MPC40S Date: Answer Key

Review – CHAPTER 1 Questions

1.1 – Horizontal and Vertical Translations

1. The following points lie on the graph of $f(x)$: $(-2,\ 5)$, $(0,-1)$ and $(3,4)$. Determine the coordinate of the image point that would be included in each of the following graphs.

a) $g(x) = f(x+1)$

$(-2,5) \rightarrow (-3,5)$

$(0,-1) \rightarrow (-1,-1)$

$(3,4) \rightarrow (2,4)$

b) $g(x) = f(x) - 3$

$(-2,5) \rightarrow (-2,2)$

$(0,-1) \rightarrow (0,-4)$

$(3,4) \rightarrow (3,1)$

c) $g(x) = f(x-2) + 5$

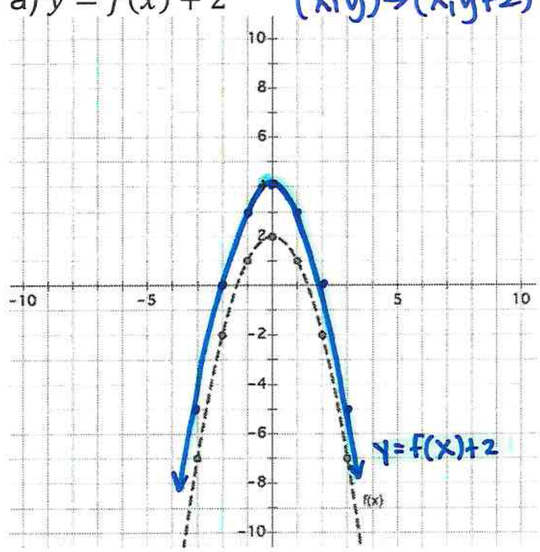
$(-2,5) \rightarrow (0,10)$

$(0,-1) \rightarrow (2,4)$

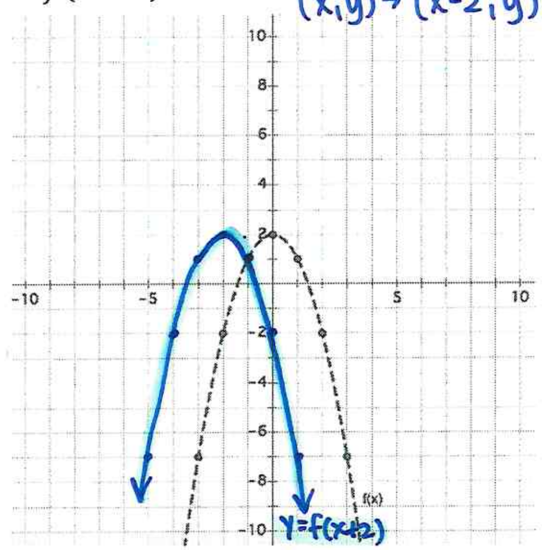
$(3,4) \rightarrow (5,9)$

2. Given the function $f(x) = -x^2 + 2$, sketch the following graphs:

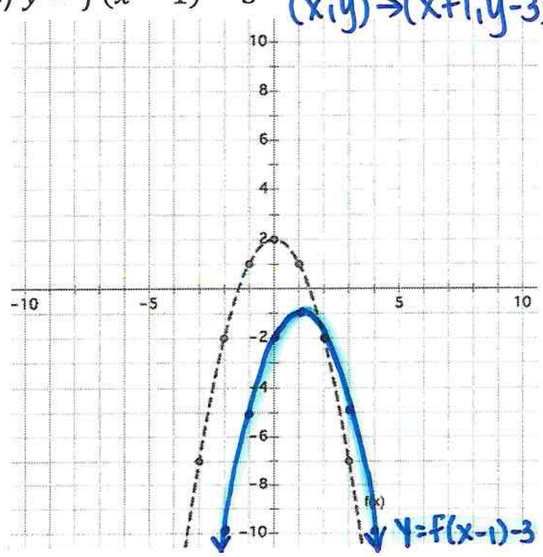
a) $y = f(x) + 2$ $(x,y) \rightarrow (x,y+2)$



b) $y = f(x+2)$ $(x,y) \rightarrow (x-2,y)$



c) $y = f(x-1) - 3$ $(x,y) \rightarrow (x+1,y-3)$



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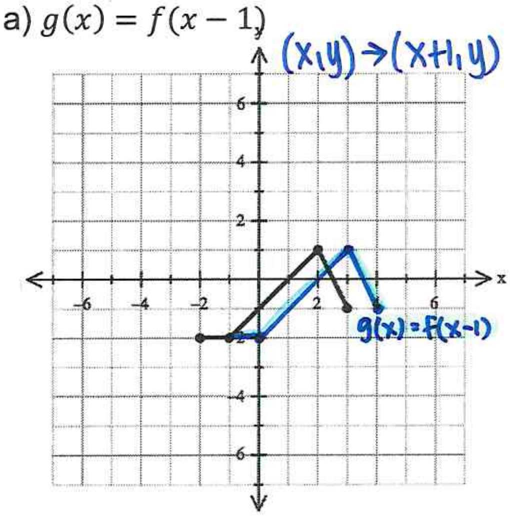
3. Using the following graph of $f(x)$, sketch the graphs of the following function:

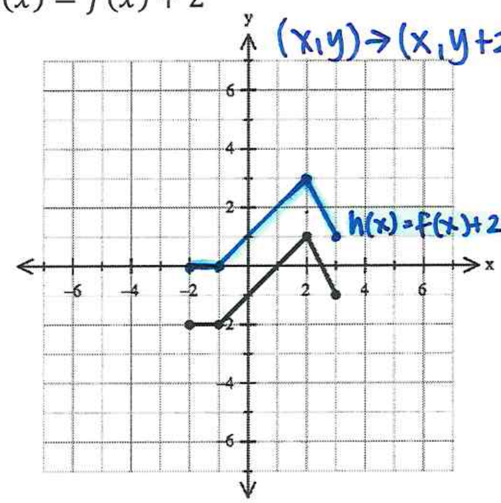
a) $g(x) = f(x-1)$

$(x,y) \rightarrow (x+1, y)$



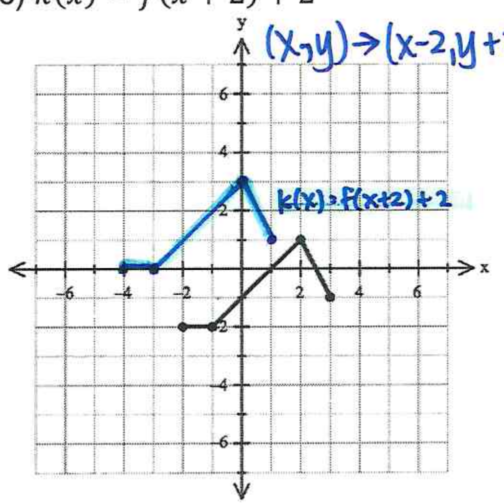
b) $h(x) = f(x) + 2$

$(x,y) \rightarrow (x, y+2)$



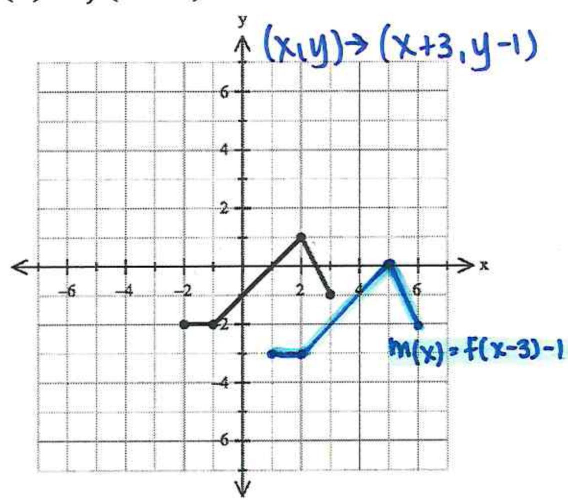
c) $k(x) = f(x+2) + 2$

$(x,y) \rightarrow (x-2, y+2)$



d) $m(x) = f(x-3) - 1$

$(x,y) \rightarrow (x+3, y-1)$



4. Given the function $f(x) = (x-1)^2 + 3$, write the equation for each of the following functions:

a) $g(x) = f(x+4) - 7$

$g(x) = (x+3)^2 - 4$

b) $h(x) = f(x-6) + 5$

$h(x) = (x-7)^2 + 8$

c) $p(x) = f(x+1) - 3$

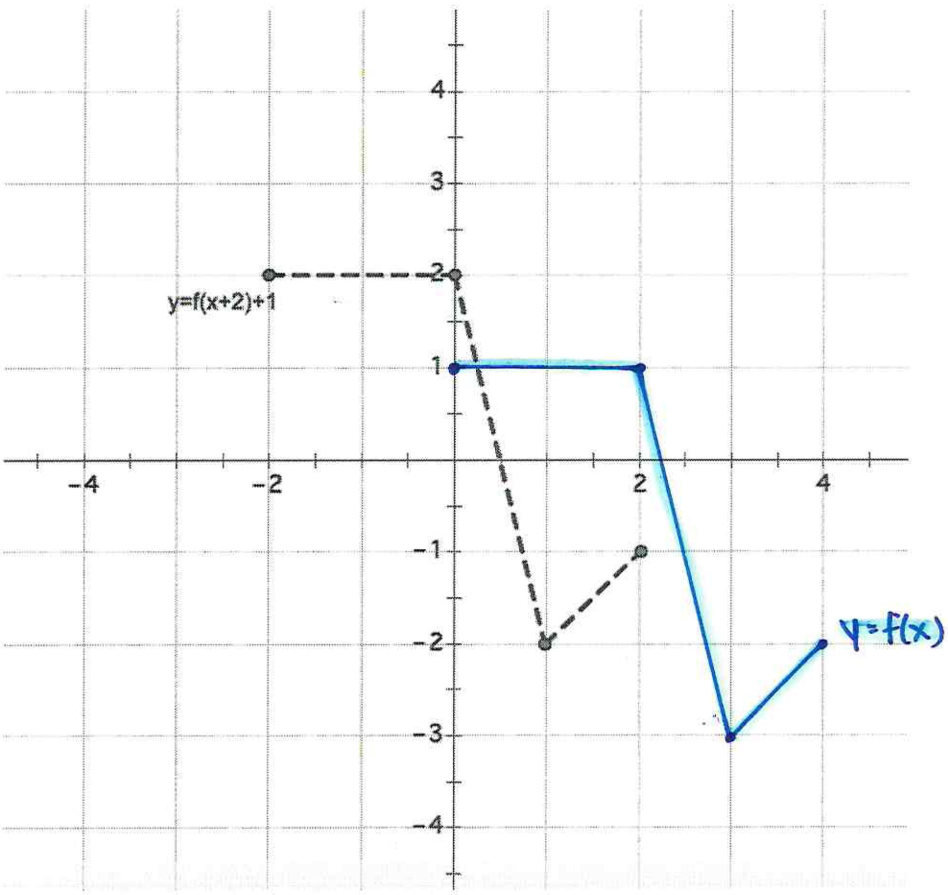
$p(x) = (x)^2$

5. Explain, in words, the type of translations that would be applied to the original graph of $f(x)$ if the graph $y = f(x-4)+3$ is sketched.

↳ Horizontal shift 4 units right

↳ Vertical shift 3 units up

6. Given the graph of $y = f(x+2)+1$ below, sketch the graph of $y = f(x)$.



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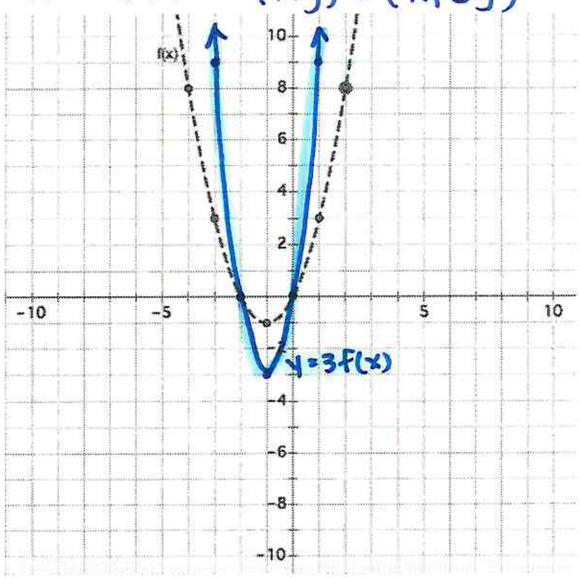
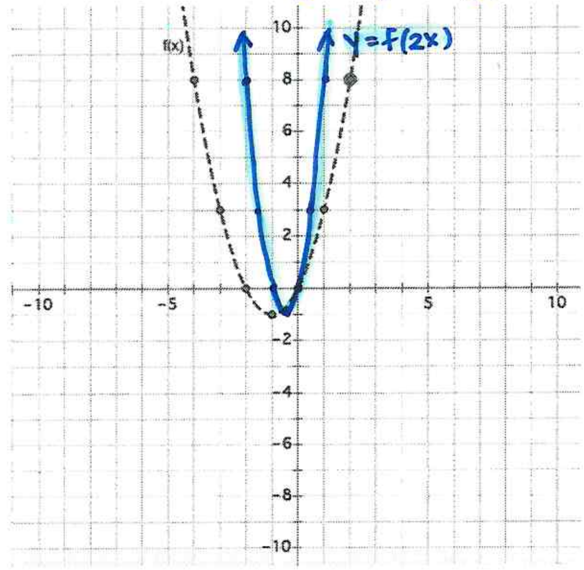
1.2 – Reflections and Stretches/Compressions

1. Sketch the function $f(x) = (x + 1)^2 - 1$ and use this graph to sketch the functions below.

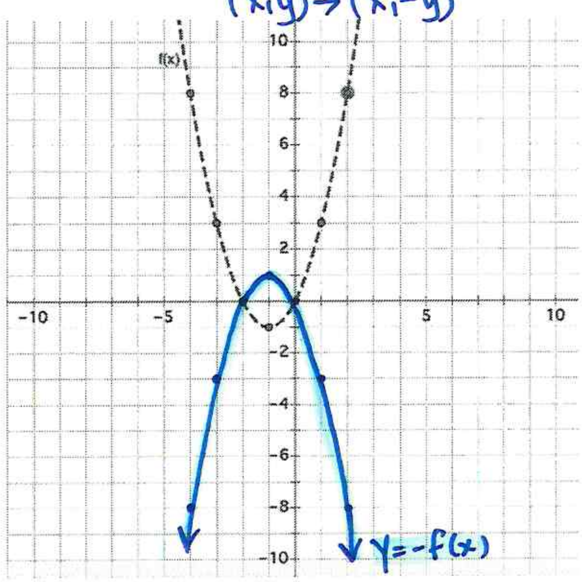
a) $y = 3f(x)$ $(x, y) \rightarrow (x, 3y)$



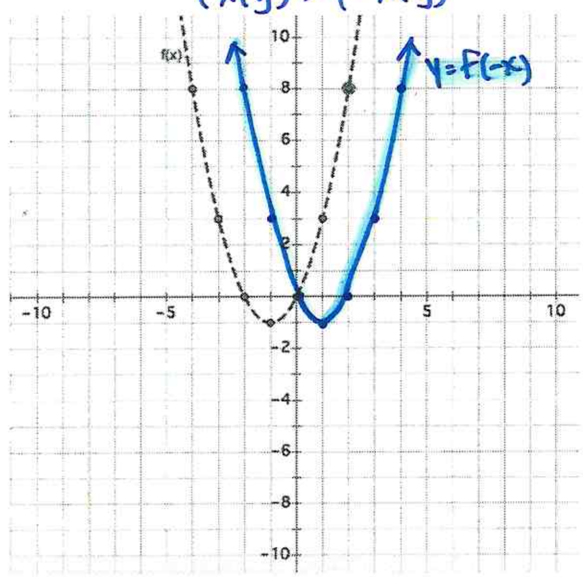
b) $y = f(2x)$ $(x, y) \rightarrow (\frac{1}{2}x, y)$



c) $y = -f(x)$ $(x, y) \rightarrow (x, -y)$



d) $y = f(-x)$ $(x, y) \rightarrow (-x, y)$



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2. The domain of the function $f(x)$ is $[-2, 6]$. The range of the same function is $[0, 4]$. Use this information to determine the domain and range of the following graphs.

a) $y = \frac{1}{2}f(x)$

D: $[-2, 6]$

R: $[0, 2]$

b) $y = f\left(\frac{x}{2}\right)$

D: $[-4, 12]$

R: $[0, 4]$

c) $y = f(-x)$

D: $[-6, 2]$

R: $[0, 4]$

d) $y = -f(4x)$

D: $[-1/2, 3/2]$

R: $[-4, 0]$

e) Explain, in words, why in parts a), b) and c), only the domain OR range changes but not both at the same time.

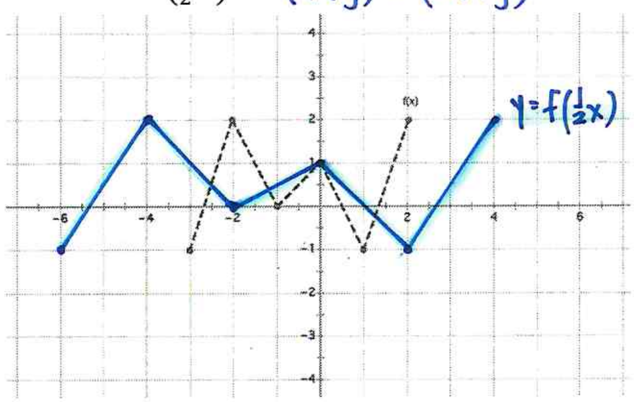
a) Vertical stretch only affects y-values (range)

b) Horizontal stretch only affects x-values (domain)

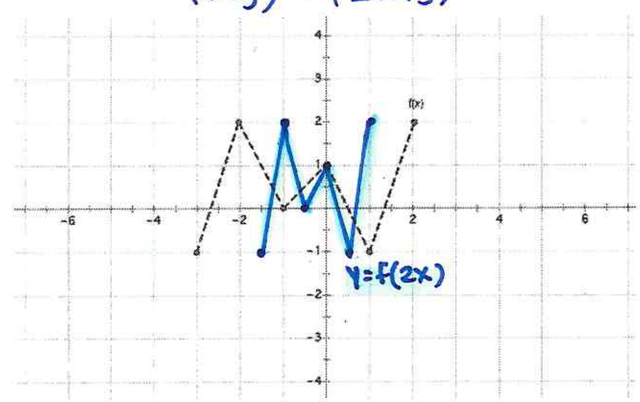
c) Reflection over y-axis affects x-values only (domain)

3. Using the graphs of $f(x)$ below, sketch the following graphs:

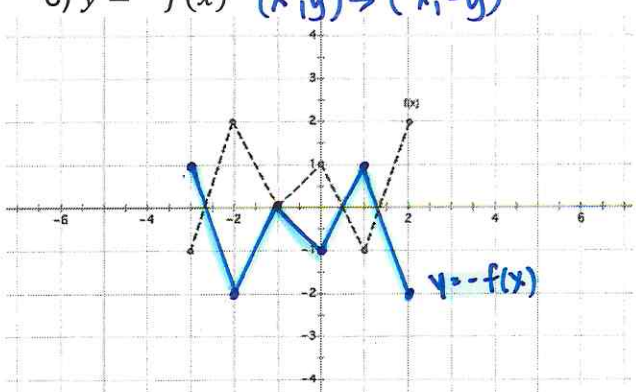
a) $y = f\left(\frac{1}{2}x\right)$ $(x, y) \rightarrow (2x, y)$



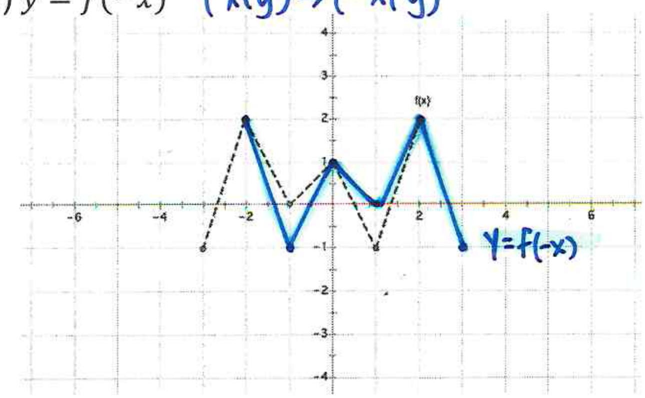
b) $y = f(2x)$ $(x, y) \rightarrow (\frac{1}{2}x, y)$



c) $y = -f(x)$ $(x, y) \rightarrow (x, -y)$



d) $y = f(-x)$ $(x, y) \rightarrow (-x, y)$



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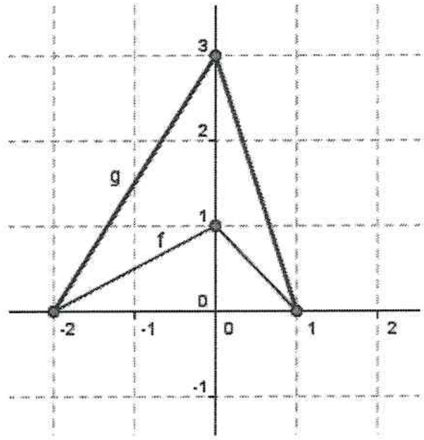
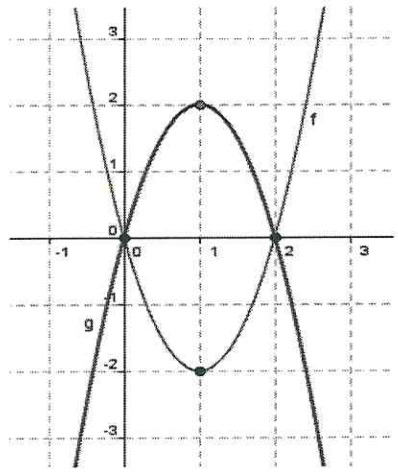
4. Determine the transformation that must be applied to the graph of $f(x)$ to obtain the graph of $g(x)$.

a)



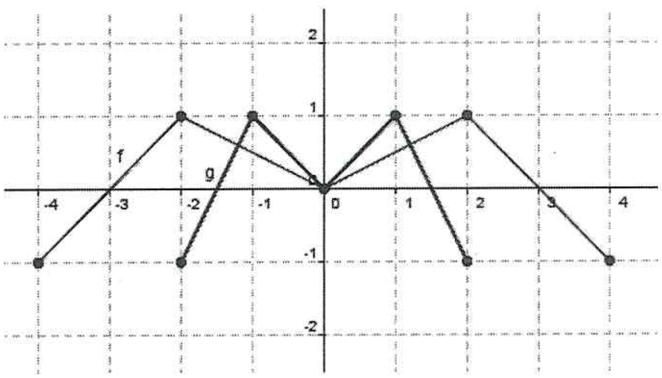
$g(x) =$ 3f(x)

b)



$g(x) =$ -f(x)

c)



$g(x) =$ f(2x)

d) in part b), the equation of $f(x)$ is $f(x) = 2(x-1)^2 - 2$. Determine the equation of $g(x)$.

$g(x) = -f(x)$

$g(x) = -(2(x-1)^2-2)$

$g(x) = -2(x-1)^2+2$

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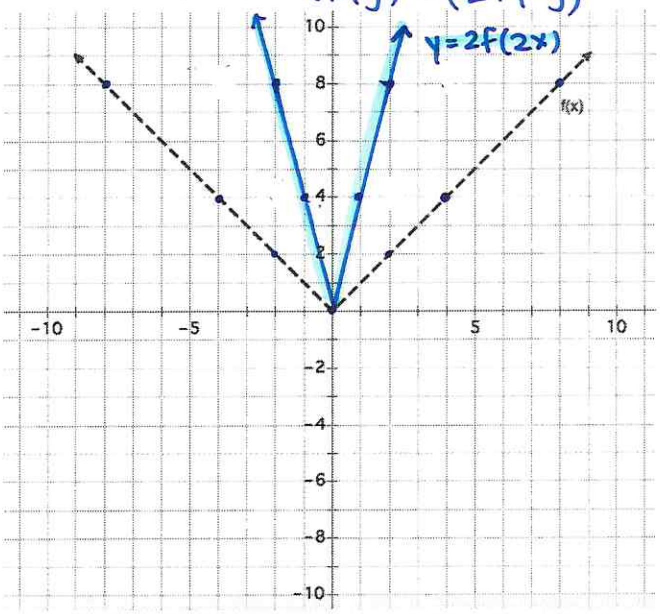
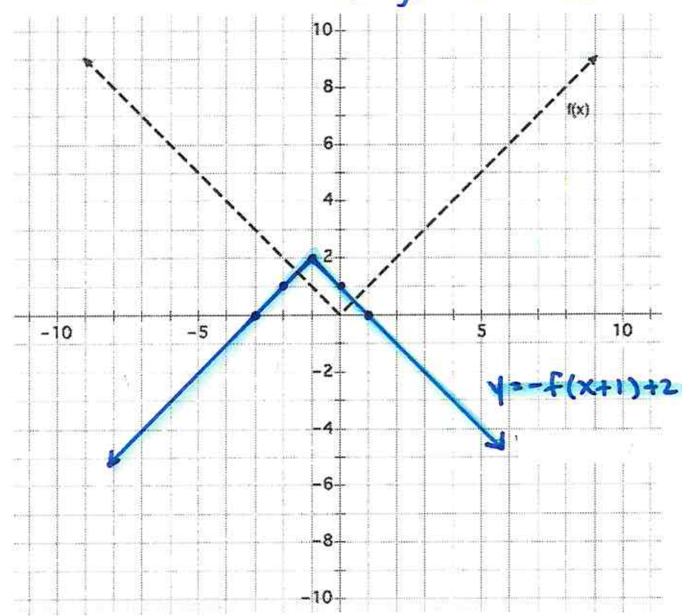
1.3 – Combining Transformations

1. Given $f(x) = |x|$, sketch the following graphs:

a) $y = 2f(2x)$ $(x, y) \rightarrow (\frac{1}{2}x, 2y)$

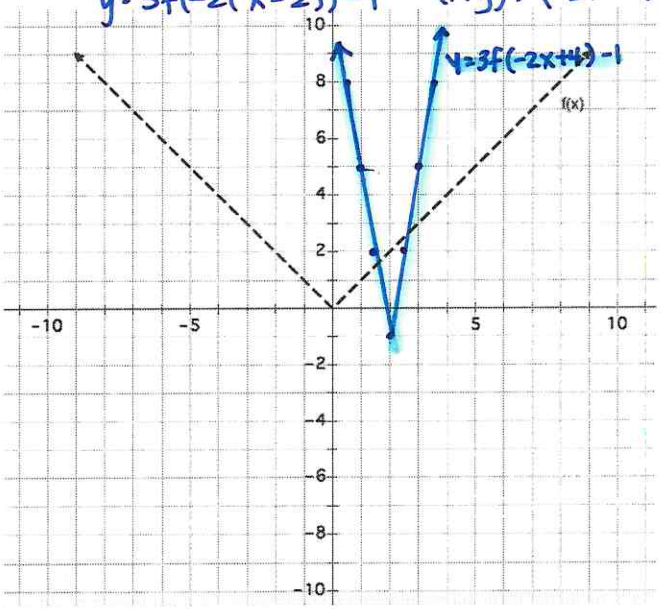


b) $y = -f(x+1) + 2$ $(x, y) \rightarrow (x-1, -y+2)$

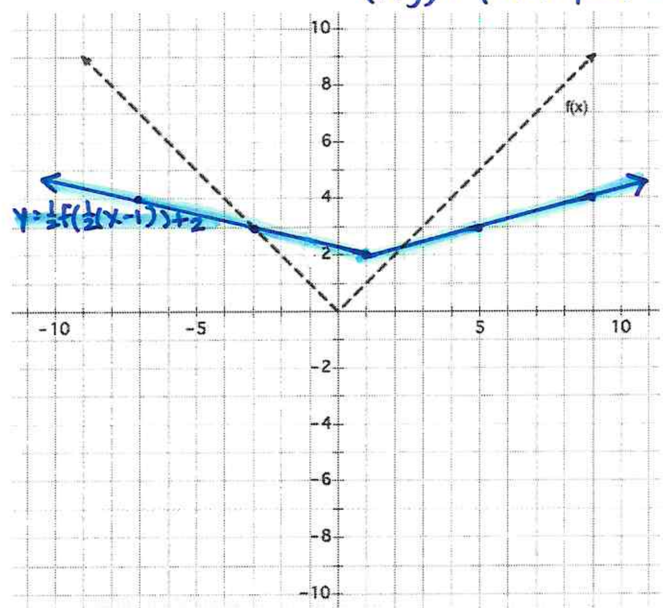


c) $y = 3f(-2x+4) - 1$

$y = 3f(-2(x-2)) - 1$ $(x, y) \rightarrow (-\frac{1}{2}x+2, 3y-1)$



d) $y = \frac{1}{2}f\left(\frac{1}{2}(x-1)\right) + 2$ $(x, y) \rightarrow (2x+1, \frac{1}{2}y+2)$



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2. Suppose that the graph of $f(x)$ is given. Explain, in words, how to obtain the graphs of the following functions.

a) $y = 4f(x-2)+3$

1.) Vertical stretch by a factor of 4

2.) Shift 2 units right

3.) Shift 3 units up

b) $y = -f(2x)-1$

1.) Reflect over x-axis

2.) Horizontal stretch by a factor of $\frac{1}{2}$

3.) Shift down 1 unit.

c) $y = -\frac{1}{2}\left(\frac{1}{3}(x-6)\right)$

1.) Reflect over x-axis

2.) Vertical stretch by a factor of $\frac{1}{2}$

3.) Horizontal stretch by a factor of 3

4.) Shift 6 units right

d) $y = f(-2x+4)+5$ $y=f(-2(x-2))+5$

1.) Reflect over y-axis

2.) Horizontal stretch by a factor of $\frac{1}{2}$

3.) Shift 2 units right

4.) Shift 5 units up

3. The point $(10,-16)$ is on the graph of $y = f(x)$. Determine the coordinates of the new point after performing the following transformations.

a) $y = f(x+3)+1$

$(10,-16) \rightarrow (7,-15)$

b) $y = 2f(-x+1)+5$

$y = 2f(-(x-1))+5$

$(10,-16) \rightarrow (-9,-27)$

c) $= -\frac{1}{2}f(x-4)+6$

$(10,-16) \rightarrow (14,14)$

d) $y = \frac{1}{4}f\left(\frac{1}{2}x-6\right)-9$

$y = \frac{1}{4}f(\frac{1}{2}(x-12))-9$

$(10,-16) \rightarrow (32,-13)$

e) $y = -3f(2x+8)-1$

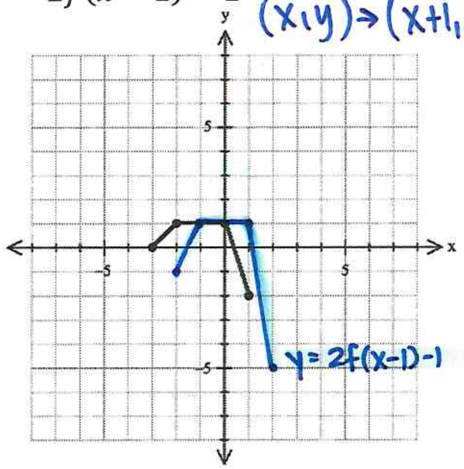
$y = -3f(2(x+4))-1$

$(10,-16) \rightarrow (1,47)$

4. The following graph represents the function $y = f(x)$. Sketch the graph of the function below.

a) $y = 2f(x-1) - 1$

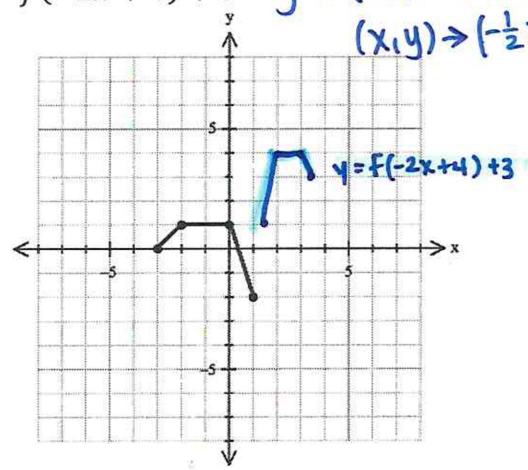
$(x,y) \to (x+1, 2y-1)$



b) $y = f(-2x+4) + 3$

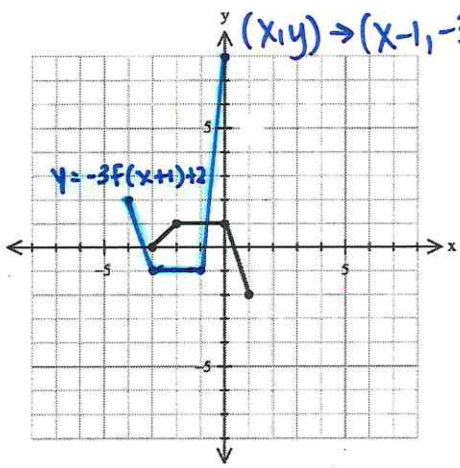
$y = f(-2(x-2)) + 3$

$(x,y) \to (-\frac{1}{2}x + 2, y+3)$



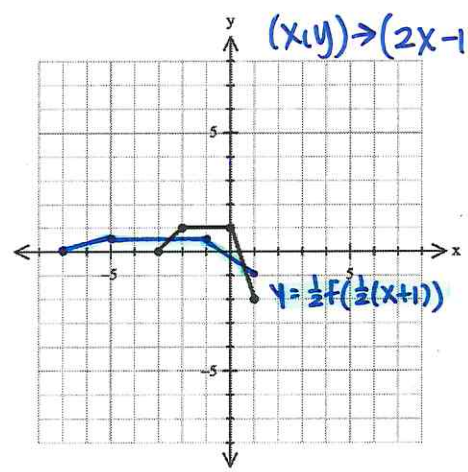
c) $y = -3f(x+1) + 2$

$(x,y) \to (x-1, -3y+2)$



d) $y = \frac{1}{2}f\left(\frac{1}{2}(x+1)\right)$

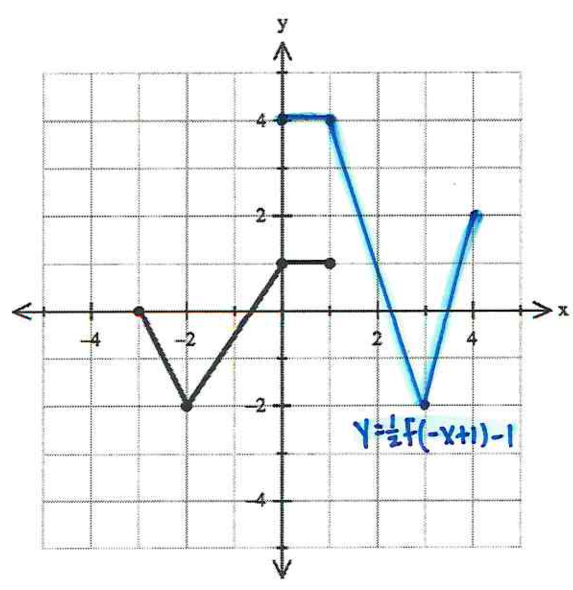
$(x,y) \to (2x-1, \frac{1}{2}y)$



5. Given the graph of $y = \frac{1}{2}f(-x+1) - 1$, sketch the graph of $y = f(x)$.

$y = \frac{1}{2}f(-(x-1)) - 1$

$(x,y) \to (-x+1, \frac{1}{2}y - 1)$



1.4 – Inverse of a Relation

1. The following points are found on the graph of $y = f(x)$: $(-3, 7)$, $(0, 5)$, $(-4, -4)$ and $(3, 9)$.

a) Determine the points that must be on the graph of $y = f^{-1}(x)$.

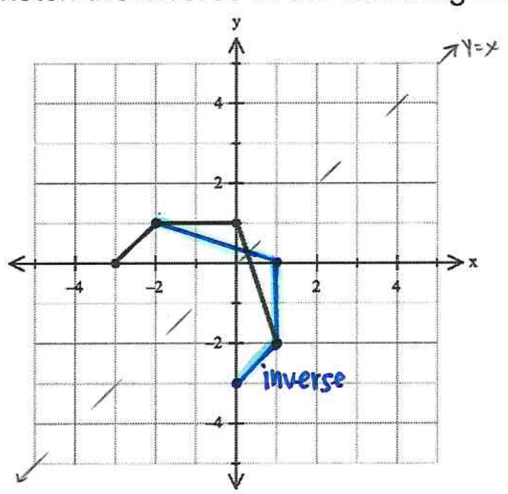
(7, -3), (5, 0), (-4, -4) and (9, 3)

b) Using the line $y = x$, explain why the point $(-4, -4)$ is invariant.

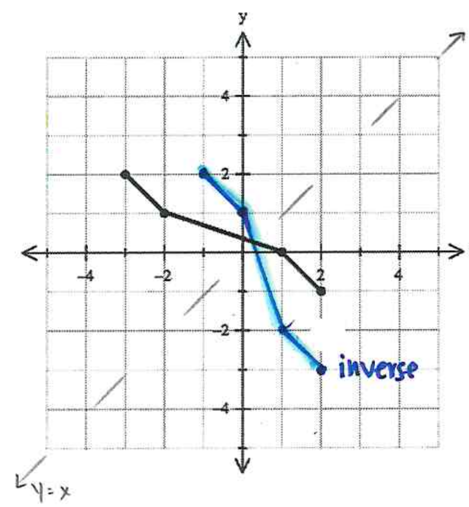
↳ When reflected over the line y=x, the point (-4, -4) is invariant because when the x and y values interchange, they have the same value as they are both -4.

2. Sketch the inverse of the following relations.

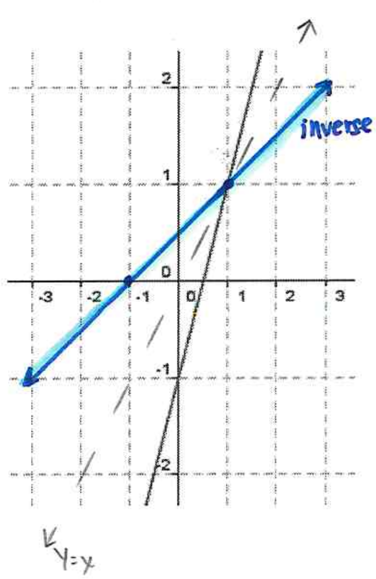
a)



b)



c)



d) Using the graphs, which one of the inverse relations is not a function? Explain how this same information can be determined without graphing the inverse.

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3. Determine the equation of the inverse for each equation given below.

a) $f(x) = 3x$

$y = 3x$

$\frac{x}{3} = \frac{3y}{3}$

$\frac{1}{3}x = y$

b) $f(x) = -2x + 4$

$y = -2x + 4$

$x = -2y + 4$

$\frac{x-4}{-2} = \frac{-2y}{-2}$

$-\frac{1}{2} + 2 = y$

c) $f(x) = \frac{x-1}{5}$

$y = \frac{x-1}{5}$

$x = \frac{y-1}{5}$

$5x = y - 1$

$5x + 1 = y$

d) $f(x) = \frac{3x}{4} - 2$

$y = \frac{3x}{4} - 2$

$x = \frac{3y}{4} - 2$

$x + 2 = 3y$

$4x + 8 = 3y$

$\frac{4}{3}x + \frac{8}{3} = y$

e) $f(x) = x^2 - 9$

$y = x^2 - 9$

$x = y^2 - 9$

$x + 9 = y^2$

$\pm\sqrt{x+9} = y$

f) $f(x) = 3(x-1)^2 + 2$

$y = 3(x-1)^2 + 2$

$x = 3(y-1)^2 + 2$

$x - 2 = 3(y-1)^2$

$\frac{1}{3}(x-2) = (y-1)^2$

$\pm\sqrt{\frac{1}{3}(x-2)} = y - 1$

$1 \pm\sqrt{\frac{1}{3}(x-2)} = y$

g) $f(x) = 3 - x^2$

$y = 3 - x^2$

$x = 3 - y^2$

$y^2 = 3 - x$

$y = \pm\sqrt{3-x}$

h) Restrict the domain of the original functions in e), f), and g) so that each of the inverse relations are functions.

e) $x \geq 0$ or $x \leq 0$

f) $x \geq 1$ or $x \leq 1$

g) $x \geq 0$ or $x \leq 0$