1. If $\log_a 2 = x$ and $\log_a 5 = y$, find an algebraic expression for $\log_a\left(\frac{\sqrt[3]{10}}{2}\right)$ in terms of x and y. [3 marks]

$\log_a 10^{1/3} - \log_a 2$

$\frac{1}{3}\log_a(5\cdot 2) - \log_a 2$

$\frac{1}{3}(\log_a 5 + \log_a 2) - \log_a 2$

$\frac{1}{3}y + \frac{1}{3}x - x$

$\frac{1}{3}(y - 2x)$ or $\frac{1}{3}y - \frac{2}{3}x$

2. Solve for x algebraically.
Show all steps.
$\ln\left(e^{\sqrt{x}}\right) = 8$ [2 marks]

$\sqrt{x}\ln e = 8$

$\sqrt{x} = 8$

$x = 64$

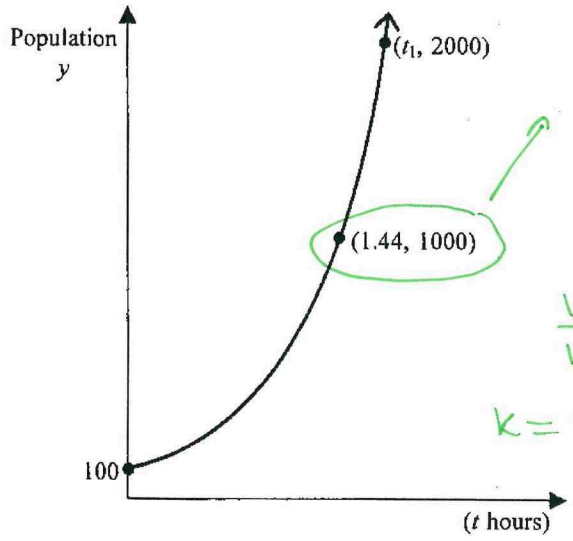
* Never round early *

$\ln e = 1$

3. The population growth of a bacterial culture follows an exponential curve $y = 100e^{kt}$ as shown in the diagram.

Find the value of t_1, the time that the population reaches 2000.

Round your final answer to two decimal places. [3 mark]



① Find k

$1000 = 100e^{k(1.44)}$

$10 = e^{1.44k}$

$\ln 10 = \ln e^{1.44k}$

$\ln 10 = 1.44k$

$\frac{\ln 10}{1.44} = k$

$k = 1.599017426$

② Find t_1

$2000 = 100e^{kt}$

$20 = e^{kt}$

$\ln 20 = \ln e^{kt}$

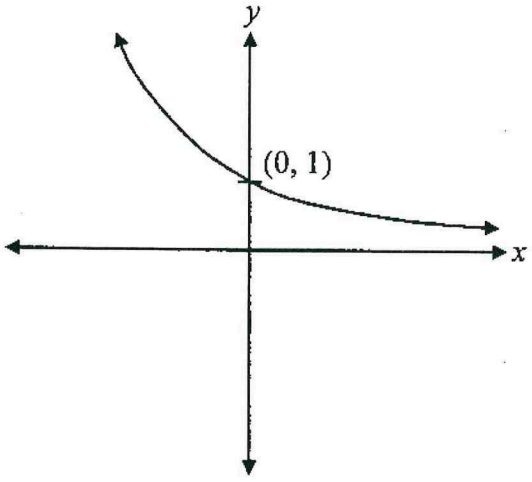
$\ln 20 = kt$

$\frac{\ln 20}{k} = t$

$1.87 = t$

hours.

4. The graph of the exponential function $y = a^x$ is shown below. State a possible value for a. [1 mark]



$a = \frac{1}{2}$

Any answer where

$0 < a < 1$

5. Solve for x:

$125 = 5^{2x-1}$ [1 mark]

$5^3 = 5^{2x-1}$

$3 = 2x - 1$

$4 = 2x$

$2 = x$

6. Find the zero(s), in simplified form, of the following function:

$h(x) = \log_2(\log_{16} x) + 2$ [2 marks]

$0 = \log_2(\log_{16} x) + 2$

exponential form

$-2 = \log_2(\log_{16} x)$

$2^{-2} = \log_{16} x$

$16^{2^{-2}} = x$

$16^{+1/4} = x$

$\sqrt[4]{16^1} = x$

$2 = x$

7. If $f(x) = 5\ln x$, find $f^{-1}(x)$. [2 marks]

$y = 5\ln x$

$x = 5\ln y$

$\frac{x}{5} = \ln y$

$e^{x/5} = y$

OR

$y = \ln x^5$

$x = \ln y^5$

$e^x = y^5$

$\sqrt[5]{e^x} = y$

8. Solve for x, algebraically. State the answer in simplest numerical form. Show all steps.

$32^{x+3} = 16^{-2}$ [3 marks]

$(2^5)^{x+3} = (2^4)^{-2}$

$2^{5x+15} = 2^{-8}$

$5x + 15 = -8$

$5x = -2$

$x = \frac{-23}{5}$

9. What is the range of the function $f(x) = 2^x + 1$? [1 mark]

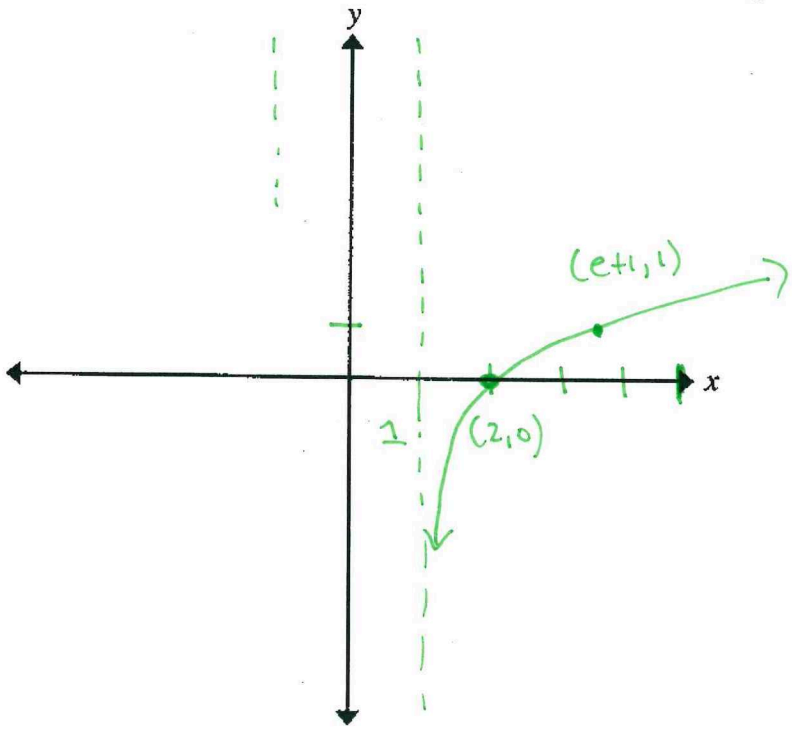
$(1, \infty)$

10. Evaluate

$5^{\log_5 6}$ [1 mark]

6

11. Sketch a clearly labeled graph of $y = \ln(x-1)$, showing any intercepts and asymptotes. [3 marks]



12. Solve for x: $\log_2(x-2) - \log_2 x = 3$ [3 marks]

$\log_2 \frac{(x-2)}{x} = 3$

$2^3 = \frac{x-2}{x}$

$8x = x-2$

$7x = -2$

$x = -2/7$

NO SOLUTION

13. The lights are left on when a car is parked. The battery discharges and the voltage, V volts, of the battery is given at any time by: $V = V_0 e^{-kt}$

$V_0 = 12$ volts, $k = 0.01$ and t is measured in minutes.

Find, to the nearest minute, the time it takes for the battery charge to reduce to 9 volts. [3 marks]

$9 = 12e^{-0.01t}$

$\frac{9}{12} = e^{-0.01t}$

$\ln\left(\frac{9}{12}\right) = \ln e^{-0.01t}$ $\quad \ln e = 1$

$\ln\left(\frac{9}{12}\right) = -0.01t \ln e$

$\frac{\ln(9/12)}{-0.01} = t$

$28.768 = t$ $\quad t = 29$ mins

14. State the domain of the function:
$f(x) = \log_3(x+2)$ [1 mark]

$x > -2$

15. Solve for x in terms of a and b:
$e^x = a^b$ [1 mark]

$\ln e^x = \ln a^b$

$x \ln e = \ln a^b$

$x = \ln a^b$

or $x = b \ln a$

16. Consider the function: $y = 8^x - 2$.
Find the x-intercept of this function. [1 mark]

$0 = 8^x - 2$

$2 = 8^x$

$2^1 = 2^{3x}$

$1 = 3x$ $x = \frac{1}{3}$

17. If $\log_x 125 = 3$, find the exact value of x. [1 mark]

exp. form.

$x^3 = 125$

$x = 5$

18. Solve for x. Express your answer correct to 3 decimal places

$e^x = 8^{1-x}$ [4 marks]

$\ln e^x = \ln 8^{1-x}$

$x \ln e = (1-x)\ln 8$

$x = 1\ln 8 - x\ln 8$

$x + x\ln 8 = \ln 8$

$x(1 + \ln 8) = \ln 8$

$x = \frac{\ln 8}{1+\ln 8}$

$x = 0.675$

19. Find the numerical value of:

$\frac{\log_3 9}{\log_9 3}$ [2 marks]

$\frac{2}{\frac{1}{2}}$

$2\left(\frac{2}{1}\right) = 4$

20. If $f(x) = \log_{16} x$, find the value of $f(4)$. [1 mark]

$f(4) = \log_{16} 4$

$f(4) = \frac{1}{2}$

"16 to what power gives 4^4"

note.

$y = \log_{16} 4$

$16^y = 4$

$4^{2x} = 4^1$

$2x = 1$

$x = 1/2$

21. If $x = e^{2\ln 3}$ then find the value of x as a whole number. [1 mark]

$x = e^{\ln 3^2}$

$x = e^{\ln 9}$

$x = 9$

We could change to log form.

$\ln x = \ln 9$

$x = 9$

Both are base e

22. Solve for x:

$\log_2(2-x) = 1 - \log_2(3-x)$ [4 marks]

$\log_2(2-x) + \log_2(3-x) = 1$

$\log_2(2-x)(3-x) = 1$

$2^1 = (2-x)(3-x)$

$2 = 6 - 5x + x^2$

$0 = x^2 - 5x + 4$

$0 = (x-4)(x-1)$

$x = 4, 1$

* we will give you the formula

23. An investment earns interest at an annual rate of 7% compounded semi-annually. How long will it take, in years, for the investment to triple? [3 marks]

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$3 = 1\left(1 + \frac{0.07}{2}\right)^{2t}$

$3 = (1.035)^{2t}$

$\log 3 = \log 1.035^{2t}$

$\log 3 = 2t(\log 1.035)$

$\frac{\log 3}{2 \log 1.035} = t$

$t \approx 16$ years. (15.96753963)

* we can let P = any value as long as A is triple that value!

24. Solve the equation $3^{(x+1)} = 4(5^x)$ **algebraically**.
Express your final answer correct to 3 decimal places. [4 marks]

$\log 3^{(x+1)} = \log 4(5^x)$

$(x+1)\log 3 = \log 4 + \log 5^x$

$x\log 3 + \log 3 = \log 4 + x\log 5$

$x\log 3 - x\log 5 = \log 4 - \log 3$

$x(\log 3 - \log 5) = \log 4 - \log 3$

$x = \frac{\log 4 - \log 3}{\log 3 - \log 5}$

$x = -0.563$

$x = -0.563173$

25. If $\log_a 2 = p$ and $\log_a 3 = q$, find an expression for $\log_a 6$ in terms of p and q. [1 mark]

$\log_a(2 \cdot 3)$

$\log_a 2 + \log_a 3$

$p + q$

26. Find $f^{-1}(x)$, if $f(x) = e^x$. [1 mark]

$y = e^x$

$x = e^y$

$\ln x = y$

we can go straight to $y = \ln x$

$f^{-1}(x) = \ln x$

27. Solve for x:

$\log_2(\log_{81} x) = -2$ [3 marks]

exp. form

$2^{-2} = \log_{81} x$

$81^{2^{-2}} = x$

$81^{1/4} = x$

$3 = x$

28. Solve for x:

$\left(\frac{1}{4}\right)^{2x-1} = 8^x$ [2 marks]

$(2^{-2})^{2x-1} = 2^{3x}$

$2^{-4x+2} = 2^{3x}$

$-4x + 2 = 3x$

$-7x = -2$

$x = +2/7$

29. Solve the following equation algebraically. State your answer correct to 3 decimal places

$5^{2x-1} = 7^{x+4}$ [3 marks]

$\log 5^{(2x-1)} = \log 7^{(x+4)}$

$(2x-1)\log 5 = (x+4)\log 7$

$2x\log 5 - \log 5 = x\log 7 + 4\log 7$

$2x\log 5 - x\log 7 = 4\log 7 + \log 5$

$x(2\log 5 - \log 7) = 4\log 7 + \log 5)$

$x = \dfrac{4\log 7 + \log 5}{2\log 5 - \log 7}$

$x \approx 7.379$

30. In June 1998, the black bear population in Manitoba was estimated at 1500 bears. The population can be modeled by the equation $A = Pe^{rt}$ where r is the annual rate of increase and t represents the number of years.
Find the annual rate of increase in the bear population if it was estimated at 1740 bears in June 2001.
State your answer correct to 3 decimal places. [3 marks]

$A = Pe^{rt}$

$1740 = 1500\, e^{r(3)}$

$\dfrac{1740}{1500} = e^{3r}$

$\ln\left(1740/1500\right) = 3r \ln e$

$\dfrac{\ln\left(1740/1500\right)}{3} = r$

$r = 0.0494$

0.050

35. Solve for x **algebraically**.
Give your answer correct to 3 decimal places. [3 marks]

$3^{x+4} = 7^{2x+1}$

$\log 3^{(x+4)} = \log 7^{(2x+1)}$

$(x+4)\log 3 = (2x+1)\log 7$

$x\log 3 + 4\log 3 = 2x\log 7 + \log 7$

$x\log 3 - 2x\log 7 = \log 7 - 4\log 3$

$x(\log 3 - 2\log 7) = \log 7 - 4\log 3$

$x = \frac{\log 7 - 4\log 3}{\log 3 - 2\log 7}$

$x \approx 0.877$

36. If $\log_a 2 = 0.3562$ and $\log_a 5 = 0.8271$ show that $\log_a 40 = 1.8957$.
[4 marks]

$\log_a 40$

$\log_a 8 \cdot 5$

$\log_a 2^3 \cdot 5$

$\log_a 2^3 + \log_a 5$

$3\log_a 2 + \log_a 5$

$3(0.3562) + 0.8271$

$1.0686 + 0.8271$

1.8957

31. If $\log_a x = 16$, find the value of $\log_a \sqrt{x}$. [1 mark]

$\log_a x^{1/2}$

$\frac{1}{2}\log_a x$

$\frac{1}{2}(16) = 8$

32. Find $f^{-1}(x)$ if $f(x) = \log x$. Base 10 [1 mark]

$y = 10^x$

$f^{-1}(x) = 10^x$

33. Solve for x. Give you answer in simplest form.

$\log_9(\log_4 x) = \frac{1}{2}$ [2 marks]

$9^{1/2} = \log_4 x$

$4^{9^{1/2}} = x$

$4^3 = x$

$64 = x$

34. Find the x-intercept(s) of the following function:

$y = \log(10-3x) - 2\log x$ [4 marks]

$0 = \log(10-3x) - \log x^2$

$0 = \log\left(\frac{10-3x}{x^2}\right)$

$10^0 = \frac{10-3x}{x^2}$

$1 = \frac{10-3x}{x^2}$

$x^2 = 10 - 3x$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x = 5 \quad x = -2$

37. A new automobile cost \$24,000. Its value after t years is given by: $V = 24000(0.8)^t$.

a) Determine the value after 8 years. [1 mark]

$V = 24000(0.8)^8$

$V = \$4026.53$

b) How many years will it take for its value to decrease to one-eighth of its initial value? State your answer to 3 decimal places. [3 marks]

$3000 = 24000(0.8)^t$

you could start with this step → $\frac{1}{8} = (0.8)^t$

$\log\left(\frac{1}{8}\right) = \log 0.8^t$

$\frac{\log(1/8)}{\log 0.8} = t$

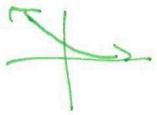
$t =$ 9.319 years.

38. Evaluate $\log\left(100\sin\frac{\pi}{2}\right)$. [1 mark]

$\log 100(1)$

2

39. State the range of the function $f(x) = 2^{-x}$. [1 mark]



$(0, \infty)$

40. Solve for x:

$$\left(\frac{1}{3}\right)^{2x} = 27^{x-5}$$ [3 marks]

$$(3^{-1})^{2x} = (3^3)^{x-5}$$

$$3^{-2x} = 3^{3x-15}$$

$$-2x = 3x - 15$$

$$-5x = -15$$

$$x = 3$$