Grade 12
Pre-Calculus Mathematics
Notebook

Chapter 10

Keyed

Function Operations

Outcomes: R1

12P.R.1. Demonstrate an understanding of operations on, and compositions of, functions.

10.1 Sums and Differences of Functions

R1

Pages 472-483

Ex1: Given $f(x) = x + 1$ and $g(x) = 2x - 3$.

a) Write an equation to represent $h(x)$ if $h(x) = f(x) + g(x)$.

$h(x) = x + 1 + 2x - 3$

$h(x) = 3x - 2$

also written as

$h(x) = (f+g)(x)$

b) Write an equation to represent $k(x)$ if $k(x) = f(x) - g(x)$.

$k(x) = x + 1 - (2x - 3)$

$k(x) = x + 1 - 2x + 3$

$k(x) = -x + 4$

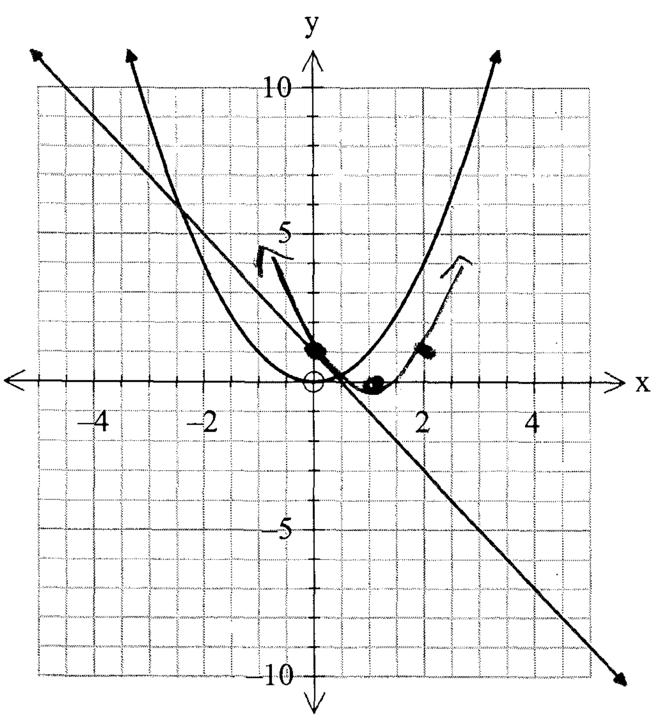
Also written as

$k(x) = (f-g)($

Ex2: Given the graphs of $f(x) = x^2$ and $g(x) = -2x + 1$

Sketch $(f+g)(x)$ using only the graphs of f and g.

Note: We simply need to add the y-values of f and g to do so.



note: the equation would be

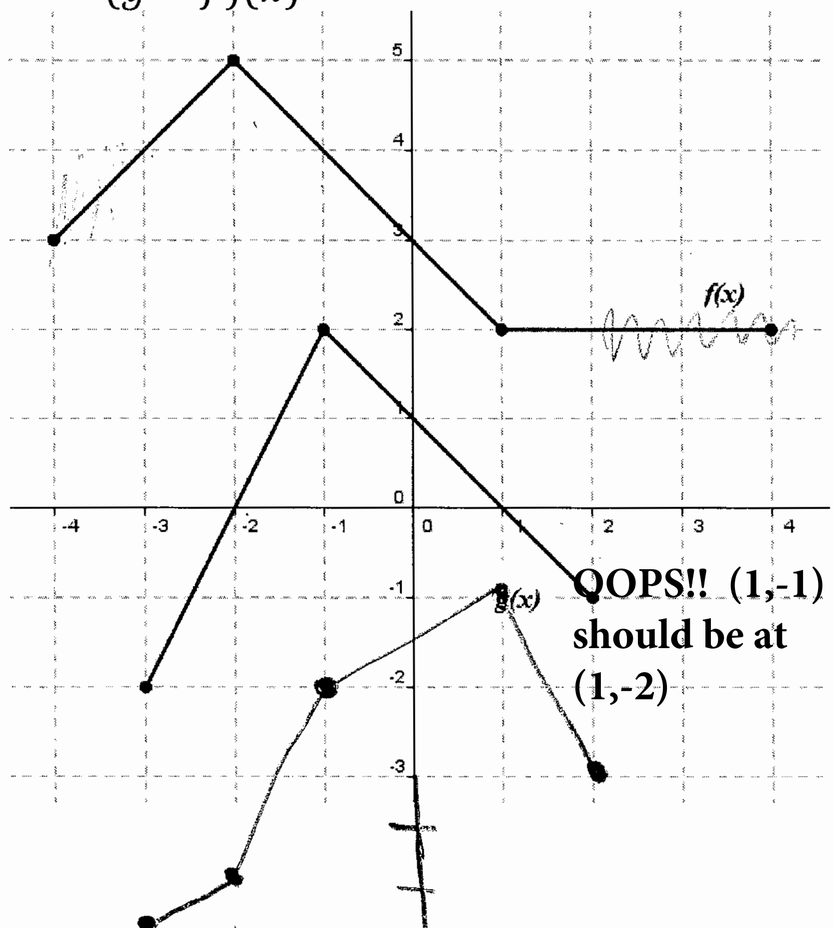
$(f+g)(x) = x^2 + (-2x+1)$

$= x^2 - 2x + 1$

$= (x-1)(x-1)$

$= (x-1)^2$

Ex3. Given the graph of $f(x)$ and $g(x)$. Sketch the graph of $(g - f)(x)$



Domain matters

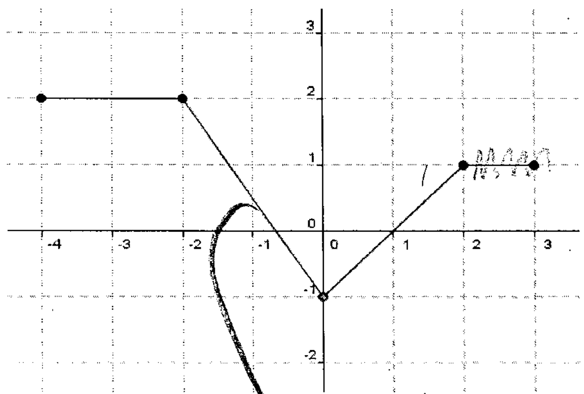
We graph by subtracting the y coordinates.

Note:

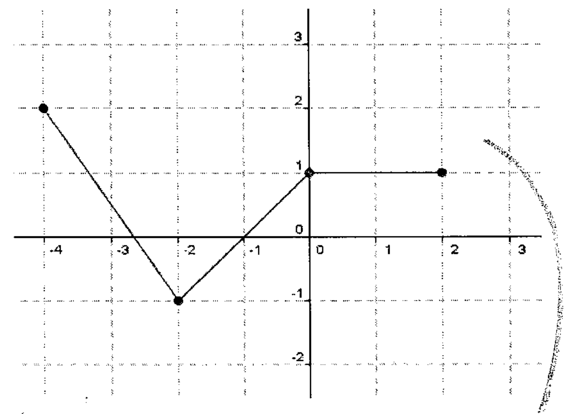
BACKWARDS

Ex4: Use the following graphs to sketch the graph of $g(x)$.

$f(x)$

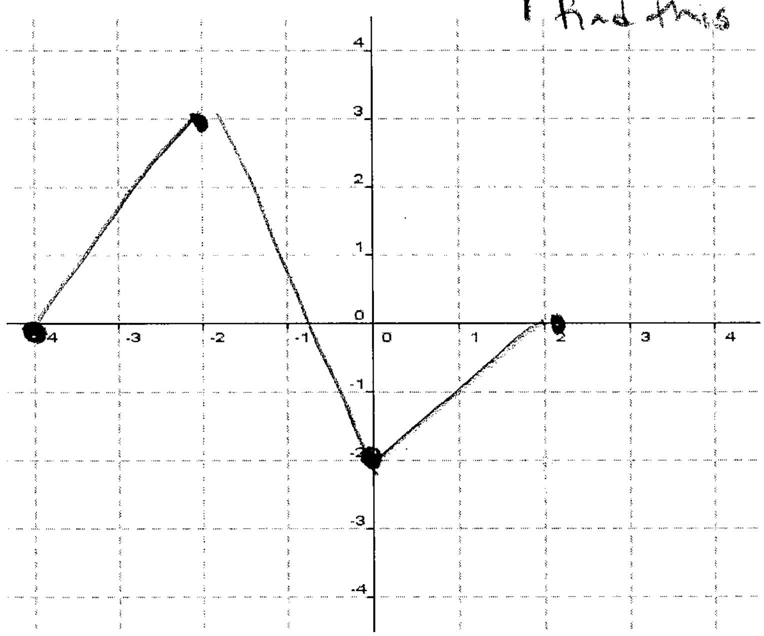


$(f-g)(x)$



x	$f(x)$	$g(x)$	$(f-g)(x)$
-4	2	0	2
-2	2	+3	-1
0	-1	-2	1
2	1	0	1

find this



Homework: Page 483 #1-4 (choose 2 from each), 5, 6 (choose 2), 7, 9-11(choose 2 each)

10.2 Products and Quotients of Functions **R1**

Pages 488-495

Ex1: Given $f(x) = x + 1$ and $g(x) = 2x - 3$.

a) Write an equation for $h(x)$ if $h(x) = f(x) \cdot g(x)$

$h(x) = (x+1)(2x-3)$

$= 2x^2 - x - 3$

Simplify

b) Write an equation for $k(x)$ if $k(x) = \dfrac{g(x)}{f(x)}$

$k(x) = \dfrac{2x-3}{x+1}$

c) Identify the domain of the graphs of $h(x)$ and $k(x)$.

Note: The function in the denominator can never be equation to zero. Therefore, $f(x) \neq 0$.

Domain $h(x)$

$\rightarrow x \in \mathbb{R}$

Domain $k(x)$

$x \in \mathbb{R}, \; x \neq -1$

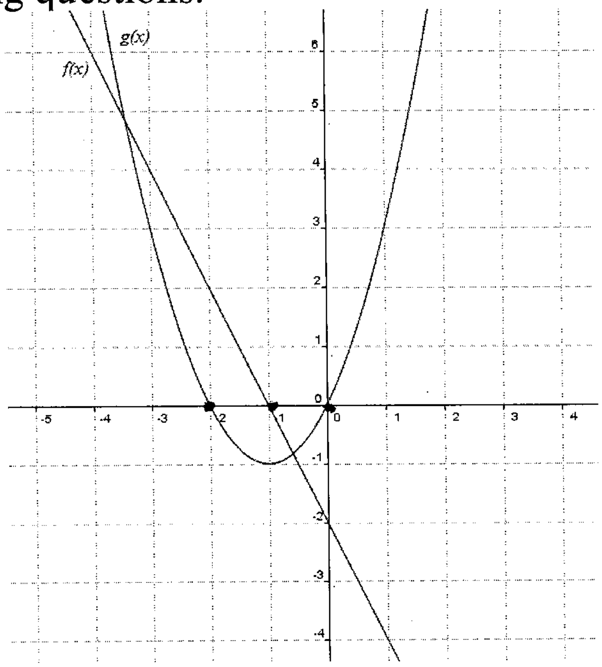
Ex2: If $h(x) = 2x^2 - 5x + 2$ and $h(x) = f(x) \cdot g(x)$, write two possible solutions for $f(x)$ and $g(x)$.

$h(x) = (2x-1)(x-2)$

$f(x) = 2x - 1$

$g(x) = x - 2$

Ex3: Using the graphs of $f(x)$ and $g(x)$ given below, answer the following questions:



a) Identify the zeros of the function $h(x)$ if $h(x) = (f \cdot g)(x)$.

Note: When $f(x) = 0$ and $g(x) = 0$, the function $h(x)$ will also equal zero.

$\therefore$ when $x = -2, 1, 0$.

b) Evaluate the following expressions:

$\left(\frac{f}{g}\right)(1)$

$\frac{f(1)}{g(1)} = \frac{-4}{3}$

$(f \cdot g)(-3)$

$f(-3) \cdot g(-3)$
$4(3)$
$= 12$

$\left(\frac{f}{g}\right)(0)$

$\frac{f(0)}{g(0)} = \frac{-2}{0}$

undefined

Homework: Page 496 #1, 2, 3, 4(a, b), 5(a,b), 6, 7, 8

Don't confuse $(f \circ g)(x)$ with $(f \cdot g)(x)$

10.3 Composite Functions R1

Pages 499-506

Ex1: Given $f(x) = x^2 + 1$ and $g(x) = 2x - 3$.

a) Evaluate the composite function $f(g(2))$.

Also written as $(f \circ g)(2)$

$g(2) = 2(2) - 3$
$= 1$

$f(1) = 1^2 + 1$
$= 2$

Note: We could also find $f(g(x))$ and then substitute $x = 2$.

$f(2x-3) = (2x-3)^2 + 1$
$= 4x^2 - 12x + 10$

$\therefore f(g(2)) = 4(2)^2 - 12(2) + 10$
$= 2$

b) Write an equation for $g(g(x))$.

$g(g(x)) = 2(2x-3) - 3$
$= 4x - 6 - 3$
$= 4x - 9$

c) Write an equation for $(g \circ f)(x)$.

$g(f(x)) = 2(x^2+1) - 3$
$= 2x^2 + 2 - 3$
$= 2x^2 - 1$

d) Evaluate $(g \circ f)(-1)$.

$g(f(-1))$

$f(-1) = (-1)^2 + 1$
$= 2$

So $g(2) = 2(2) - 3$
$= 1$

Ex2: Use the table below to respond to the following questions.

x	1	2	3	4	5	6
$f(x)$	3	1	4	2	2	5
$g(x)$	6	3	2	1	2	3

a) $f(g(2))$

$f(3) = 4$

b) $g(\underline{f(0)})$

~~NOT~~ Does not exist

c) $(g \circ f)(3)$

$g(f(3))$

$g(4) = 1$

d) $(f \circ f)(1)$

$f(f(1))$

$f(3) = 4$

Ex3: Given $f(x) = 2x - 3$ and $g(x) = \frac{x+3}{2}$, determine $f(g(x))$ and $g(f(x))$.

$f(g(x)) = \cancel{2}\left(\frac{x+3}{\cancel{2}}\right) - 3$

$= x + 3 - 3$

$= x$

$g(f(x)) = \frac{(2x-3)+3}{2}$

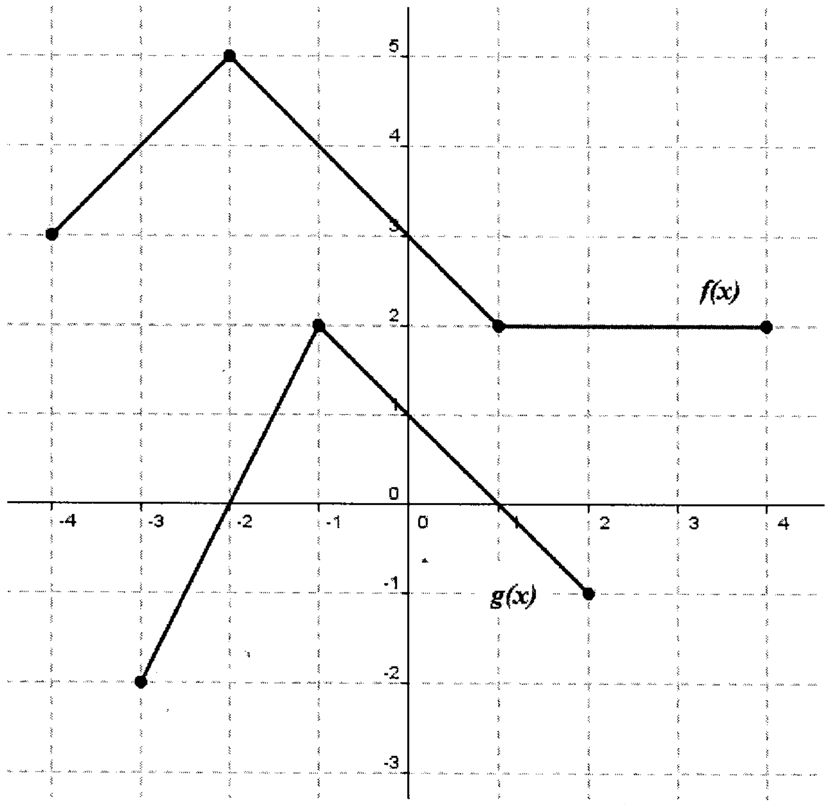
$= \frac{2x}{2}$

$= x$

Note: When $f(g(x)) = x$ or $g(f(x)) = x$, this means that $f(x)$ and $g(x)$ are inverses of each other.

COOL!

Ex4: Use the graphs below to answer the following questions:



a) $f(g(2))$

$f(-1)$

$= 4$

b) $g(f(4))$

$g(2)$

$= -1$

c) Determine the value of x if $f(g(x)) = 3$.

first: $f(x) = 3$

$y = 3$ when $x = 0$ and -4

<u>Next:</u> $g(x) = 0$ and $g(x) = -4$

true when

$x = -2$ and

$x = 1$

Never happens!

note:

Domain: $x \geq -3$ Domain $x \in \mathbb{R}$.

Ex5: Given $f(x)=\sqrt{x+3}$ and $g(x)=x^2-4$.

a) Determine $f(g(x))$ and identify the domain.

$f(g(x))$

$f(x^2-4) = \sqrt{(x^2-4)+3}$

$= \sqrt{x^2-1}$

Domain

$x^2-1 \geq 0$

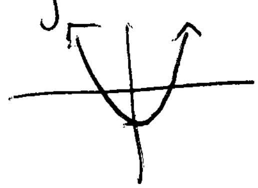
$x^2 \geq 1$

$x \geq 1, \ x \leq -1$

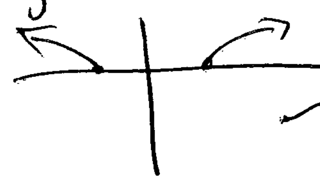
Recall

Graphing $y=\sqrt{f(x)}$

$y = x^2-1$



$y=\sqrt{x^2-1}$



b) Determine $g(f(x))$ and identify the domain.

$g(f(x))$

$g(\underbrace{\sqrt{x+3}}_{\text{restricted domain.}}) = (\sqrt{x+3})^2 - 4$

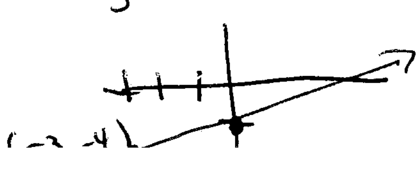
$= x+3-4$

$g(\sqrt{x+3}) = x-1$

Domain $x \geq -3$

Note: We must consider the domain of the original graph.

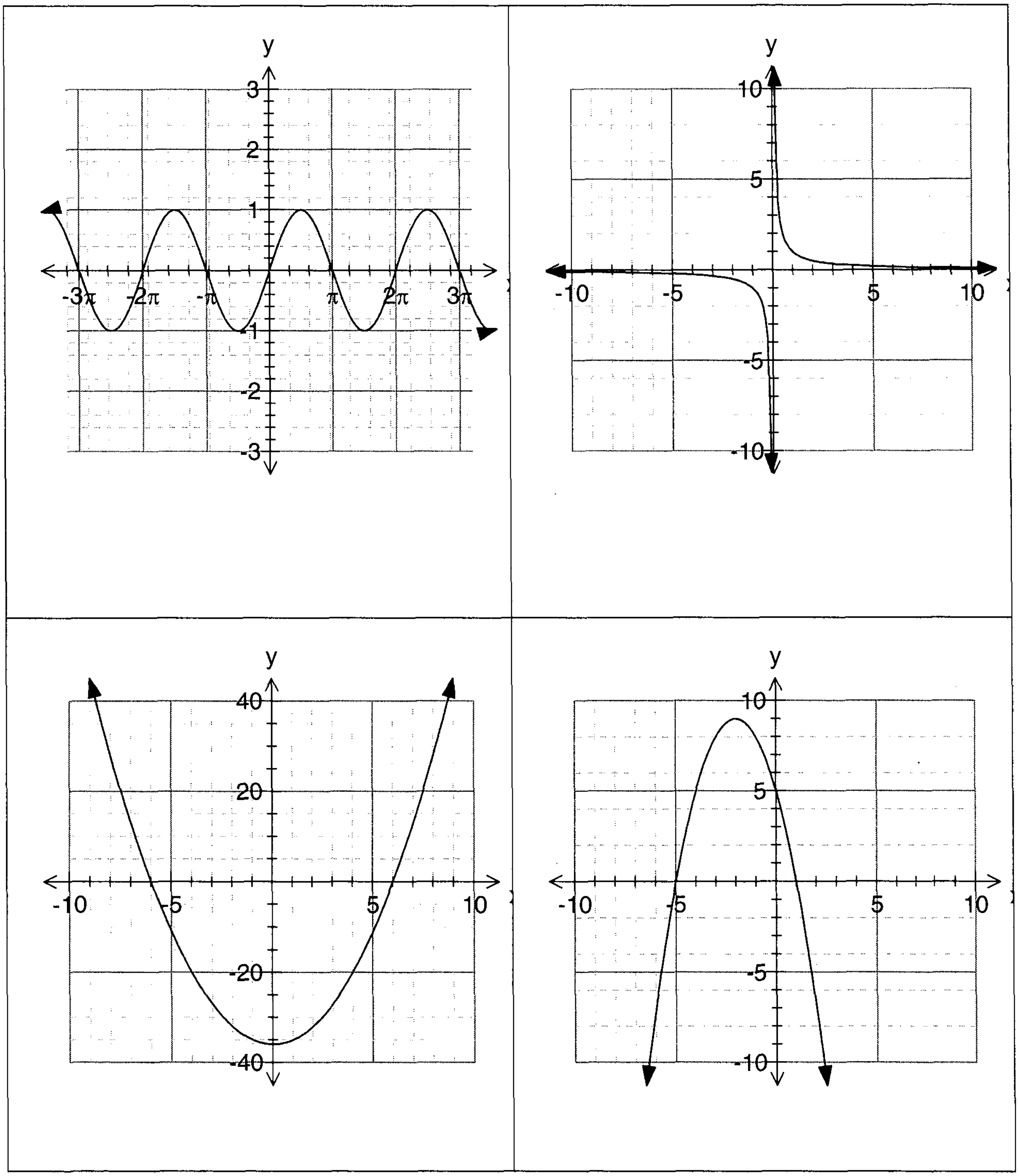
This graph would ~~look~~ like



We will cover this later — not on next test

Graphing Absolute Value and Reciprocal Functions — R1

For each graph, draw in the absolute value function. $y = |f(x)|$



For each graph, draw in the reciprocal function, state $y = \frac{1}{g(x)}$, and determine the **equation** of the **asymptote(s)** of the **reciprocal function**.