0.2t}$$
$$\ln(1/2) = \ln e^{-0.2t}$$
$$\ln(1/2) = -0.2t(\ln e)$$
$$\ln(1/2) = -0.2t$$
$$\frac{\ln(1/2)}{-0.2} = t$$
$$3.466 = t$$

It will take 3.466 years

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5. If you invest $2500 at 11.25% compounded quarterly, how long will it take for the investment to double? Use the compound interest formula $A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = 5000$
$P = 2500$
$r = 0.1125$
$n = 4$

$A = P(1+\frac{r}{n})^{nt}$

$5000 = (2500)(1+\frac{0.1125}{4})^{4t}$

$2 = (1.028125)^{4t}$

$\log 2 = \log(1.028125)^{4t}$

$\log 2 = (4t)\log(1.028125)$

$\frac{\log 2}{4\log 1.028125} = t$

$6.248 = t$

It will take 6.248 years to double the money

P = initial value
A = final value
n = # of times the interest is compounded in one year
r = interest rate, as a decimal
t = time, in years

6. The magnitude formula of Richter, M, of an earthquake is $M = \log\frac{A}{A_0}$ where A is the recorded amplitude of ground movement and A_0 is the reference amplitude.

In 2012, on February 1, a location on the Manitoba and Saskatchewan border sustained a magnitude 3.3 earthquake. One of the largest earthquakes in Manitoba's history. I

In 2010, Haiti was the center of a 7.0 magnitude earthquake.

How many times stronger was the Earthquake in Haiti compared to the Manitoba earthquake?

2012 Manitoba

$$M = \log\left(\frac{A}{A_0}\right)$$
$$3.3 = \log\left(\frac{A}{A_0}\right)$$
$$10^{3.3} = \frac{A}{A_0}$$
$$(10^{3.3})A_0 = A$$

2010 Haiti

$$M = \log\left(\frac{A}{A_0}\right)$$
$$7.0 = \log\left(\frac{A}{A_0}\right)$$
$$10^{7.0} = \frac{A}{A_0}$$
$$(10^{7.0})A_0 = A$$

Compare

$$\frac{\text{Haiti}}{\text{Manitoba}} = \frac{10^{7.0}(\cancel{A_0})}{10^{3.3}(\cancel{A_0})}$$
$$= \frac{10^{7.0}}{10^{3.3}}$$
$$= 10^{7.0-3.3}$$
$$= 10^{3.7}$$
$$= 5\,011.872$$

The Earthquake in Haiti was 5011.872 times stronger than the one in Manitoba.

7. The pH of a solution is defined by the $pH = -\log(H^+)$ where H^+ is the concentration of hydrogen ions in moles per litre (mol/L).

a) Lactic acidosis is a disease characterized by excess lactate and a blood pH of less than 7.35. A person severely affected by this disease has a blood pH of 7.1.

Determine the concentration of hydrogen in that person's blood.

$pH = -\log(H^+)$

$7.1 = -\log(H^+)$

$-7.1 = \log(H^+)$

$10^{-7.1} = H^+$

7.9×10^{-8} mol/L $= H^+$

b) A rain is said to be acidic if it has a pH less than 5.3. In some places in Ontario, rain can have a pH of 4.5.

How many more times is a rain with a pH of 4.5 more acidic than a normal rain at a pH of 5.3?

pH 4.5

$4.5 = -\log(H^+)$

$-4.5 = \log(H^+)$

$10^{-4.5} = H^+$

pH 5.3

$5.3 = -\log(H^+)$

$-5.3 = \log(H^+)$

$10^{-5.3} = H^+$

compare : $\frac{\text{pH } 4.5}{\text{pH } 5.3}$

$= \frac{10^{-4.5}}{10^{-5.3}}$

$= 10^{-4.5-(-5.3)}$

$= 10^{0.8}$

$=$ 6.310 times more acidic

c) Michelle's conditioner is 100 times more acidic than her shampoo. If the pH of are shampoo is 6.2, what is the pH of the conditioner?

Shampoo

$6.2 = -\log(H^+)$

$-6.2 = \log(H^+)$

$10^{-6.2} = H^+_s$

Conditioner

$pH = -\log(H^+)$

$-pH = \log(H^+)$

$10^{-pH} = H^+_c$

compare : $\frac{H^+_c}{H^+_s} = 100$

$\frac{10^{-pH}}{10^{-6.2}} = 100$

$10^{-pH-(-6.2)} = 100$

$10^{-pH+6.2} = 10^2$

$-pH + 6.2 = 2$

$4.2 = pH$