 Date: Answer key

Review – CHAPTER 3 Questions

3.2 – The Remainder Theorem

1. Using the **Remainder Theorem**, determine the **remainder** when each of the following polynomials is divided by $(x+2)$. State whether or not $(x+2)$ is a factor of each.

a) $P(x) = -4x^4 - 3x^3 + 2x^2 - x + 5$

$P(-2) = -4(-2)^4 - 3(-2)^3 + 2(-2)^2 - (-2) + 5$

$P(-2) = -64 + 24 + 8 + 4 + 5$

$P(-2) = -23$

↳ The remainder is -23

$\therefore$ $(x+2)$ is not a factor

b) $P(x) = 7x^5 + 5x^4 + 23x^2 + 8$

$P(-2) = 7(-2)^5 + 5(-2)^4 + 23(-2)^2 + 8$

$P(-2) = -224 + 80 + 92 + 8$

$P(-2) = -44$

↳ The remainder is -44

$\therefore$ $(x+2)$ is not a factor

c) $P(x) = 8x^3 - 1$

$P(-2) = 8(-2)^3 - 1$

$P(-2) = -64 - 1$

$P(-2) = -65$

↳ The remainder is -65

$\therefore$ $(x+2)$ is not a factor

2. Given $(2x^3 + 5x^2 - kx + 9) \div (x+3)$, determine the value of k if the remainder is 6.

Let $P(x) = 2x^3 + 5x^2 - kx + 9$

$P(-3) = 6$

$2(-3)^3 + 5(-3)^2 - k(-3) + 9 = 6$

$-54 + 45 + 3k + 9 = 6$

$\frac{3k}{3} = \frac{6}{3}$

$k = 2$

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3. **Divide** the following polynomials.

a) $(4w^4 + 3w^3 - 7w^2 + 2w - 1) \div (w + 2)$

-2	4	3	-7	2	-1
+	↓	-8	10	-6	8
x	4	-5	3	-4	7

$$\therefore \frac{4w^4+3w^3-7w^2+2w-1}{w+2} = 4w^3-5w^2+3w-4+\frac{7}{w+2}$$

b) $\frac{x^4+2x^3-8x^2-5x-2}{x-2}$

2	1	2	-8	-5	-2
+	↓	2	8	0	-10
x	1	4	0	-5	-12

$$\therefore \frac{x^4+2x^3-8x^2-5x-2}{x-2} = x^3+4x^2-5-\frac{12}{x-2}$$

c) $5y^4 + 2y^2 - y + 4 \div y + 1$

-1	5	0	2	-1	4
+	↓	-5	5	-7	8
x	5	-5	7	-8	12

$$\therefore \frac{5y^4+2y^2-y+4}{y+1} = 5y^3-5y^2+7y-8+\frac{12}{y+1}$$

d) $3x^2 - 16x + 5 \div x - 5$

5	3	−16	5
+	↓	15	−5
x	3	−1	0

$$\therefore \frac{3x^2-16x+5}{x-5} = 3x-1$$

e) $2x^4 - 3x^3 - 5x^2 + 6x - 1 \div x + 3$

−3	2	−3	−5	6	−1
+	↓	−6	27	−66	180
x	2	−9	22	−60	179

$$\therefore \frac{2x^4-3x^3-5x^2+6x-1}{x+3} = 2x^3-9x^2+22x-60+\frac{179}{x+3}$$

f) $(4x^3 + 5x^2 - 7) \div (x - 2)$

2	4	5	0	−7
+	↓	8	26	52
x	4	13	26	45

$$\therefore \frac{4x^3+5x^2-7}{x-2} = 4x^2+13x+26+\frac{45}{x-2}$$

4. Determine if, when divided by each binomial, there will be a resulting remainder or not.

a) $(3x^3 - 4x^2 + 6x - 9) \div (x + 1)$

Let $P(x) = 3x^3 - 4x^2 + 6x - 9$

$P(-1) = 3(-1)^3 - 4(-1)^2 + 6(-1) - 9$

$P(-1) = -3 - 4 - 6 - 9$

$P(-1) = -22$

$\therefore$ yes, the remainder is -22

b) $(3x^2 - 8x + 4) \div (x - 2)$

Let $P(x) = 3x^2 - 8x + 4$

$P(2) = 3(2)^2 - 8(2) + 4$

$P(2) = 12 - 16 + 4$

$P(2) = 0$

$\therefore$ No remainder

c) $(x^3 + 5x^2 - 7x + 9) \div (x + 5)$

Let $P(x) = x^3 + 5x^2 - 7x + 9$

$P(-5) = (-5)^3 + 5(-5)^2 - 7(-5) + 9$

$P(-5) = -125 + 125 + 35 + 9$

$P(-5) = 44$

$\therefore$ yes, the remainder is 44

5. When $4x^2 - 8x - 20$ is divided by $(x + k)$, the remainder is 12. Determine the possible values for k.

Let $P(x) = 4x^2 - 8x - 20$

$P(-k) = 12$

$4(-k)^2 - 8(-k) - 20 = 12$

$4k^2 + 8k - 32 = 0$

$4(k^2 + 2k - 8) = 0$

$4(k + 4)(k - 2) = 0$

$k = -4$ OR $k = 2$

3.3 – The Factor Theorem

6. Determine if $(x-1)$ is a **factor** of each polynomial.

a) $-4x^4-3x^3+2x^2-x+5$ Let $P(x)=-4x^4-3x^3+2x^2-x+5$

$P(1)=-4(1)^4-3(1)^3+2(1)^2-(1)+5$

$P(1)=-4-3+2-1+5$

$P(1)=-1$

$\therefore$ since $P(1)\neq 0$, no, $(x-1)$ is not a factor

b) $7x^5+5x^4+23x^2+8$ Let $P(x)=7x^5+5x^4+23x^2+8$

$P(1)=7(1)^5+5(1)^4+23(1)^2+8$

$P(1)=7+5+23+8$

$P(1)=43$

$\therefore$ since $P(1)\neq 0$, no, $(x-1)$ is not a factor

c) $2x^4-3x^3-5x^2+7x-1$ Let $P(x)=2x^4-3x^3-5x^2+7x-1$

$P(1)=2(1)^4-3(1)^3-5(1)^2+7(1)-1$

$P(1)=2-3-5+7-1$

$P(1)=0$

$\therefore$ since $P(1)=0$, yes, $(x-1)$ is a factor

7. Determine if each polynomial has a **factor** of $(x+2)$.

a) $-3x^3+2x^2+10x+5$ Let $P(x)=-3x^3+2x^2+10x+5$

$P(-2)=-3(-2)^3+2(-2)^2+10(-2)+5$

$P(-2)=24+8-20+5$

$P(-2)=17$

$\therefore$ since $P(-2)\neq 0$, no, $(x+2)$ is not a factor

b) $2x^4-3x^3-5x^2+36$ Let $P(x)=2x^4-3x^3-5x^2+36$

$P(-2)=2(-2)^4-3(-2)^3-5(-2)^2+36$

$P(-2)=32+24-20+36$

$P(-2)=72$

$\therefore$ since $P(-2)\neq 0$, no, $(x+2)$ is not a factor

c) $3x^3-12x-2$ Let $P(x)=3x^3-12x-2$

$P(-2)=3(-2)^3-12(-2)-2$

$P(-2)=-24+24-2$

$P(-2)=-2$

$\therefore$ since $P(-2)\neq 0$, no, $(x+2)$ is not a factor

8. Explain how you know that $(x+2)$ is not a factor of the polynomial $y = 4x^6 + 3x^4 - x^3 + 9x^2 - 15$ without having to calculate the remainder.

↳ The possible integral zeroes are $\pm 1, \pm 3, \pm 5, \pm 15$

↳ -2 is not a factor of the constant term.

9. Determine the possible **integral zeros** of each polynomial.

a) $P(x) = x^3 - 5x^2 + x + 4$

$\pm 1, \pm 2, \pm 4$

b) $P(x) = x^4 - 3x^2 + x + 12$

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

c) $P(x) = 2x^3 + 3x^2 - 17x + 30$

$\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

10. **Factor** each polynomial below completely.

a) $x^3 + 2x^2 - 13x + 10$

$$\begin{array}{c|cccc} 1 & 1 & 2 & -13 & 10 \\ + & \downarrow & 1 & 3 & -10 \\ \hline \times & 1 & 3 & -10 & 0 \end{array}$$

$$\therefore x^3 + 2x^2 - 13x + 10 = (x-1)(x^2+3x-10)$$

$$= (x-1)(x+5)(x-2)$$

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b) $2x^3 + 3x^2 - 3x - 2$

$$\begin{array}{c|cccc} 1 & 2 & 3 & -3 & -2 \\ + & \downarrow & 2 & 5 & 2 \\ \hline \times & 2 & 5 & 2 & 0 \end{array}$$

$$\therefore 2x^3 + 3x^2 - 3x - 2 = (x-1)(2x^2+5x+2)$$

$$= (x-1)(2x+1)(x+2)$$

c) $x^3 - x^2 - 26x - 24$

$$\begin{array}{c|cccc} -1 & 1 & -1 & -26 & -24 \\ + & \downarrow & -1 & 2 & 24 \\ \hline \times & 1 & -2 & -24 & 0 \end{array}$$

$$\therefore x^3 - x^2 - 26x - 24 = (x+1)(x^2-2x-24)$$

$$= (x+1)(x-6)(x+4)$$

d) $x^4 - 26x^2 + 25$

1	1	0	-26	0	25
+	↓	1	1	-25	-25
x	1	1	-25	-25	0

-1	1	1	-25	-25
+	↓	-1	0	25
x	1	0	-25	0

$$\therefore x^4 - 26x^2 + 25 = (x-1)(x^3 + x^2 - 25x - 25)$$

$$= (x-1)(x+1)(x^2 - 25)$$

$$= (x-1)(x+1)(x+5)(x-5)$$

e) $3x^3 - 5x^2 - 26x - 8$

-2	3	-5	-26	-8
+	↓	-6	22	8
x	3	-11	-4	0

$$\therefore 3x^3 - 5x^2 - 26x - 8 = (x+2)(3x^2 - 11x - 4)$$

$$= (x+2)(3x+1)(x-4)$$

11. Each polynomial has $(x-3)$ as a factor. Determine the value of k in each case.

a) $kx^3 - 10x^2 + 2x + 3$

Let $P(x) = kx^3 - 10x^2 + 2x + 3$

if $(x-3)$ is a factor, then $P(3) = 0$

$$k(3)^3 - 10(3)^2 + 2(3) + 3 = 0$$

$$27k - 90 + 6 + 3 = 0$$

$$\frac{27k}{27} = \frac{81}{27}$$

$$k = 3$$

b) $4x^4 - 3x^3 - 2x^2 + kx - 9$

Let $P(x) = 4x^4 - 3x^3 - 2x^2 + kx - 9$

if $(x-3)$ is a factor, then $P(3) = 0$

$$4(3)^4 - 3(3)^3 - 2(3)^2 + k(3) - 9 = 0$$

$$324 - 81 - 18 + 3k - 9 = 0$$

$$\frac{3k}{3} = \frac{-216}{3}$$

$$k = -72$$

3.1 & 3.4 – Characteristics and Graphing Polynomial Functions

12. State the **zeros** and the **y – intercept** of each of the following polynomials functions.

a) $y = (x+3)(x-5)(x+1)$

↳ Zeros : $x = -3, 5, -1$

↳ y-int : $y = (0+3)(0-5)(1)$

$y = -15$ @ $(0, -15)$

b) $y = -(x-2)(x+2)(x-5)(x+1)$

↳ zeros : $x = 2, -2, 5, -1$

↳ y-int : $y = -(0-2)(0+2)(0-5)(0+1)$

$y = -20$ @ $(0, -20)$

c) $y = (3-x)(x+1)(x-8)$

↳ zeros : $x = 3, -1, 8$

↳ y-int : $y = (3-0)(0+1)(0-8)$

$y = -24$ @ $(0, -24)$

13. **Explain** what happens to the graph of a polynomial function when the leading coefficient is negative.

↳ Odd degree, graph extends up in Quadrant 2 and down in Quadrant 4.

↳ Even degree, graph extends down in Quadrant 3 and down in Quadrant 4.

14. **Explain** the end behaviour of each function.

a) $f(x) = 4x^5 - 3x^4 + 2x^3$

↳ Down in Quadrant 3 and up in Quadrant 1

b) $y = -x^4 + x^3 - x^2 + x - 1$

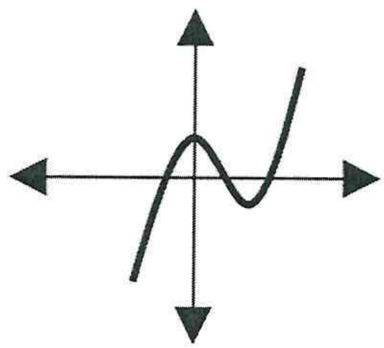
↳ Down in Quadrant 3 and down in Quadrant 4

15. For each of the following graphs, determine the possible **degree** of the polynomial function and the sign of the **leading coefficient**.

a)

Degree: 3

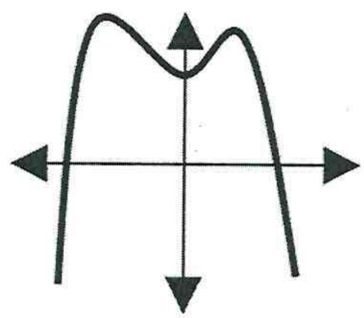
Leading Coefficient: positive



b)

Degree: 4

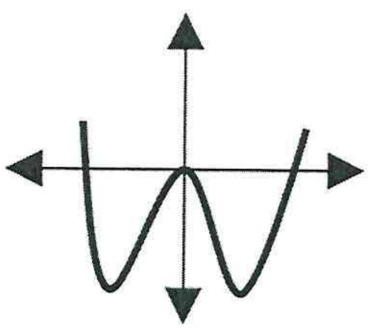
Leading Coefficient: negative



c)

Degree: 4

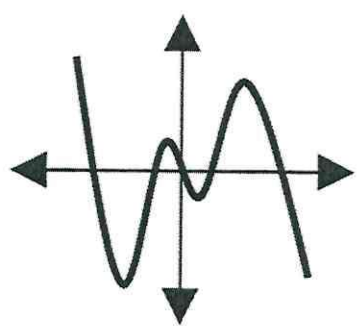
Leading Coefficient: positive



d)

Degree: 5

Leading Coefficient: negative



16. Write the **equation** of a cubic function with roots at -2, -3, and 1. The point $(2, 40)$ is on the graph of this function.

$y = a(x+2)(x+3)(x-1)$

$40 = a(2+2)(2+3)(2-1)$

$40 = a(4)(5)(1)$

$\frac{40}{20} = \frac{a(20)}{20}$

$2 = a$

$\therefore\ y = -2(x+2)(x+3)(x-1)$

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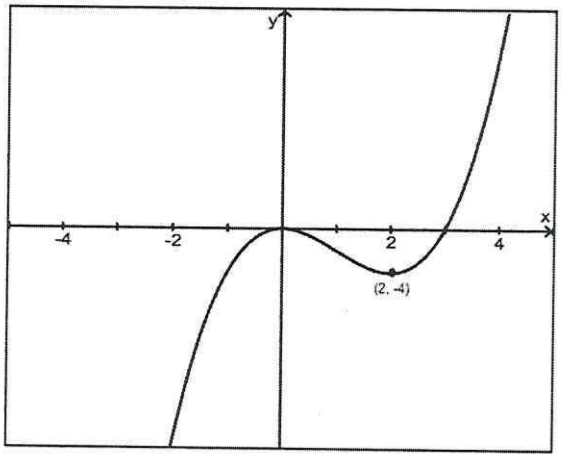
17. Write the **equation** of each polynomial function given below.

a)



$y = a(x)^2(x-3)$

$-4 = a(2)^2(2-3)$

$-4 = a(4)(-1)$

$-4 = a(-4)$

$\frac{-4}{-4} = \frac{a(-4)}{-4}$

$1 = a$

$\therefore\ y = x^2(x-3)$

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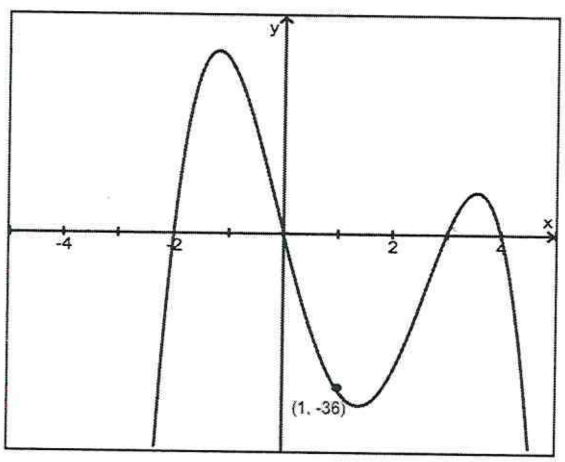
b)



$y = a(x+2)(x)(x-3)(x-4)$

$-36 = a(1+2)(1)(1-3)(1-4)$

$-36 = a(3)(1)(-2)(-3)$

$\frac{-36}{18} = \frac{a(18)}{18}$

$-2 = a$

$\therefore\ y = -2(x+2)(x)(x-3)(x-4)$

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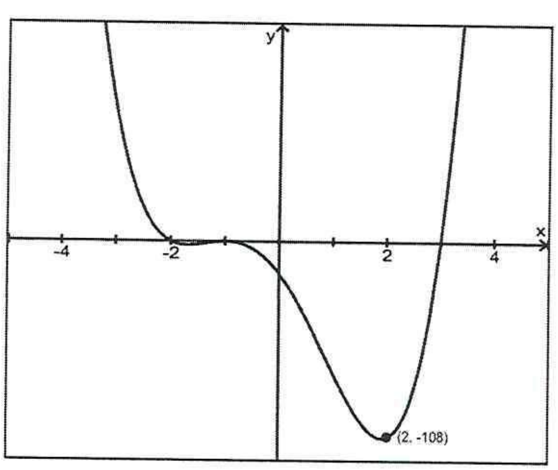
c)



$y = a(x+2)(x+1)^2(x-3)$

$-108 = a(2+2)(2+1)^2(2-3)$

$-108 = a(4)(9)(-1)$

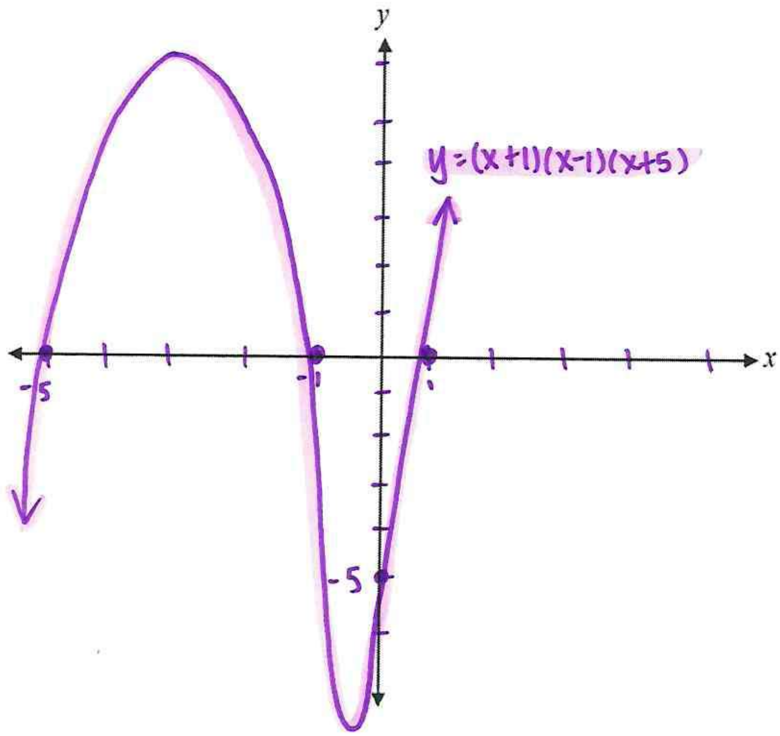
$\frac{-108}{-36} = \frac{a(-36)}{-36}$

$3 = a$

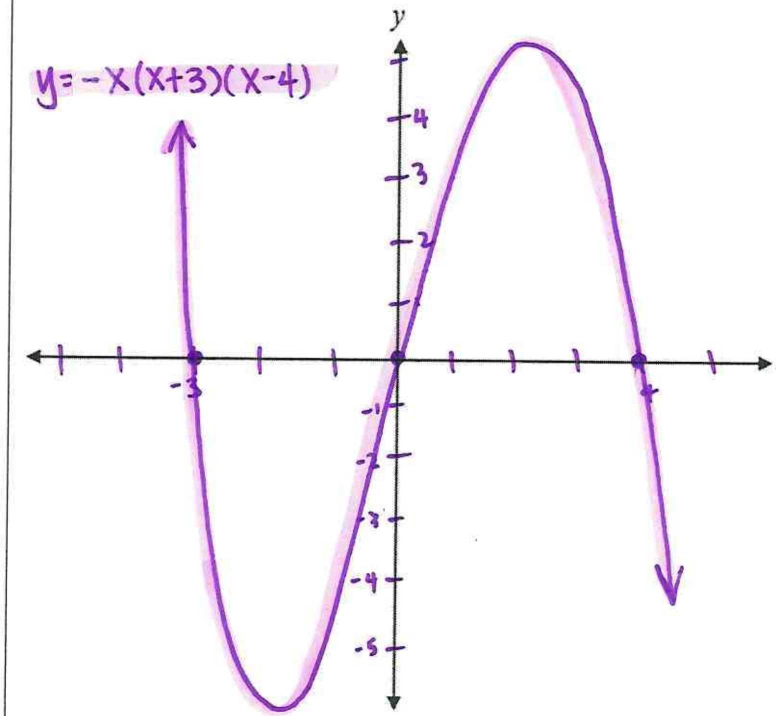
$\therefore\ y = 3(x+2)(x+1)^2(x-3)$

18. **Sketch** each of the following polynomial functions. De sure to include all intercepts on your graph.

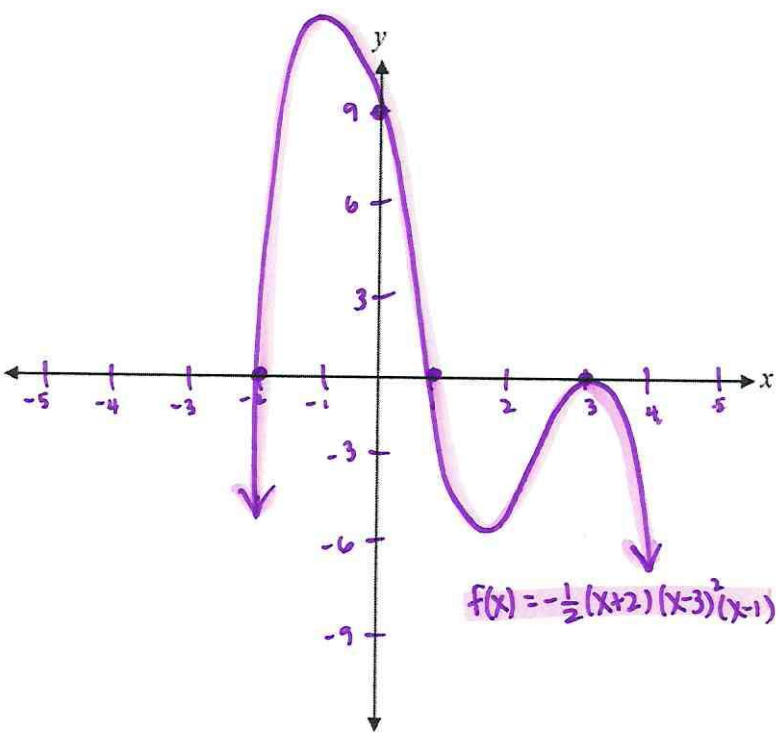
a) $y = (x+1)(x-1)(x+5)$



b) $y = -x(x+3)(x-4)$



c) $f(x) = -\frac{1}{2}(x+2)(x-3)^2(x-1)$



d) $y = (x-1)^3(x+2)^2$