Chapter 11 Review Pre-Calculus 40s Old Exam Questions

1. A mathematics class consists of 7 girls and 6 boys. 13 people.

   a) In how many ways can a group of 8 students be selected from this class? Sate your final answer as a whole number.

   b) How many groups of 8 students consist of 5 girls and 3 boys? State your final answer as a whole number. [3 marks]

(a) $_{13}C_8 = 1287$ groups.

(b) $_7C_5 \cdot {}_6C_3 = 21 \cdot 20 = 420$ groups

girls boys

2. Find a simplified expression for the fourth term in the expansion of $\left(x^2 - \frac{2}{x}\right)^{10}$. [3 marks]

$t_{k+1} = {}_nC_k \, a^{n-k} \, b^k$

k=3
n=10

$t_4 = {}_{10}C_3 (x^2)^{10-3} \left(\frac{-2}{x}\right)^3$

$= 120\left(x^{14}\right)\left(\frac{-8}{x^3}\right)$

$= -960x^{11}$

3. In how many ways can 7 people be seated in a row table if 2 people refuse to sit next to each other? [2 marks]

$\boxed{\text{Not together}}$ = total arrangements − together.

together 1 group of 2
5 groups of 1
6 groups.

$= 7! - 6!2!$

$= 5040 - 1440 = \underline{\underline{3600}}$

4. There are 6 finalists in the 100-meter dash at the Snow Lake Track Meet. In how many different ways can first and second place medals be awarded? Assume that there are no ties.

State your answer as a whole number. [1 mark]

$\underset{1st}{\underline{6}} \cdot \underset{2nd}{\underline{5}} \cdot$ [illegible] = [illegible] ways

30

5. Find and simplify the 4th term in the expansion of $\left(\frac{x^2}{2}+\frac{4}{x}\right)^8$. [2 marks]

k = 3

n = 8

$$t_4 = {}_8C_3\left(\frac{x^2}{2}\right)^{8-3}\left(\frac{4}{x}\right)^3$$

$$= 56\left(\frac{x^{10}}{2^5}\right)\left(\frac{4^3}{x^3}\right)$$

$$= 56\left(\frac{64}{32}\right)\left(\frac{x^{10}}{x^3}\right) = 112x^7$$

6. A bookshelf ~~has 16 different books. There are~~ 3 Algebra books, 6 chemistry books, 5 History books and 2 English books.

How many ways can they be arranged on the shelf if the books are to be kept together by subject?

Leave you answer in factorial notation. [2 marks]

4 Groups

$$\boxed{\underset{\#\text{groups}}{4!} \cdot \underset{A}{3!}\,\underset{C}{6!}\,\underset{H}{5!}\,\underset{E}{2!}}$$

7. Susan has 10 friends she would like to invite to a dinner party. However, she only has room for 6 guests. Jack and Jill, two of these friends, insist on either attending the party together or not at all. In how many ways can Susan select the 6 guests to be invited?

[3 marks]

Case 1: Jack and Jill attend

$$\underset{J \& J}{{}_2C_2} \cdot \underset{\text{others}}{{}_8C_4} = 1 \cdot 70 = 70$$

Case 2: Jack and Jill do not attend

$$\underset{J \& J}{{}_2C_0} \cdot \underset{\text{others}}{{}_8C_6} = 1 \cdot 28 = 28$$

$70 + 28 = \boxed{98 \text{ ways}}$

8. How many positive 3-digit integers less than 360 have no repetition of digits?

[3 marks] cases

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

case 1: Begin 1 or 2

$$\underset{1,2}{\underline{2}} \cdot \underline{9} \cdot \underline{8} = 144$$

9 remaining

case 2: Begin 3

$$\underset{3}{\underline{1}} \cdot \underset{0,1,2,\cancel{3},4,5}{\underline{5}} \cdot \underline{8} = 40$$

8 remaining

$144 + 40 = \boxed{\cancel{192}}$ integers. 184

9. Pizazz Pizza offers 6 different meat toppings, 4 different vegetable toppings and 2 different types of cheese.

If one topping is selected from each of the above groups, how many different pizzas with 3 toppings are possible?

State your answer as a whole number.

[1 mark]

$$\underset{M}{\underline{6}} \cdot \underset{V}{\underline{4}} \cdot \underset{C}{\underline{2}} = 48 \text{ pizzas}$$

10. Find the simplified numerical coefficient of the term containing a^9 in the expansion of $(a-2b)^{10}$. $\rightarrow a^{10} + ?\,a^9(-2b)^1 + \cdots$

[2 marks]

$$t_2 = {}_{10}C_1 (a)^9 (-2b)^1$$

$k=1$
$n=10$

$$= 10\, a^9 (-2b)$$

$$= -20 a^9 b$$

11. In a class there are 15 boys and 10 girls. 25 people.
A sample of 5 students is to be selected to represent this class.
In how many ways can this be done? [2 marks]

$_{25}C_5 = 53130$ ways.

12. Consider the numbers between 100 and 400.
How many of these numbers are even and have no repetition of digits? [3 marks]

Could combine

Case 1: End in 0

$$\underset{1,2,3}{\underline{3}} \cdot \underline{8} \cdot \underset{0}{\underline{1}} = 24$$

Case 2: End in X4,6,8

$$\underset{1,2,3}{\underline{3}} \cdot \underline{8} \cdot \underset{4,6,8}{\underline{3}} = 72$$

Case 3: End in 2

$$\underset{1,3}{\underline{2}} \ \underline{8} \ \underset{2}{\underline{1}} = 16$$

$\boxed{112 \text{ ways}}$

13. Consider the expansion of $\left(2x^3 - \frac{1}{x}\right)^{15}$.

Determine if there is a term, in simplified form, containing x^{30} in this expansion.
Explain your answer. [2 marks]

Ignore coefficients

$$t_{k+1} = \cancel{{}_nC_k}\, a^{n-k} b^k$$

$$(2x^3)^{15-k}\left(-\frac{1}{x}\right)^k = x^{30}$$

$$(x^3)^{15-k}\left(\frac{1}{x}\right)^k = x^{30}$$

$$\left(x^{45-3k}\right)\left(x^{-k}\right) = x^{30}$$

$$x^{45-4k} = x^{30}$$

$$45-4k = 30$$

$$-4k = -45$$

$$k = \frac{45}{4}$$

There is no term since k is not an integer.

14. Find the first term in the expansion of $(2-x)^5$. [1 mark]

$2^5 = 32$

15. Simplify: $\frac{n!}{(n+1)!}$ $= \frac{\cancel{n!}}{(n+1)(\cancel{n!})}$ [1 mark]

$= \frac{1}{n+1}$

16. Four boys and three girls are to be seated in a row. The boys and girls sit in alternate positions.

a) How many seating arrangements are possible if Suney, one of the four boys, must be seated at the right end of the row? [2 marks]

b) How many seating arrangements are possible if Suney does not sit at the right end of the row? [1 mark]

(a) $\underset{B}{\underline{3}} \cdot \underset{G}{\underline{3}} \cdot \underset{B}{\underline{2}} \cdot \underset{G}{\underline{2}} \cdot \underset{B}{\underline{1}} \cdot \underset{G}{\underline{1}} \cdot \underset{\overset{S}{\uparrow}}{\underline{1}} = 36$ ways.

(b) ✱ Still need to alternate: $4!3! = 144$ → Total Alternate

Total − At end.

Not at End $= 4!3! - 3!3!$

$= 144 - 36 =$ $\boxed{108 \text{ ways}}$

17. Consider the following digits: 0, 1, 2, 3, 4, 5. How many three-digit positive integers can be formed if the three digits are **different** and the number is **even**? [3 marks]

Case 1 last digit 0

$\underline{5} \cdot \underline{4} \cdot \underset{0}{\underline{1}} = 20$

Case 2: last digit 2, 4

$\underline{4} \cdot \underline{4} \ \underset{2,4}{\underline{2}} = 32$

Not 0
1st gone

52 different digits.

18. There are 9 different gifts placed on the table.
They are to be divided amongst Anna, Betty and Connie so that each receives 3 gifts.
In how many ways can this be done? [3 marks]

$$\underset{\text{Anna}}{\underline{{}_9C_3}} \quad \underset{\text{Betty}}{\underline{{}_6C_3}} \quad \underset{\text{Connie}}{\underline{{}_3C_3}} = 84 \cdot 20 \cdot 1$$

"Choose gifts"

$$= 1680 \text{ ways.}$$

19. Consider the expansion of $\left(x^2 + \frac{2}{x}\right)^{10}$.

a) How many terms are in the expansion? [1 mark]

b) Simplify the term which contains x^{11}. [3 marks]

(a) $10 + 1 = 11$ terms.

(b) ignore coefficients.

$$t_{k+1} = \left(x^2\right)^{10-k}\left(\frac{1}{x}\right)^{k} = x^{11}$$

$$\left(x^{20-2k}\right)\left(x^{-k}\right) = x^{11}$$

$$20 - 3k = 11$$
$$-3k = -9$$
$$k = 3 \quad \text{(term 4)}$$

$$t_4 = {}_{10}C_3\left(x^2\right)^7\left(\frac{2}{x}\right)^3$$

$k = 3$
$n = 10$

$$= 120\left(x^{14}\right)\left(\frac{8}{x^3}\right)$$

$$\boxed{= 960x^{11}}$$

20. There are 8 points placed on a circle. Triangles are formed by using 3 of these points as vertices. How many different triangles are determined by these 8 points? (Leave your answer in factorial form.) why? [1 mark]

$$_8C_3 = \frac{8!}{5!3!} = \frac{8\cdot7\cdot6\cdot\cancel{5!}}{\cancel{5!}\,\cancel{3!}}$$

order not important!

$= 56$ ways.

21. The binomial $(x+y)^9$ is expanded using the Binomial theorem.
What is the value of a in the term $_9C_6x^ay^6$? [1 mark]

$a = 9-6 = 3$

22. Find the number of permutation of all the letters of the word PEPPER.
State your answer as a whole number. [1 mark]

$$\frac{6!}{3!2!} = \frac{\overset{3}{\cancel{6}}\cdot5\cdot4\cdot\cancel{3!}}{\cancel{3!\cdot2!}}$$

$= 60$

23. Solve for *n*:

$$\frac{(n+4)!}{(n+2)!}=2$$

$$\frac{(n+4)(n+3)\cancel{(n+2)!}}{\cancel{(n+2)!}} = 2$$

$$n^2+7n+12=2$$

$$n^2+7n+10=0$$

$$(n+5)(n+2)=0$$

$\cancel{n=-5}$ $n=-2$

$\frac{-1!}{-3!}$

$\frac{2!}{0!}$

$=2$

$\boxed{n=-2}$

24. Find and simplify the middle term of $(2a-b)^8$. [3 marks]

9 terms t_5 is middle.

$$t_5 = {}_8C_4(2a)^4(-b)^4$$

$k=4$

$n=8$

$$=70(16a^4)(b^4)$$

$$=1120a^4b^4$$

25. Ron is having a dinner party. He invites 7 out of his 10 friends.

a) In how many ways can he choose his 7 guests? [1 mark]

b) In how many ways can he choose his guests if two of them, Al and Sue, must attend together or not at all?
Briefly explain your calculations. [3 marks]

(a) ${}_{10}C_7 = 120$

(b) Case 1: A & S attend

${}_2C_2 \cdot {}_8C_5$

A&S

$=1\cdot 56$

$=56$

Case 2: A & S do not attend

${}_2C_0 \cdot {}_8C_7$

$=1\cdot 8$

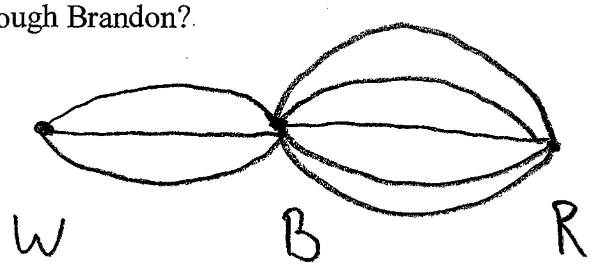
$=8$

$56+8=\boxed{64 \text{ ways}}$

26. How many five-digit arrangement are possible using all the digits 1, 2, 3, 4, 5, if the middle digit must be odd and repetition of the digits is not allowed? [1 mark]

$\underline{4}\cdot\underline{3}\cdot\underset{1,3,5}{\underline{3}}\cdot\underline{2}\cdot\underline{1} = 72$ ways.

27. There are 3 roads between Winnipeg and Brandon and 5 roads between Brandon and Roblin. How many different routes are there from Winnipeg to Roblin through Brandon? [1 mark]



$3 \cdot 5 = 15$ different routes.

28. How many numbers between 99 and 999 are divisible by 5 and have no repetition of digits? [3 marks]

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

cases

Case 1: End in 0

$$\underline{9}\cdot\underline{8}\;\underset{0}{\underline{1}} = 72$$

Case 2: End in 5

$$\underline{8}\cdot\underline{8}\;\underset{5}{\underline{1}} = 64$$

Not 5 or 0

$72 + 64$

$\boxed{136 \text{ numbers}}$

29. Find the simplified expression for the 6th term in the expansion of:

$$\left(x^3-\frac{4}{x}\right)^8$$

[3 marks]

k = 5

n = 8

$$t_6 = {}_8C_5\,(x^3)^3\left(\frac{-4}{x}\right)^5$$

$$= 56\,x^9\left(\frac{-4^5}{x^5}\right)$$

$$= 56\,x^9\left(\frac{-1024}{x^5}\right)$$

$$= -57344\,x^4$$

30. In how many ways can 8 people be seated in a row with 8 chairs if three of the people insist on being seated together? Leave your answer in factorial form. [1 mark]

1 Group of 3

5 Groups of 1

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6 Groups.

$\boxed{6!\;3!}$

# of groups

# of arrangements of each group.

31. In the binomial expansion of $(a+b)^{15}$, what is the numerical coefficient of the term containing a^2b^{13}? [1 mark]

$k = 13$

$n = 2 + 13$

$= 15$

$_nC_k$

$_{15}C_{13} = 105$

32. In how many ways can 3 people be seated in a row of 5 chairs?
State your answer as a whole number. [1 mark]

$\underline{5} \cdot \underline{4} \cdot \underline{3} = 60$ ways.

33. Solve for n: Must use Algebra; Not guess & check

$_nP_2 = 12$ [2 marks]

$$\frac{n!}{(n-2)!} = 12$$

$$\frac{n \cdot (n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 12$$

$$n^2 - n = 12$$

$$n^2 - n - 12 = 0$$

$$(n-4)(n+3) = 0$$

$\boxed{n=4}$ $\cancel{n=-3}$

34. A baseball coach needs to assign one player to each of the following positions: pitcher, catcher and first base. In how many ways can Dean, Dolores, Carlos, Carmine, Gus and Olga be assigned to those positions?
Give your answer as a whole number. 6 people. [1 mark]

$$\underset{P}{\underline{6}} \cdot \underset{C}{\underline{5}} \cdot \underset{F}{\underline{4}} = \boxed{120 \text{ ways}}$$

35. Karl has written 20 songs and must chose 12 of them to record in his studio.

a. In how many ways can he chose 12 songs for his CD?
Express your answer as a whole number. [1 mark]

$${}_{20}C_{12} = \boxed{125970 \text{ ways}}$$

b. The songs *Miracle* and *Bright Beginning* are very similar. If Karl uses **no more than one** of these two songs, in how many ways can he chose 12 songs for his CD?
Briefly describe you calculations. [2 marks]

Another way
Total - Both.

$125970 - {}_2C_2 \cdot {}_{18}C_{10}$

$125,970 - 43758$

$= 82212$

Case 1: Uses None

$${}_2C_0 \cdot {}_{18}C_{12} = 18564$$
$$(1) \cdot (18564)$$

Case 2: USES 1

$${}_2C_1 \cdot {}_{18}C_{11} = 63648$$
$$(2) \cdot (31824)$$

$$\underline{\text{Total}}\ 18564 + 63648 = \boxed{82212 \text{ ways}}$$

36. Using the letters from the word PORTAGE:

a. How many 5 letter arrangements are possible?
Express your answer as a whole number. [1 mark]

No Rep: $\underline{7}\cdot\underline{6}\cdot\underline{5}\cdot\underline{4}\cdot\underline{3} = \boxed{2520}$

(or) $_7P_5$

b. How many 7 letter arrangements are possible if "P" must be the first letter and the letters "T" and "E" must be together?
Briefly describe your calculations. IGNORE P [2 marks]

$\cancel{\underline{P}}\ \underline{O}\ \underline{R}\ \underline{T}\ \underline{A}\ \underline{G}\ \underline{E}$

1 Group of 2 (TE)
4 Groups of 1
5 Groups.

$\boxed{5!2!}$ (or) 240 ways

37. There are 3 different roads connecting St. Malo with Rosa and 4 different roads connecting Rosa with Tolstoi. In how many different ways can a person drive from St. Malo to Tolstoi, passing through Rosa on the way? [1 mark]

$\underline{3}\cdot\underline{4} = 12$ ways.

38. You have 2 different pictures and 5 different frames. In how many different ways can you from the 2 pictures? [1 mark]
form with Frames.

$2\cdot 5 = 10$ ways.

39. Solve for n **algebraically**:

$$(n-1)! = 6(n-3)!$$ [3 marks]

$$\frac{(n-1)!}{(n-3)!} = 6$$

$$\frac{(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 6$$

$$n^2 - 3n + 2 = 6$$

$$n^2 - 3n - 4 = 0$$

$$(n-4)(n+1) = 0$$

$$\boxed{n=4} \quad \cancel{n = -3}$$

LHS

$\cancel{-4!}$

LHS:	RHS
3!	6·1!
=3·2·1	=6
=6	