MPC40S Date: Answer key

Review – CHAPTER 4 Questions

4.1 – Angles and Their Measures

1. Express the following angles in **radian measure** and state which quadrant each angle terminates.

a) $120° \times \left(\frac{\pi}{180°}\right)$

$= \frac{2\pi}{3}$

Quadrant: II

b) $225° \times \left(\frac{\pi}{180°}\right)$

$= \frac{5\pi}{4}$

Quadrant: III

c) $-300° \times \left(\frac{\pi}{180°}\right)$

$= -\frac{5\pi}{3}$

Quadrant: I

d) $-100° \times \left(\frac{\pi}{180°}\right)$

$= -\frac{5\pi}{9}$

Quadrant: III

e) $190° \times \left(\frac{\pi}{180°}\right)$

$= \frac{19\pi}{18}$

Quadrant: III

f) $500° \times \left(\frac{\pi}{180°}\right)$

$= \frac{25\pi}{9}$

Quadrant: II

2. Express the following angles in **degrees** and state which quadrant each angle terminates.

a) $\frac{\pi}{3} \times \left(\frac{180°}{\pi}\right)$

$= 60°$

Quadrant: I

b) $8 \times \left(\frac{180°}{\pi}\right)$

$= 458.366°$

Quadrant: II

c) $-\frac{3\pi}{2} \times \left(\frac{180°}{\pi}\right)$

$= -270°$

Quadrant: III / IV

d) $-\frac{7\pi}{6} \times \left(\frac{180°}{\pi}\right)$

$= -210°$

Quadrant: II

e) $\frac{9\pi}{5} \times \left(\frac{180°}{\pi}\right)$

$= 324°$

Quadrant: IV

f) $\frac{15\pi}{4} \times \left(\frac{180°}{\pi}\right)$

$= 675°$

Quadrant: IV

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3. Determine one positive and one negative **coterminal angle** that corresponds to each of the given angles. (Note, there can be more than one possible answer).

a) $\frac{\pi}{2}$

$\frac{5\pi}{2}$, $-\frac{3\pi}{2}$

b) $-\frac{4\pi}{3}$

$\frac{2\pi}{3}$, $-\frac{10\pi}{3}$

c) 2.567

8.850 , -3.716

d) 350°

= 710° , -10°

e) –77°

283° , -437°

f) –400°

320° , -760°

4. Determine all of the **coterimal angles** over the interval $[-720°, 720°]$

a) 40°

400° , -320° , -680°

b) –257°

85° , 445° , -617°

c) 515°

155° , -205° , -565°

d) 180°

540° , -180° , -540°

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5. Determine all of the **coterminal angles** over the interval $[-4\pi, 4\pi]$

a) $\frac{3\pi}{4}$

$\frac{11\pi}{4}, \frac{-5\pi}{4}, \frac{-13\pi}{4}$

b) $-\frac{11\pi}{6}$

$\frac{\pi}{6}, \frac{13\pi}{6}, \frac{-23\pi}{6}$

c) $\frac{23\pi}{10}$

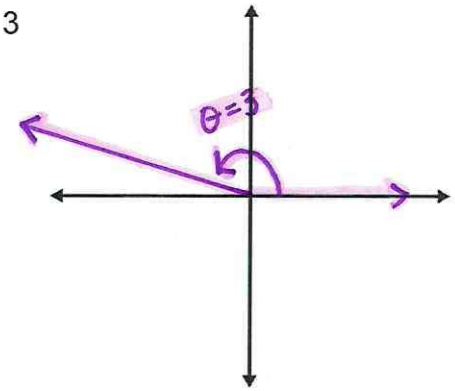
$\frac{3\pi}{10}, \frac{17\pi}{10}, \frac{-37\pi}{10}$

d) –4.2

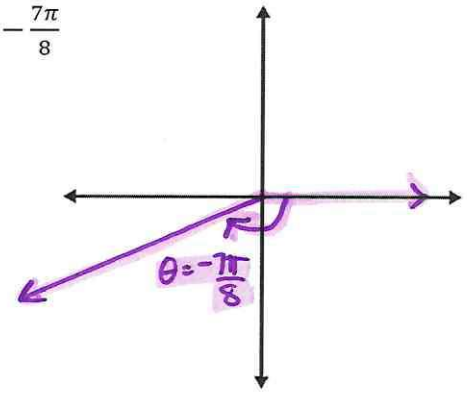
2.083, 8.366, –10.483

6. **Sketch** the following angles in standard position.

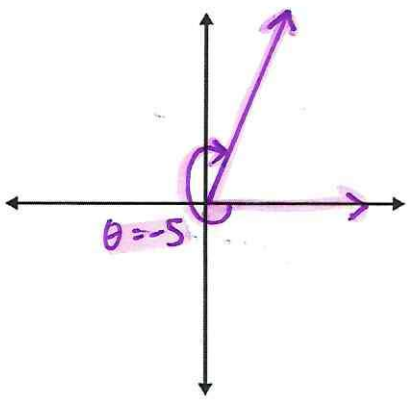
a) 3



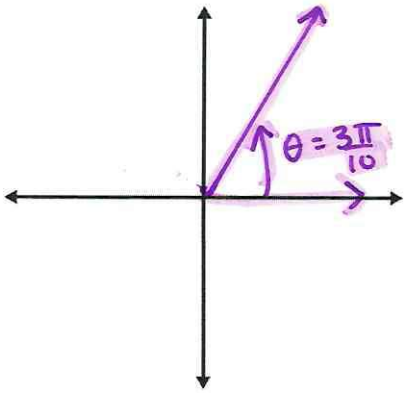
b) $-\frac{7\pi}{8}$



c) –5



d) $\frac{3\pi}{10}$



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7. A circle with radius 16.2 cm is traced on a large piece of cardboard. Next, a central angle of 74° is sketched. Determine the length of the arc intercepted by this angle, rounded to the nearest tenth of a centimetre.

$74° \times \left(\frac{\pi}{180°}\right)$

$= \frac{37\pi}{90}$

$S = \theta r$

$S = \left(\frac{37\pi}{90}\right)(16.2)$

$S = 20.92 \text{ cm}$

8. An arc on a circle of radius 7 cm is 10 cm in length. Determine the measure, in radians, of the central angle that subtends this arc.

$S = \theta r$

$\frac{10}{7} = \frac{\theta(7)}{7}$

$\frac{10}{7} = \theta$

$1.429 = \theta$

9. A tire with a circumference of 30 m rolls 5 m. How many radians does it turn? How many degrees does it roll?

$C = 2\pi r$

$\frac{30}{2\pi} = \frac{2\pi r}{2\pi}$

$\frac{30}{2\pi} = r$

$\frac{15}{\pi} = r$

$S = \theta r$

$5 = \theta\left(\frac{15}{\pi}\right)$

$\frac{\pi}{15}(5) = \theta$

$\frac{\pi}{3} = \theta$

$60° = \theta$

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4.2 – The Unit Circle

1. The following points are found on the unit circle. Find the missing value.

a) $\left(\frac{4}{5}, y\right)$, not in QI

$x^2+y^2=1$

$\left(\frac{4}{5}\right)^2+y^2=1$

$\frac{16}{25}+y^2=1$

$y^2=\frac{25}{25}-\frac{16}{25}$

$y^2=\frac{9}{25}$

$y=\pm\sqrt{\frac{9}{25}}$ $\therefore \left(\frac{4}{5}, -\frac{3}{5}\right)$

in Q4 $\therefore$ $y=-\frac{3}{5}$

b) $\left(x, \frac{3}{7}\right)$, in QII

$x^2+y^2=1$

$x^2+\left(\frac{3}{7}\right)^2=1$

$x^2+\frac{9}{49}=1$

$x^2=\frac{49}{49}-\frac{9}{49}$

$x^2=\frac{40}{49}$

$x=\pm\sqrt{\frac{40}{49}}$ $\therefore \left(-\frac{\sqrt{40}}{7}, \frac{3}{7}\right)$

in Q2 $\therefore$ $x=-\frac{\sqrt{40}}{7}$

c) $\left(x, -\frac{\sqrt{5}}{6}\right)$, not in QIII

$x^2+y^2=1$

$x^2+\left(-\frac{\sqrt{5}}{6}\right)^2=1$

$x^2+\frac{5}{36}=1$

$x^2=\frac{36}{36}-\frac{5}{36}$

$x^2=\frac{31}{36}$

$x=\pm\sqrt{\frac{31}{36}}$ $\therefore \left(\frac{\sqrt{31}}{6}, -\frac{\sqrt{5}}{6}\right)$

in Q4 $\therefore$ $x=\frac{\sqrt{31}}{6}$

d) $\left(-\frac{5}{12}, y\right)$, in QIII

$x^2+y^2=1$

$\left(-\frac{5}{12}\right)^2+y^2=1$

$\frac{25}{144}+y^2=1$

$y^2=\frac{144}{144}-\frac{25}{144}$

$y^2=\frac{119}{144}$

$y=\pm\sqrt{\frac{119}{144}}$

in Q3 $\therefore$ $y=-\frac{\sqrt{119}}{12}$ $\therefore \left(-\frac{5}{12}, -\frac{\sqrt{119}}{12}\right)$

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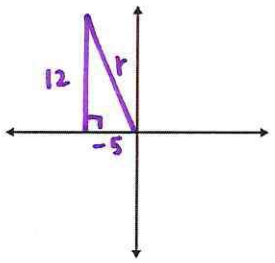
2. The point $P(\theta)$ lies on the intersection of the unit circle and a line joining the origin to the given point. Determine the coordinates of $P(\theta)$.

a) $P(-5, 12)$



$x^2 + y^2 = r^2$

$(-5)^2 + (12)^2 = r^2$

$25 + 144 = r^2$

$169 = r^2$

$\sqrt{169} = r$

$13 = r$

$P(\theta) = (\cos\theta, \sin\theta)$

$P(\theta) = \left(\frac{-5}{13}, \frac{12}{13}\right)$

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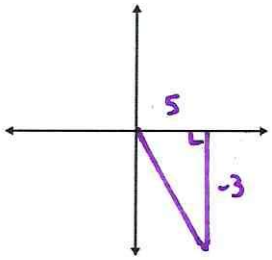
b) $P(5, -3)$



$x^2 + y^2 = r^2$

$(5)^2 + (-3)^2 = r^2$

$25 + 9 = r^2$

$34 = r^2$

$\sqrt{34} = r$

$P(\theta) = (\cos\theta, \sin\theta)$

$P(\theta) = \left(\frac{5}{\sqrt{34}}, \frac{-3}{\sqrt{34}}\right)$

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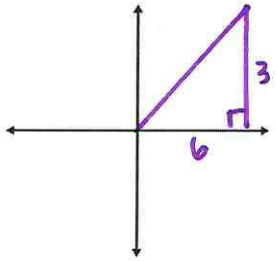
c) $P(6, 3)$



$x^2 + y^2 = r^2$

$(6)^2 + (3)^2 = r^2$

$36 + 9 = r^2$

$45 = r^2$

$\sqrt{45} = r$

$P(\theta) = (\cos\theta, \sin\theta)$

$P(\theta) = \left(\frac{6}{\sqrt{45}}, \frac{3}{\sqrt{45}}\right)$

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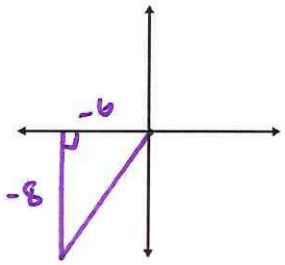
d) $P(-6, -8)$



$x^2 + y^2 = r^2$

$(-6)^2 + (-8)^2 = r^2$

$36 + 64 = r^2$

$100 = r^2$

$\sqrt{100} = r$

$10 = r$

$P(\theta) = (\cos\theta, \sin\theta)$

$P(\theta) = \left(\frac{-6}{10}, \frac{-8}{10}\right)$

$\therefore P(\theta) = \left(\frac{-3}{5}, \frac{-4}{5}\right)$

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e) Determine the exact value of the 6 trigonometric ratios in part d)

$\cos\theta = -\frac{3}{5}$ $\quad \sin\theta = -\frac{4}{5}$ $\quad \tan\theta = \frac{4}{3}$

$\sec\theta = -\frac{5}{3}$ $\quad \csc\theta = -\frac{5}{4}$ $\quad \cot\theta = \frac{3}{4}$

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3. Determine the **coordinates** of the point on the unit circle at each given angle.

a) $P\left(\frac{\pi}{6}\right)$

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

b) $P\left(-\frac{2\pi}{3}\right)$

$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

c) $P\left(-\frac{7\pi}{4}\right)$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

d) $P(-7\pi)$

$(-1, 0)$

e) $P\left(\frac{17\pi}{2}\right)$

$(0, 1)$

f) $P\left(\frac{-13\pi}{6}\right)$

$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

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4.3 – Trigonometric Ratios

1. Find the **exact value** of the following trigonometric ratios

a) $\cos 135°$ $= -\frac{\sqrt{2}}{2}$	b) $\sin\frac{4\pi}{3}$ $= -\frac{\sqrt{3}}{2}$	c) $\tan(-300°)$ $= \sqrt{3}$	d) $\csc\left(-\frac{7\pi}{4}\right)$ $= \frac{2}{\sqrt{2}}$
e) $\sec 2\pi$ $= 1$	f) $\sin\frac{17\pi}{3}$ $= -\frac{\sqrt{3}}{2}$	g) $\cot(-405°)$ $= -1$	h) $\sec\frac{3\pi}{2}$ = undefined

2. Evaluate each of the following expressions.

a) $\sin^2\left(\frac{16\pi}{3}\right) + \cos^2\left(\frac{16\pi}{3}\right)$

$= \left(-\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$= \frac{3}{4} + \frac{1}{4}$

$= \frac{4}{4} = 1$

b) $\cos\left(-\frac{5\pi}{3}\right)\sin\left(\frac{13\pi}{6}\right)$

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{1}{4}$

c) $\tan\left(\frac{4\pi}{3}\right)\cos\left(\frac{5\pi}{3}\right)\sin\left(\frac{13\pi}{4}\right)$

$= \left(\frac{\sqrt{3}}{1}\right)\left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}}{4}$

d) $\sin\left(-\frac{4\pi}{3}\right)\cos\left(\frac{11\pi}{6}\right) - \tan\left(\frac{5\pi}{6}\right)\cot\left(\frac{\pi}{3}\right)$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(-\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{3}{4} + \frac{1}{3}$

$= \frac{9}{12} + \frac{4}{12}$

$= \frac{13}{12}$

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e) $\csc\left(\frac{11\pi}{6}\right) - \sec\left(-\frac{\pi}{3}\right) + \cos(17\pi)$

$= -2 - 2 + (-1)$

$= -5$

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f) $\csc(450°)\tan(-135°) - \cos(120°)\tan(0°)$

$= (1)(1) - \left(-\frac{1}{2}\right)(0)$

$= 1 - 0$

$= 1$

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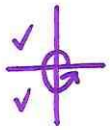
4.4 – Solving Trigonometric Equations

1. **Solve** the following equations over the given interval. State final answer as an exact value where possible, or correct to 3 decimal places.

a) $\cos\theta = -\frac{\sqrt{3}}{2}$, $[0, 2\pi)$



$\theta_R = \frac{\pi}{6}$

$\theta = \frac{5\pi}{6}, \frac{7\pi}{6}$

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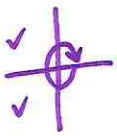
b) $\cos\theta = -\frac{\sqrt{2}}{2}$, $-360° < \theta \leq 0°$



$\theta_R = 45°$

$\theta = -135°, -225°$

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c) $\sin\theta = \frac{\sqrt{3}}{2}$, where $\theta \epsilon \mathbb{R}$

$\theta_R = \frac{\pi}{3}$

$\theta = \frac{\pi}{3} + 2\pi k, k \in Z$

$\theta = \frac{2\pi}{3} + 2\pi k, k \in Z$

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d) $\tan\theta = 3$, $(0, 4\pi)$

$\theta_R = \tan^{-1}(3)$

$\theta_R = 1.249$

$\theta = 1.249, 4.391, 7.532, 10.674$

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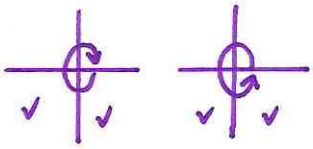
2. **Solve** the following equations over the given interval. State final answer as an exact value where possible, or correct to 3 decimal places.

a) $\sin\theta = -\frac{1}{2}, \quad -360° < \theta < 360°$



$\theta_R = 30°$

$\theta = -30°$

$\theta = -150°$

$\theta = 210°$

$\theta = 330°$

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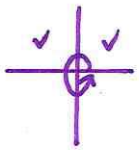
b) $5\sin\theta - 1 = 0, \quad 0° \leq \theta < 360°$

$\frac{5\sin\theta}{5} = \frac{1}{5}$



$\sin\theta = 1/5$

$\theta_R = \sin^{-1}(1/5)$

$\theta_R = 11.537°$

Q1

$\theta = \theta_R$

$\theta = 11.537°$

Q2

$\theta = 180° - \theta_R$

$\theta = 180° - 11.537°$

$\theta = 168.463°$

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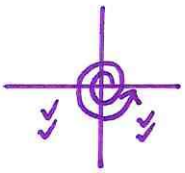
c) $4\csc\beta + 7 = 0, \quad 0° \leq \beta < 720°$

$\csc\beta = -7/4$



$\sin\beta = -4/7$

$\beta_R = \sin^{-1}(4/7)$

$\beta_R = 34.850°$

$\theta = 214.85°$

$\theta = 325.15°$

$\theta = 574.85°$

$\theta = 685.15°$

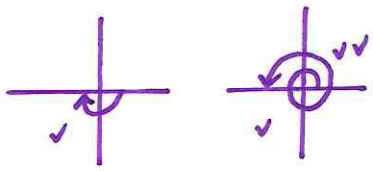
d) $5 - 3\tan\alpha = 0, \quad -\pi \leq \alpha \leq 3\pi$

$\frac{-3\tan\alpha}{-3} = \frac{-5}{-3}$

$\tan\alpha = 5/3$

$\alpha_R = \tan^{-1}(5/3)$

$\alpha_R = 1.030$



$\alpha = 1.030$

$\alpha = 4.172$

$\alpha = 7.313$

$\alpha = -2.112$

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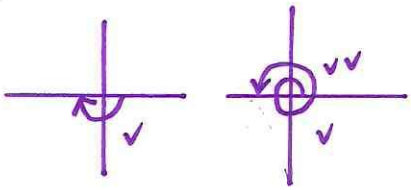
e) $\sec x - 6 = 0, \quad -\pi \leq x \leq 3\pi$

$\sec x = 6$

$\cos x = 1/6$

$x_R = \cos^{-1}(1/6)$

$x_R = 1.403$



$x = -1.403$

$x = 1.403$

$x = 4.880$

$x = 7.686$

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f) $2\cot\theta + 3 = 5\cot\theta - 1, \quad$ where $\theta \in \mathbb{R}$

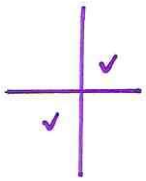
$\frac{4}{3} = \frac{3\cot\theta}{3}$

$4/3 = \cot\theta$

$\frac{3}{4} = \tan\theta$

$\tan^{-1}(3/4) = \theta_R$

$\theta_R = 0.644$



Q1

$\theta = \theta_R$

$\theta = 0.644$

Q3

$\theta = \pi + \theta_R$

$\theta = \pi + 0.644$

$\theta = 3.786$

$\theta = 0.644 + 2\pi k, \; k \in \mathbb{Z}$

$\theta = 3.786 + 2\pi k, \; k \in \mathbb{Z}$

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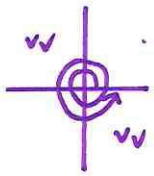
g) $\tan\theta + 1 = 0, \ [0, 4\pi]$



$\tan\theta = -1$

$\theta = \frac{3\pi}{4}$

$\theta = \frac{7\pi}{4}$

$\theta = \frac{11\pi}{4}$

$\theta = \frac{15\pi}{4}$

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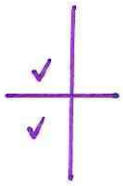
h) $3\sec\theta = 7\sec\theta + 8, \quad$ where $\theta \epsilon \mathbb{R}$

$\frac{-4\sec\theta}{-4} = \frac{8}{-4}$

$\sec\theta = -2$

$\cos\theta = -\frac{1}{2}$

$\theta_R = \frac{\pi}{3}$



$\theta = \frac{2\pi}{3} + 2\pi k, \ k \epsilon Z$

$\theta = \frac{4\pi}{3} + 2\pi k, \ k \epsilon Z$

3. Mark and Eric each solve the equation $\cot\theta = -1$, where $\theta \epsilon \mathbb{R}$.

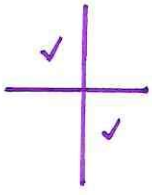
Mark's solution is $\theta = \frac{3\pi}{4} + k\pi, k \epsilon Z$

Eric's solution is $\theta = \frac{3\pi}{4} + 2k\pi, k \epsilon Z$ and $\theta = \frac{7\pi}{4} + 2k\pi, k \epsilon Z$

Who is correct? **Explain** your response.

$\cot\theta = -1$

$\therefore \tan\theta = -1$



↳ Both are correct.

↳ Mark's solution includes all the solutions in QIV since he is adding πk revolutions.

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4. **Solve** the following equations over the given interval. State final answer as an exact value where possible, or correct to 3 decimal places.

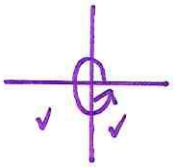
a) $2\sin\theta - 5 = 9\sin\theta - 3, \quad [0, 2\pi]$

$\frac{-7\sin\theta}{-7} = \frac{2}{-7}$

$\sin\theta = -2/7$

$\theta_R = \sin^{-1}(2/7)$

$\theta_R = 0.290$



Q3

$\theta = \pi + \theta_R$

$\theta = \pi + 0.290$

$\theta = 3.432$

Q4

$\theta = 2\pi - \theta_R$

$\theta = 2\pi - 0.290$

$\theta = 5.993$

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b) $20\csc\theta - 12\csc\theta = 5\csc\theta + 17, \quad [0, 2\pi]$

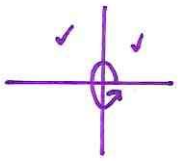
$\frac{3\csc\theta}{3} = \frac{17}{3}$

$\csc\theta = 17/3$

$\sin\theta = 3/17$

$\theta_R = \sin^{-1}(3/17)$

$\theta_R = 0.177$



Q1

$\theta = \theta_R$

$\theta = 0.177$

Q2

$\theta = \pi - \theta_R$

$\theta = \pi - 0.177$

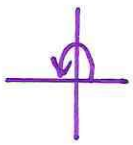
$\theta = 2.965$

---

c) $6 - \cos\theta + 3 = 9, \quad [0, \pi]$

$-\cos\theta = 0$

$\cos\theta = 0$



$\theta = \frac{\pi}{2}$

d) $-\sec\theta + 5 = -5\sec\theta + 6, \quad [0, 2\pi)$

$\frac{4\sec\theta}{4} = \frac{1}{4}$

$\sec\theta = 1/4$

$\cos\theta = 4$

$\therefore$ No solution

---

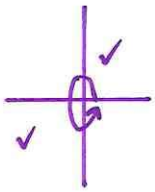
e) $11\tan\theta - 4 = \tan\theta + 11, \quad$ where $\theta \in \mathbb{R}$

$\frac{10\tan\theta}{10} = \frac{15}{10}$

$\tan\theta = \frac{3}{2}$

$\theta_R = \tan^{-1}(3/2)$

$\theta_R = 0.983$



Q1

$\theta = \theta_R$

$\theta = 0.983$

Q3

$\theta = \pi + \theta_R$

$\theta = \pi + 0.983$

$\theta = 4.125$

$\theta = 0.983 + 2\pi k, \ k \in \mathbb{Z}$

$\theta = 4.125 + 2\pi k, \ k \in \mathbb{Z}$

---

f) $\cot\theta = 3\cot\theta, \quad$ where $\theta \in \mathbb{R}$

$\frac{0}{2} = \frac{2\cot\theta}{2}$

$0 = \cot\theta$

undefined $= \tan\theta$

$\theta = \frac{\pi}{2} + 2\pi k, \ k \in \mathbb{Z}$

$\theta = \frac{3\pi}{2} + 2\pi k, \ k \in \mathbb{Z}$

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5. **Solve** the following equations over the given interval. State final answer as an exact value where possible, or correct to 3 decimal places.

a) $\cos^2\theta - \cos\theta - 6 = 0, \quad 0° \leq \theta \leq 360°$

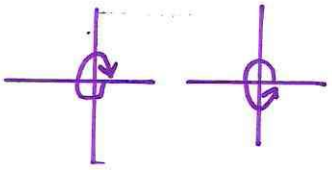
$(\cos\theta - 3)(\cos\theta + 2) = 0$

$\cos\theta - 3 = 0$	$\cos\theta + 2 = 0$
$\cos\theta = 3$	$\cos\theta = -2$
No solution	No solution

$\therefore$ No solution

---

b) $\sin^2\theta - 2\sin\theta = 0, \quad -360° \leq \theta \leq 360°$



$\sin\theta(\sin\theta - 2) = 0$

$\sin\theta = 0$	$\sin\theta - 2 = 0$
$\theta = 0°, 180°, 360°, -180°, -360°$	$\sin\theta = 2$
	No solution

$\therefore \theta = -360°, -180°, 0°, 180°, 360°$

---

c) $\sec^2\theta = 1, \quad 0 \leq \theta < 2\pi$

$\sec\theta = \pm\sqrt{1}$

$\sec\theta = \pm 1$

$\cos\theta = \pm 1$

$\theta = 0, \pi,$

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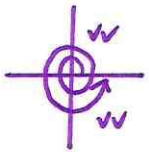
d) $2\cos^2\theta - 3\cos\theta + 1 = 0, \ [0, 4\pi]$



$(2\cos\theta - 1)(\cos\theta - 1) = 0$

$2\cos\theta - 1 = 0$	$\cos\theta - 1 = 0$
$\cos\theta = 1/2$	$\cos\theta = 1$
$\theta_R = \frac{\pi}{3}$	$\theta = 0, 2\pi, 4\pi$
$\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$	

$\therefore \theta = 0, \frac{\pi}{3}, \frac{5\pi}{3}, 2\pi, \frac{7\pi}{3}, \frac{11\pi}{3}, 4\pi$

---

e) $4\cos^2\theta - 3 = 0, \ \text{where } \theta \in \mathbb{R}$

$4\cos^2\theta = 3$

$\cos^2\theta = \frac{3}{4}$

$\cos\theta = \pm\sqrt{\frac{3}{4}}$

$\cos\theta = \pm\frac{\sqrt{3}}{2}$

$\theta_R = \frac{\pi}{6}$

$\theta = \frac{\pi}{6} + 2\pi k, k \in \mathbb{Z}$

$\theta = \frac{5\pi}{6} + 2\pi k, k \in \mathbb{Z}$

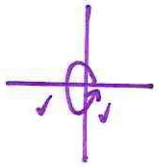
$\theta = \frac{7\pi}{6} + 2\pi k, k \in \mathbb{Z}$

$\theta = \frac{11\pi}{6} + 2\pi k, k \in \mathbb{Z}$

---

f) $7\sin^2\theta - 69\sin\theta - 10 = 0, \ \text{where } \theta \in \mathbb{R}$

$(7\sin\theta + 1)(\sin\theta - 10) = 0$



$7\sin\theta + 1 = 0$	$\sin\theta - 10 = 0$
$\sin\theta = -1/7$	$\sin\theta = 10$
$\theta_R = \sin^{-1}(1/7)$	No solution
$\theta_R = 0.143$	
Q3: $\theta = \pi + \theta_R$, $\theta = 3.285$	Q4: $\theta = 2\pi - \theta_R$, $\theta = 6.140$

$\therefore \theta = 3.285 + 2\pi k, k \in \mathbb{Z}$

$\theta = 6.140 + 2\pi k, k \in \mathbb{Z}$

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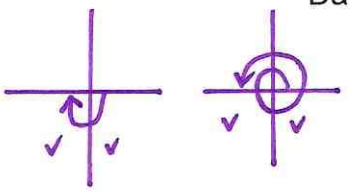
g) $6\tan x \csc x = -14\tan x, \quad [-\pi, 3\pi]$



$6\tan x \csc x + 14\tan x = 0$

$2\tan x(3\csc x + 7) = 0$

$2\tan x = 0$	$3\csc x + 7 = 0$
$\tan x = 0$	$\csc x = -7/3$
$x = 0, \pi, 2\pi, 3\pi, -\pi$	$\sin x = -3/7$
	$x_R = \sin^{-1}(3/7)$
	$x_R = 0.443$

$x_R = -0.443, -2.699, 3.585, 5.840$

$x = 0, \pi, 2\pi, 3\pi, -\pi, -0.443, -2.699, 3.585, 5.840$

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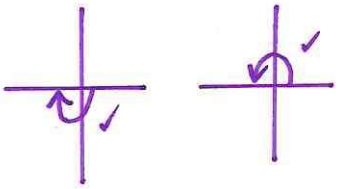
h) $12\sec^2\theta - 13\sec\theta - 4 = 0, \quad [-\pi, \pi]$



$(4\sec\theta + 1)(3\sec\theta - 4) = 0$

$4\sec\theta + 1 = 0$	$3\sec\theta - 4 = 0$
$\sec\theta = -1/4$	$\sec\theta = 4/3$
$\cos\theta = -4$	$\cos\theta = 3/4$
No solution	$\theta_R = \cos^{-1}(3/4)$
	$\theta_R = 0.723$

$\theta = -0.723, 0.723$

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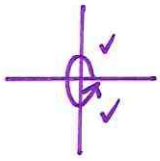
i) $5\sin\theta\cos\theta - 2\sin\theta = 0, \quad [0, 2\pi]$



$\sin\theta(5\cos\theta - 2) = 0$

$\sin\theta = 0$	$5\cos\theta - 2 = 0$
$\theta = 0, \pi, 2\pi$	$\cos\theta = 2/5$
	$\theta_R = 1.159$
	Q1: $\theta = \theta_R$, $\theta = 1.159$
	Q4: $\theta = 2\pi - \theta_R$, $\theta = 5.124$

$\therefore \theta = 0, \pi, 2\pi, 1.159, 5.124$

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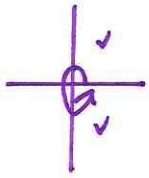
6. **Solve** the following equations over the given interval. State final answer as an exact value where possible, or correct to 3 decimal places.

a) $\sec^2\theta - 4\sec\theta = 5,\ 0° \le \theta \le 360°$

$\sec^2\theta - 4\sec\theta - 5 = 0$

$(\sec\theta + 1)(\sec\theta - 5) = 0$



$\sec\theta + 1 = 0$	$\sec\theta - 5 = 0$
$\sec\theta = -1$	$\sec\theta = 5$
$\cos\theta = -1$	$\cos\theta = 1/5$
$\theta = 180°$	$\theta_R = \cos^{-1}(1/5)$
	$\theta_R = 78.463°$
Q1	Q4
$\theta = \theta_R$	$\theta = 360° - \theta_R$
$\theta = 78.463°$	$\theta = 281.537$

$\therefore \theta = 180°, 78.463°, 281.537°$

---

b) $\cos^2\theta = \cos\theta + 1,\ 0° \le \theta \le 360°$

$\cos^2\theta - \cos\theta - 1 = 0$

$$\cos\theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\cos\theta = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

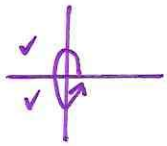
$$\cos\theta = \frac{1 \pm \sqrt{5}}{2}$$

$\cos\theta = \frac{1+\sqrt{5}}{2}$

$\cos\theta = 1.618$

No solution

$\cos\theta = \frac{1-\sqrt{5}}{2}$



$\cos\theta = -0.618$

$\theta_R = \cos^{-1}(0.618)$

$\theta_R = 51.830°$

Q2	Q3
$\theta = 180° - \theta_R$	$\theta = 180° + \theta_R$
$\theta = 128.17°$	$\theta = 231.83°$

$\therefore \theta = 128.17°, 231.83°$

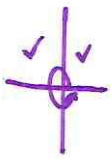
c) $2\csc^2\theta - 11 = 1, \ [0, 2\pi]$

$\frac{2\csc^2\theta}{2} = \frac{12}{2}$

$\csc^2\theta = 6$

$\csc\theta = \pm\sqrt{6}$

$\sin\theta = \pm\frac{1}{\sqrt{6}}$



$\sin\theta = \frac{1}{\sqrt{6}}$

$\theta_R = \sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

$\theta_R = 0.421$

Q1: $\theta = \theta_R$, $\theta = 0.421$

Q2: $\theta = \pi - \theta_R$, $\theta = 2.721$

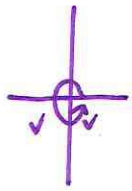
$\sin\theta = \frac{1}{-\sqrt{6}}$

$\theta_R = \sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

$\theta_R = 0.421$

Q3: $\theta = \pi + \theta_R$, $\theta = 3.563$

Q4: $\theta = 2\pi - \theta_R$, $\theta = 5.862$



$\therefore \ \theta = 0.421, 2.721, 3.563, 5.862$

---

d) $3\tan^2\theta + 5 = 3\tan\theta, \ [0, 2\pi]$

$3\tan^2\theta - 3\tan\theta + 5 = 0$

$\tan\theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\tan\theta = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(5)}}{2(3)}$

$\tan\theta = \frac{3 \pm \sqrt{-51}}{6}$

No solution

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e) $3\sin^2 x + 5\sin x + 1 = 0,\ -\pi < x < \pi$

$\sin x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

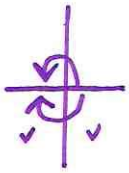
$\sin x = \frac{-(5) \pm \sqrt{(5)^2-4(3)(1)}}{2(3)}$

$\sin x = \frac{-5 \pm \sqrt{13}}{6}$

$\sin x = \frac{-5+\sqrt{13}}{6}$

$\sin x = -0.232$

$X_R = 0.234$



$X = -0.234$

$X = -2.907$

$\sin x = \frac{-5-\sqrt{13}}{6}$

$\sin x = -1.434$

No solution

$\therefore\ X = -0.234, -2.907$

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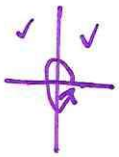
f) $12\csc^2 x + 4\csc x - 21 = 0,\ 0 \le x \le 2\pi$



$(6\csc x - 7)(2\csc x + 3) = 0$

$6\csc x - 7 = 0$

$\csc x = 7/6$

$\sin x = 6/7$

$X_R = \sin^{-1}(6/7)$

$X_R = 1.030$

$2\csc x + 3 = 0$

$\csc x = -3/2$

$\sin x = -2/3$

$X_R = \sin^{-1}(2/3)$

$X_R = 0.730$

Q1: $\theta = \theta_R$, $\theta = 1.030$

Q2: $\theta = \pi - \theta_R$, $\theta = 2.112$

Q3: $\theta = \pi + \theta_R$, $\theta = 3.872$

Q4: $\theta = 2\pi - \theta_R$, $\theta = 5.553$

$\therefore\ \theta = 1.030, 2.112, 3.872, 5.553$