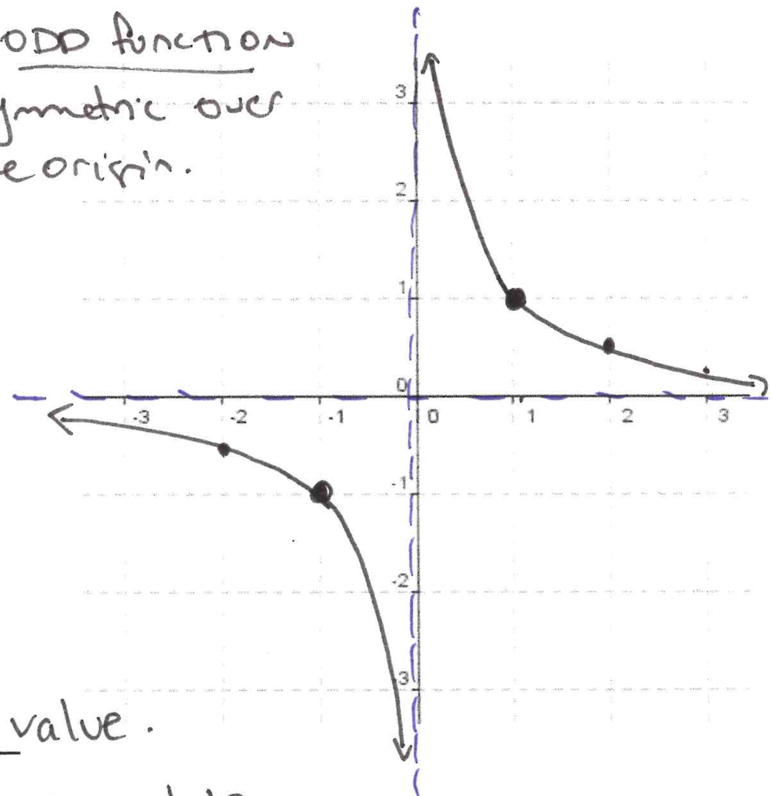
9.1 – Exploring Rational Functions: $y=\frac{a}{x-h}+k$

Sketch the graph of $f(x)=\frac{1}{x}$.

ODD function
Symmetric over the origin.

1/2 2

x	y
1	1
2	1/2
3	1/3
-1	
-2	



Note, $x = 0$ is a nonpermissible value.

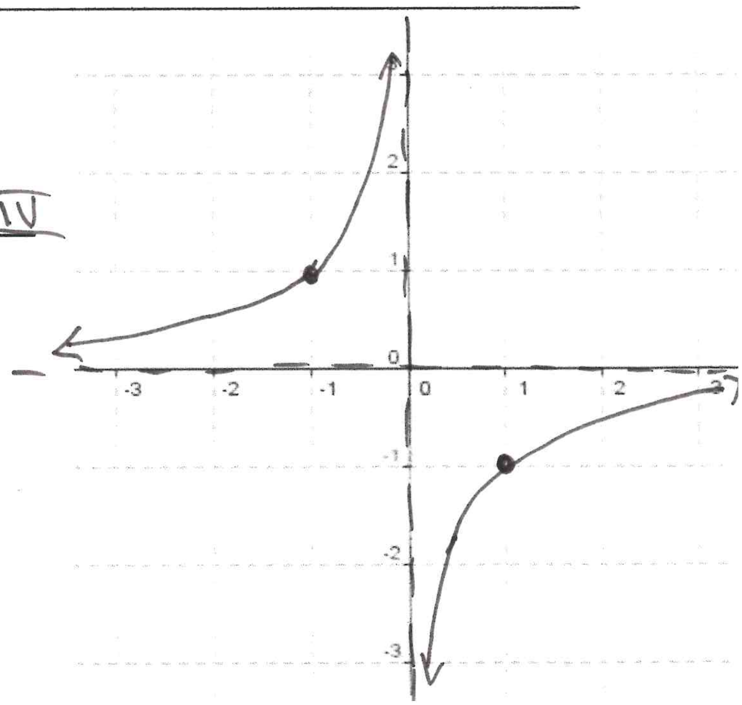
Graphically this creates a vertical asymptote

When $x \to \infty$, $y \to 0$ and when $x \to -\infty$, $y \to 0$

Thus, $y = 0$ is a horizontal asymptote. The curves are in Quadrants I, III

Sketch the graph of $f(x)=-\frac{1}{x}$.

The curves are now in Quadrants II, IV

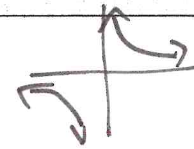


Note: $-\frac{1}{x}$ is the same as $\frac{-1}{x}$.

$\frac{1}{-x}$

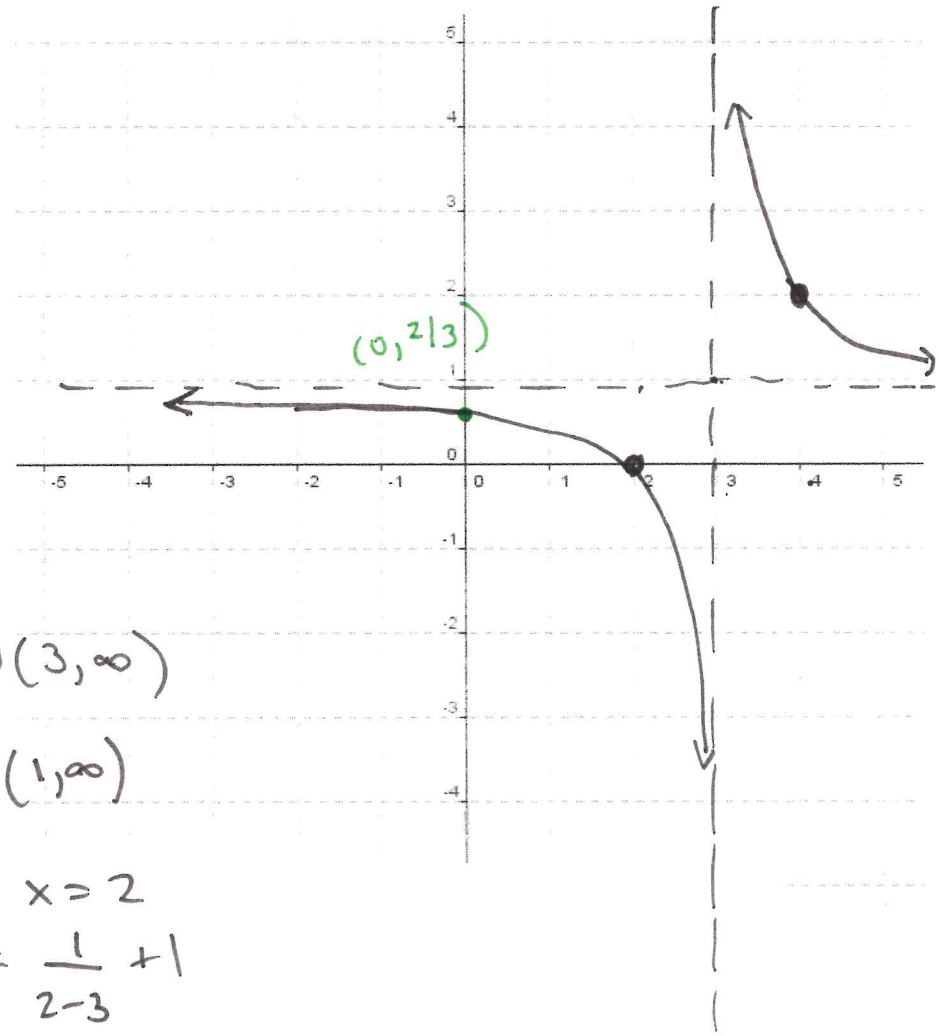
Example #1

Sketch the graph of $y = \dfrac{1}{x-3} + 1$ $\quad y = \dfrac{1}{x}$



Note : You must label a point in each section of the graph

non-permissible value	$x \neq +3$
x-intercept	2
y-intercept	2/3
vertical asymptote	$x = +3$
horizontal asymptote	$y = 1$
domain $\{x \in R,\ x \neq 3\}$	$(-\infty, 3) \cup (3, \infty)$
range $\{y \in R,\ y \neq 1\}$	$(-\infty, 1) \cup (1, \infty)$



when $x = 4$

$y = \dfrac{1}{4-3} + 1$

$y = 2$

when $x = 2$

$y = \dfrac{1}{2-3} + 1$

$y = 0$

y int:

$y = \dfrac{1}{0-3} + 1$

$= -\dfrac{1}{3} + 1$

$= -\dfrac{1}{3} + \dfrac{3}{3}$

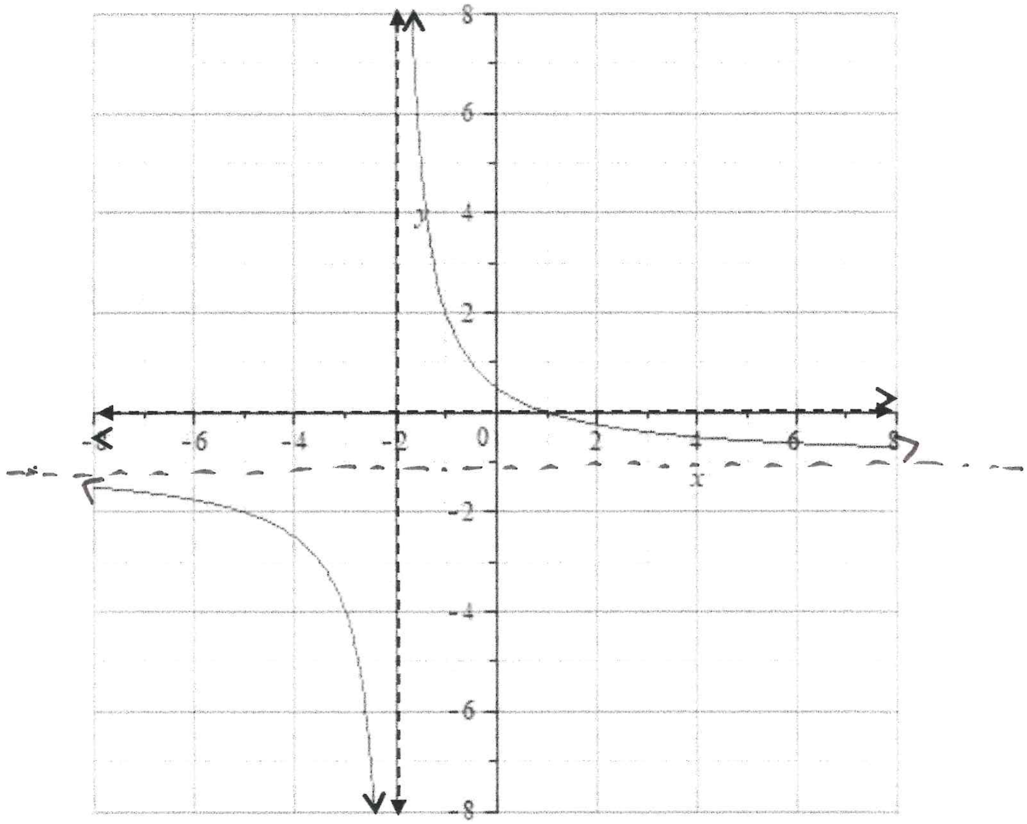
$= 2/3$

Given $f(x)=\frac{1}{x}$, we can sketch the graph of $y=3f(x+2)-1$.

Note:

The equation of the transformed graph is $y=\frac{3}{x+2}-1$.

Can you see the connection?



The general equation of a rational function is $\boxed{y=\frac{a}{x-h}+k}$.

This represents a vertical stretch by a factor of *a*, followed by a horizontal shift of *h* units, and a vertical shift of *k* units.

x=*h* is a vertical Asymptote, *y*= *k* is a horizontal asymptote.

Explain the behaviour of the graph for values of the ~~variable~~ function around x = -2.

$\lim_{x\to -2^+} f(x)=\infty$

As we approach x = -2 from the right the y values approach ∞.

As we approach x = -2 from the left the y values approach -∞.

Explain the end behaviour of the graph.

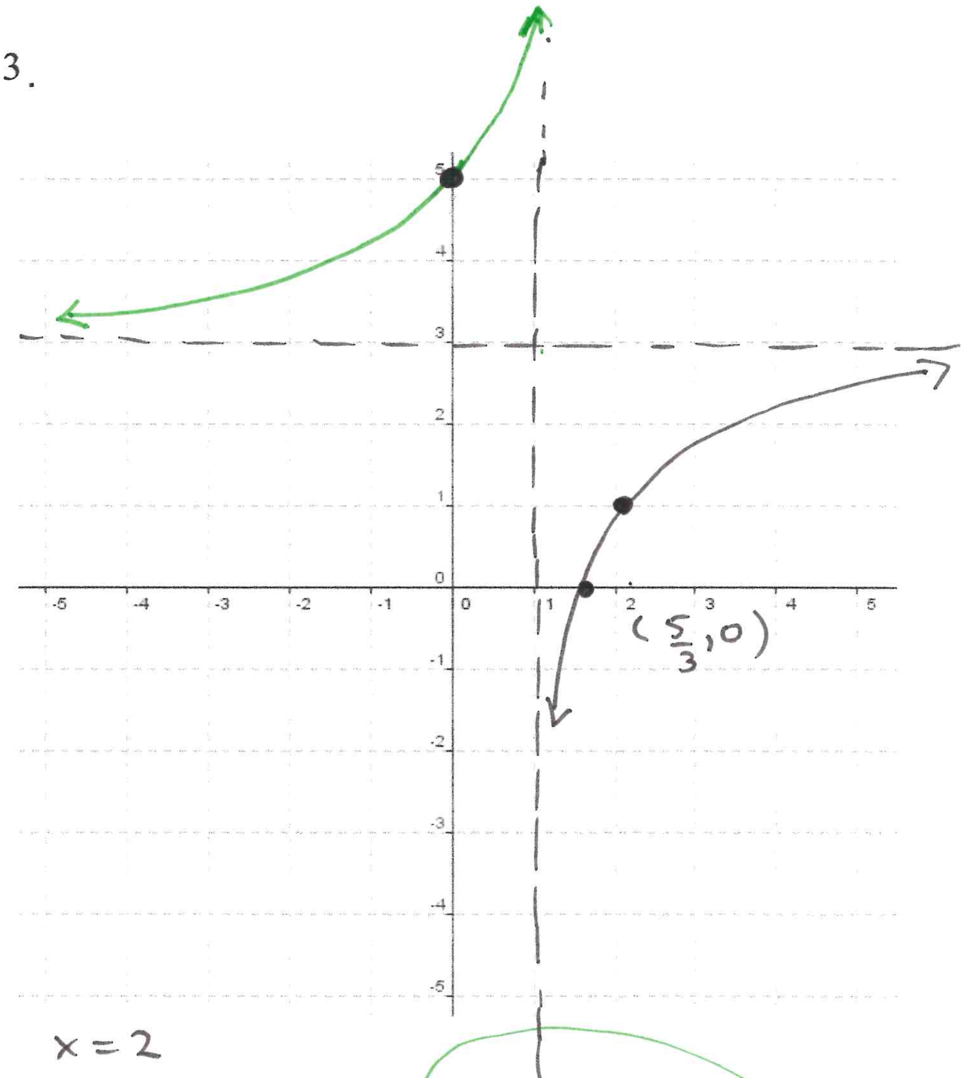
As |x| ~~op~~ get infinitely big y approaches -1

$\lim_{x\to \pm\infty} f(x) = -1$

Example #2

Sketch the graph of $y = \dfrac{-2}{x-1} + 3$.

non-permissible value	$x \neq 1$
x-intercept	$5/3$
y-intercept	5
vertical asymptote	$x = 1$
horizontal asymptote	$y = 3$
domain $\{x \in R,\ x \neq 1\}$.	
range $\{y \in R,\ y \neq 3\}$	



$x = 0$

$y = \dfrac{-2}{-1} + 3$

$y = 5$

$x = 2$

$y = \dfrac{-2}{2-1} + 3$

$y = 1$

x int:

$0 = \dfrac{-2}{x-1} + 3$

$-3 = \dfrac{-2}{x-1}$

$-3(x-1) = -2$

$-3x + 3 = -2$

$-3x = -5$

$x = 5/3$

OR

$x - 1 = \dfrac{-2}{-3}$

$x = -\dfrac{2}{3} + 1$

$x = +\dfrac{2}{3} + \dfrac{3}{3}$

$x = \dfrac{5}{3}$

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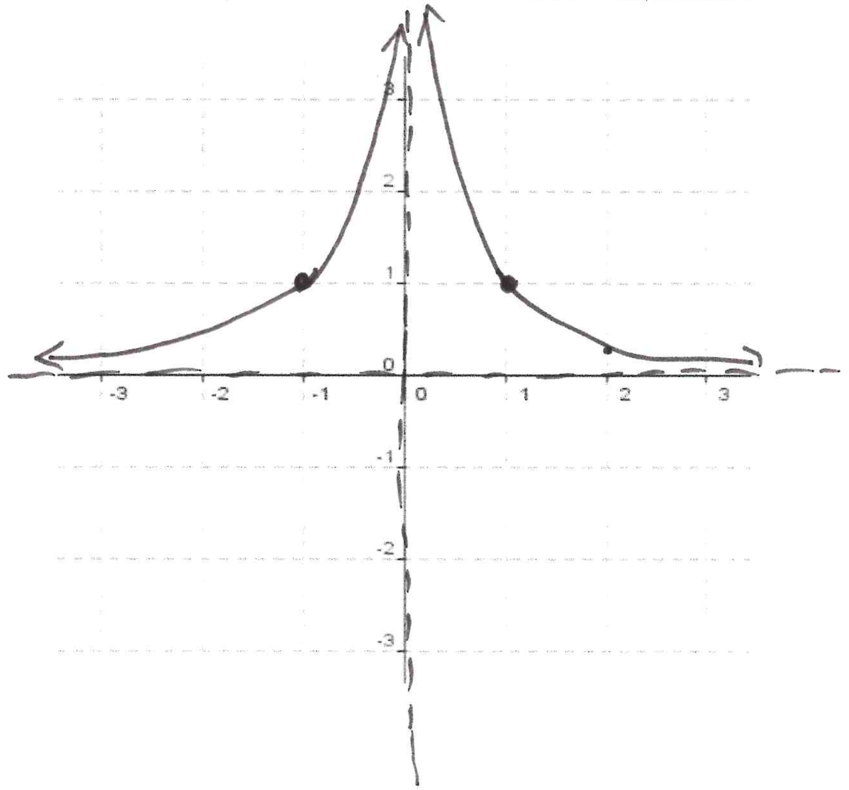
$y = \frac{a}{(x-h)^2} + k$

Example #3

even function

Sketch the graph of $f(x)=\frac{1}{x^2}$.

x	y

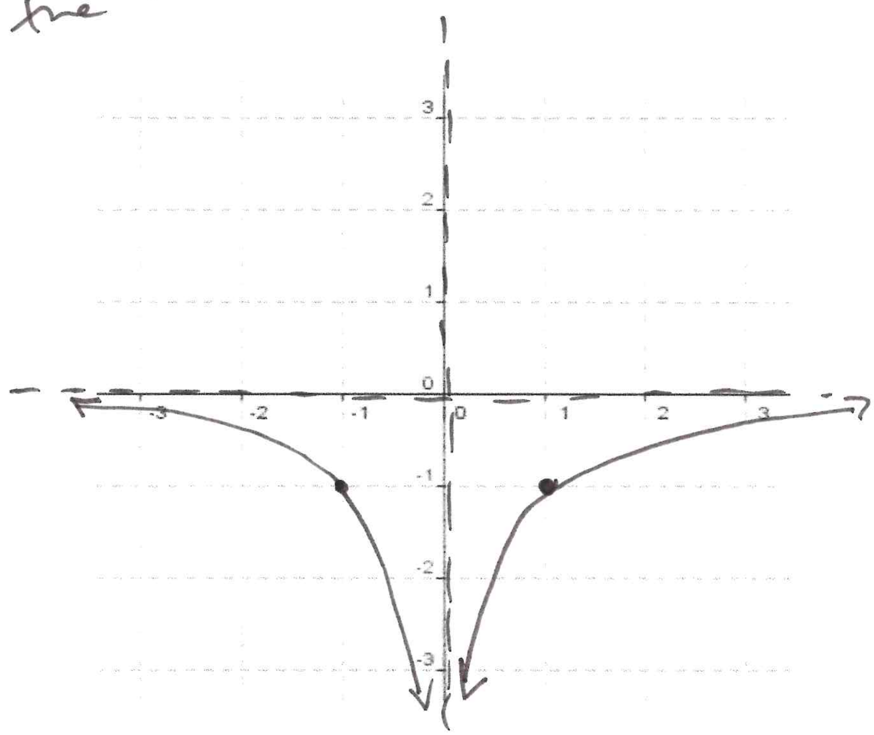


Example #4

reflects over the x-axis!

Sketch the graph of $f(x)=-\frac{1}{x^2}$.

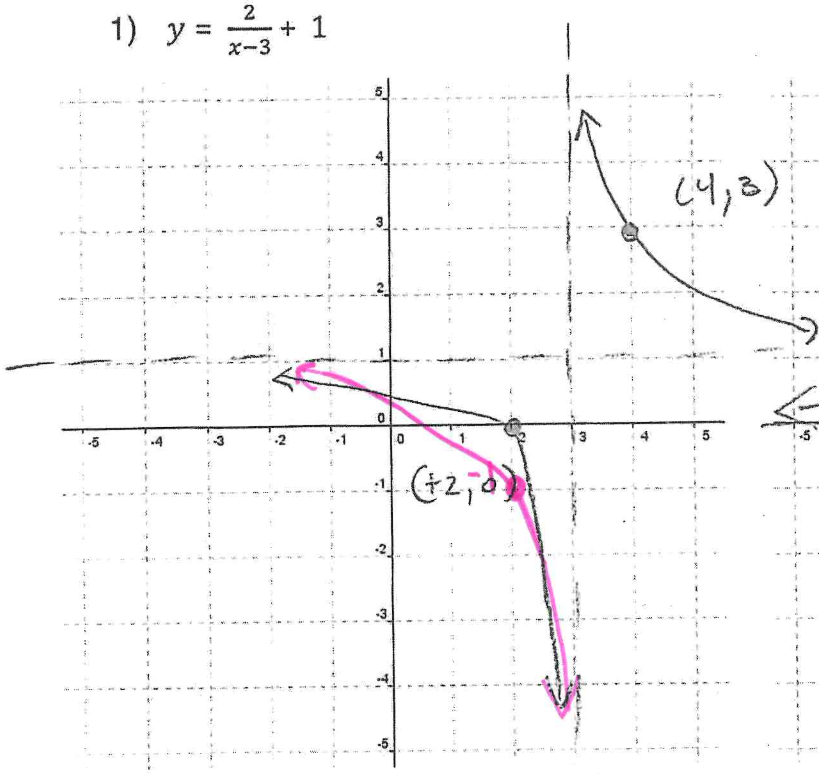
x	y



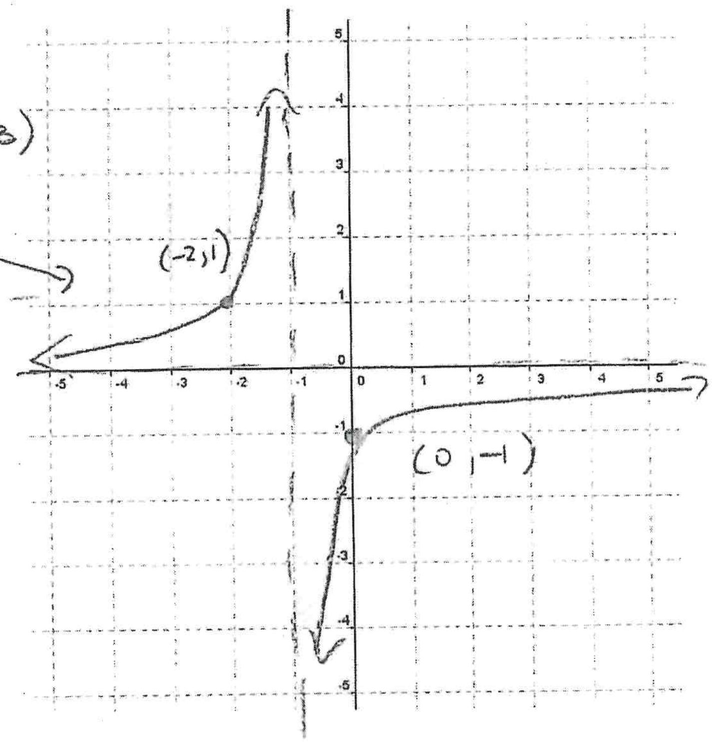
Note: We can use the previous ideas to help us graph the transformed versions of these functions.

9.1 Assignment: Graph each of the following functions

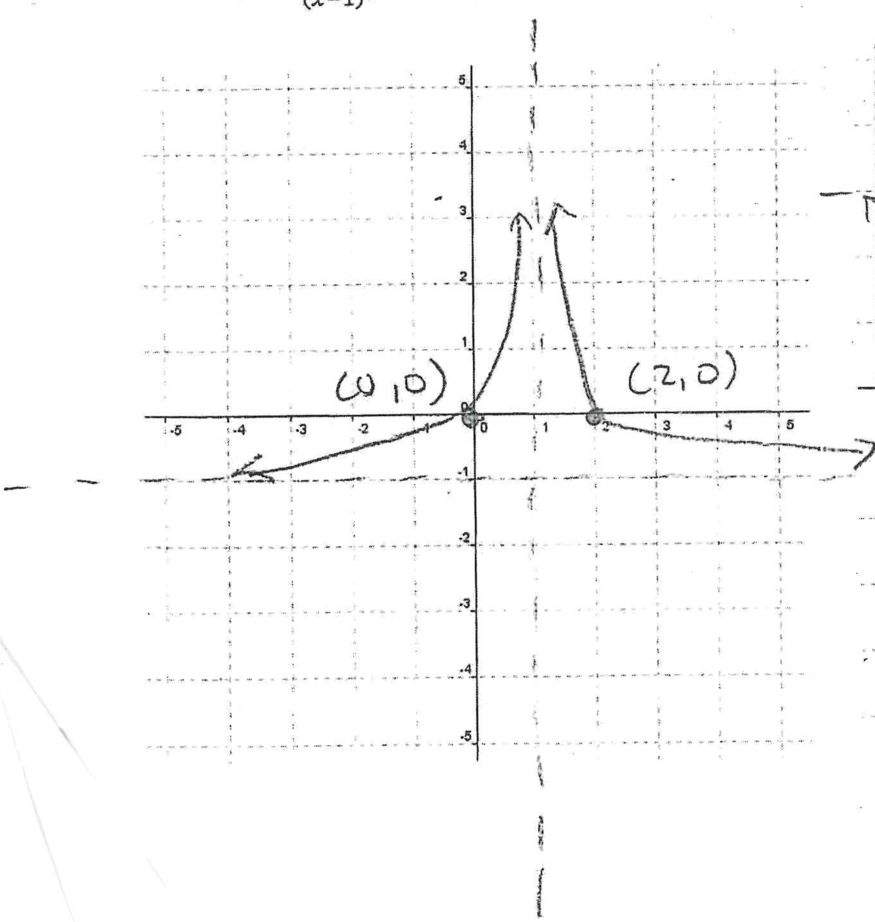
1) $y = \frac{2}{x-3} + 1$



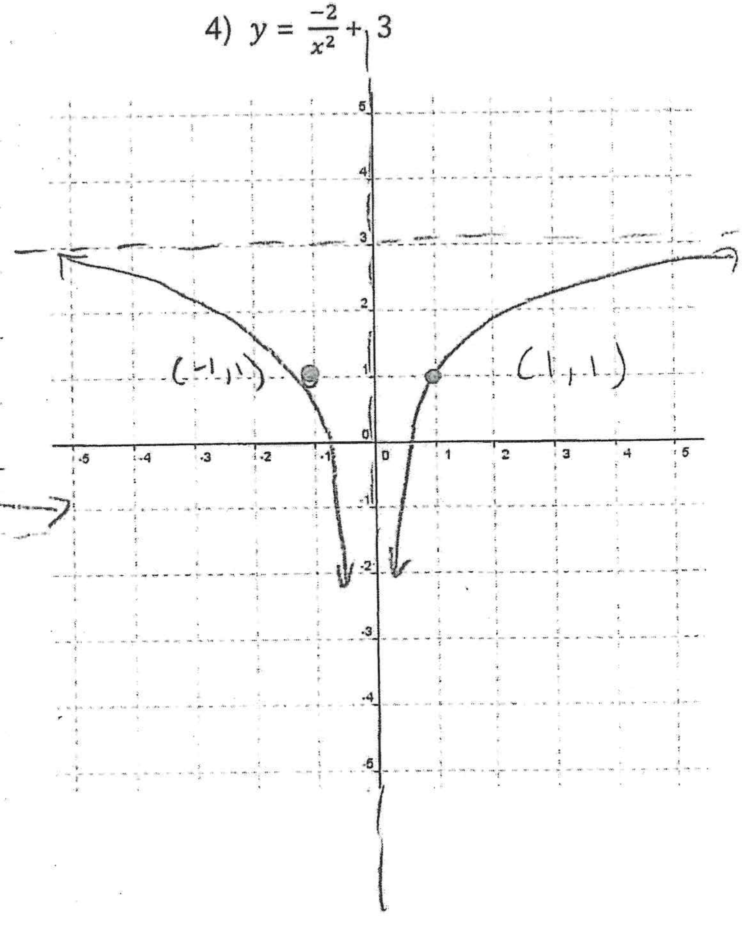
2) $y = \frac{-1}{x+1}$



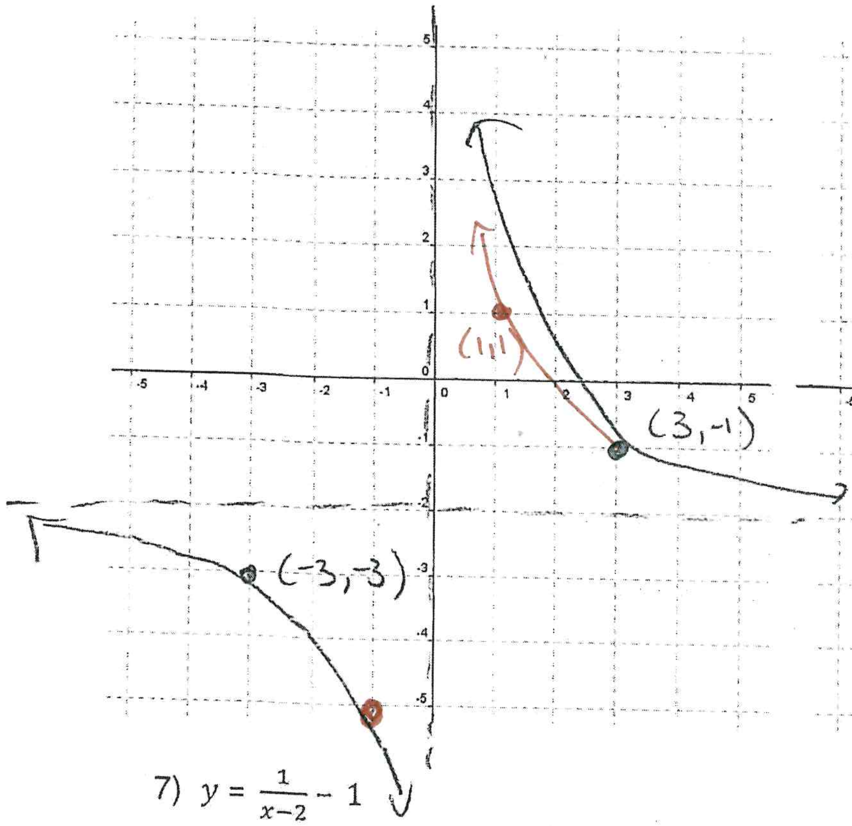
3) $y = \frac{1}{(x-1)^2} - 1$



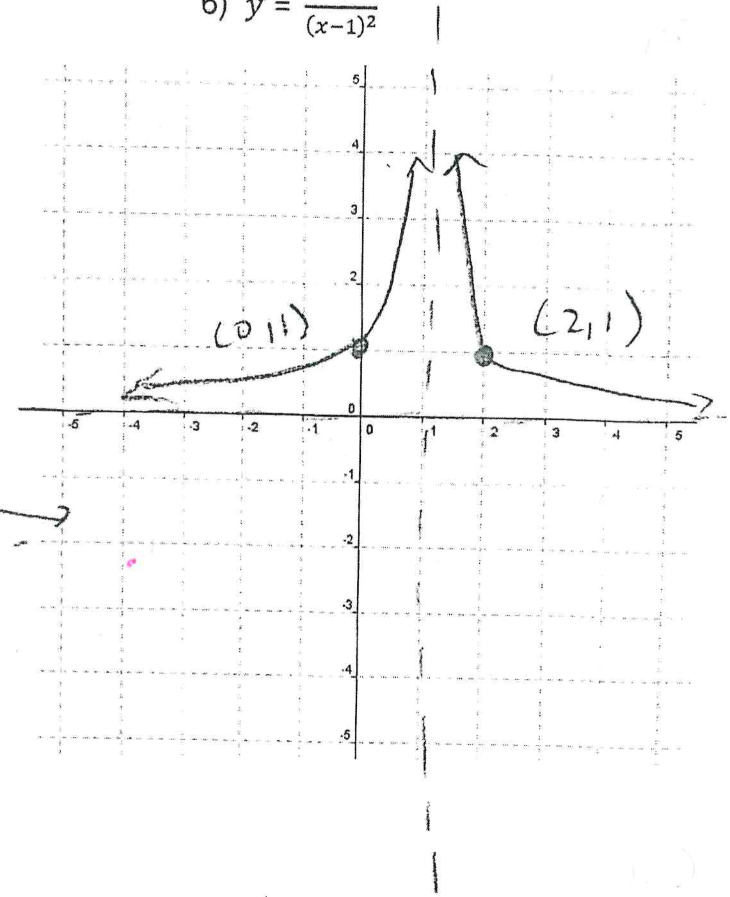
4) $y = \frac{-2}{x^2} + 3$



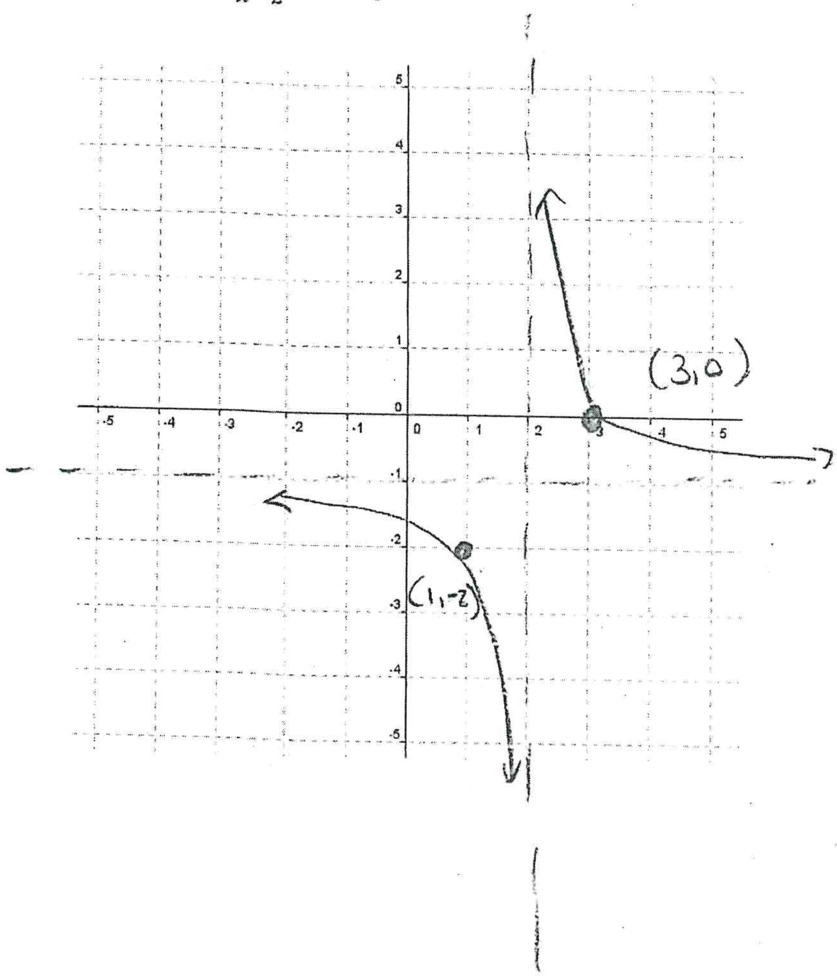
5) $y = \frac{3}{x} - 2$



6) $y = \frac{1}{(x-1)^2}$



7) $y = \frac{1}{x-2} - 1$



State a) Domain $x \in \mathbb{R},\ x \neq 2$

b) Range $y \in \mathbb{R},\ y \neq -1$

c) Vertical Asymptote $x = 2$

d) Horizontal Asymptote $y = -1$

Chapter 9: RATIONAL FUNCTIONS

9.2: Graphing Rational Functions of the form $y = \frac{f(x)}{g(x)}$

Example #1

Sketch the graph of $y = \frac{3x-4}{x-2}$.

H.A.: $y = \frac{3}{1}$

I: Determine the Vertical Asymptote(s)

Asymptotes will occur when g(x) = 0 (note: these are also NPV's)

$x = 2$

II: Determine the Horizontal Asymptote

$\lim_{x \to \pm\infty} f(x)$

- If the degree of the numerator **equals** the degree of the denominator, then the equation of the horizontal asymptote is "*y* = ratio of leading coefficients".

Ex: $y = \frac{3x^2+4}{6x^2-8}$ eqn of h.a.: $y = \frac{1}{2}$ $y = \frac{7-x+x^2-2x^3}{1x^3-5}$ eqn of h.a.: $y = \frac{-2}{1}$

- If the degree of the numerator is **less** than the degree of the denominator, then the horizontal asymptote is $y = 0$.

Ex: $y = \frac{1}{x^2+1}$ eqn of h.a.: $y = 0$ $y = \frac{x-1}{x^3+5}$ eqn of h.a.: $y = 0$

- If the degree of the numerator is **greater** than the degree of the denominator, then there is no horizontal asymptote.

Ex: $y = \frac{x+9}{3}$ eqn of h.a.: none $y = \frac{x^4-1}{x^2+1}$ eqn of h.a.: none.

$y = \frac{x+1}{1}$

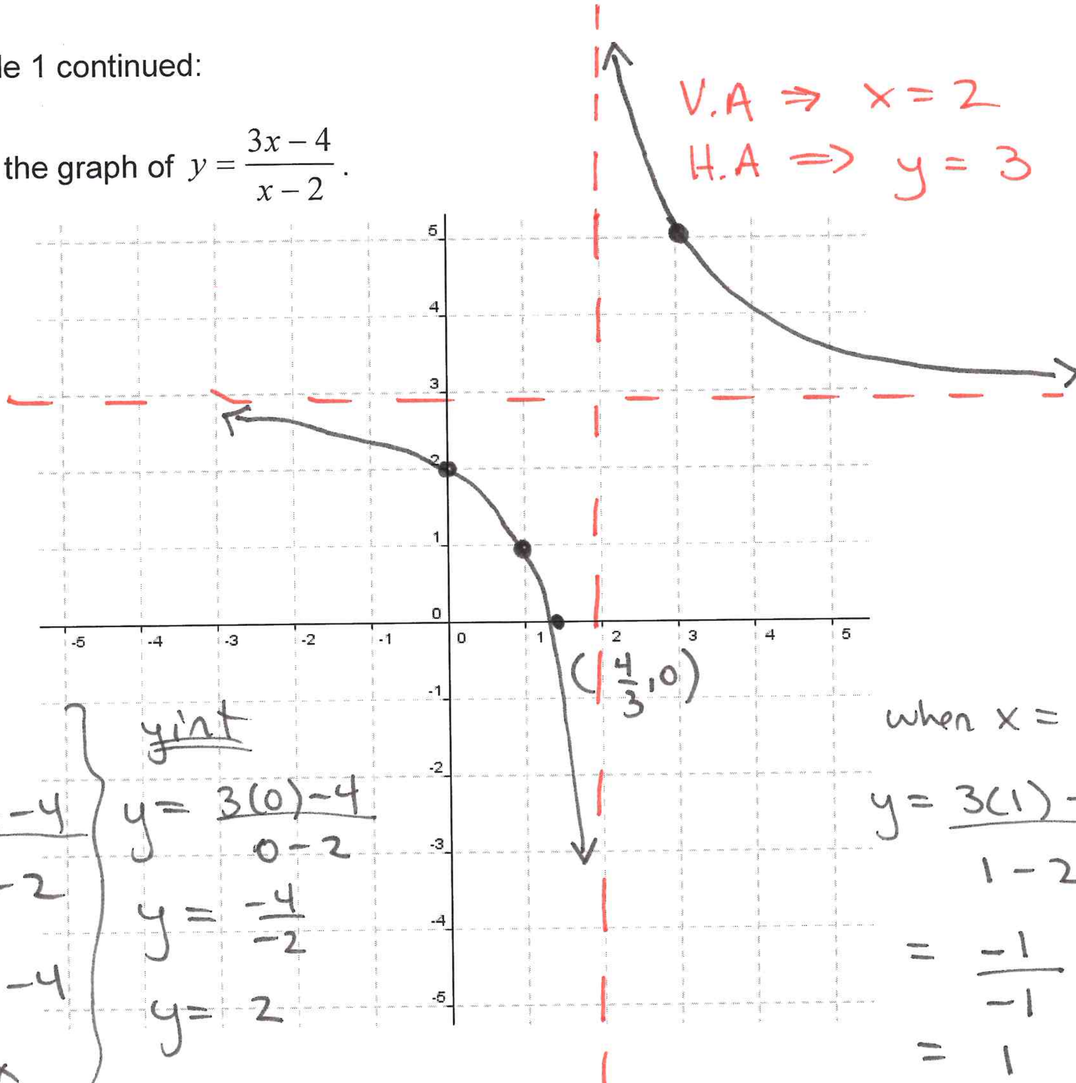
III: Determine at least one point in each section and use your knowledge of asymptotic behavior to construct the graph.

Example 1 continued:

Sketch the graph of $y = \dfrac{3x-4}{x-2}$.

V.A $\Rightarrow$ $x = 2$

H.A $\Rightarrow$ $y = 3$



xint:

$0 = \dfrac{3x-4}{x-2}$

$0 = 3x - 4$

$4 = 3x$

$\dfrac{4}{3} = x$

yint

$y = \dfrac{3(0)-4}{0-2}$

$y = \dfrac{-4}{-2}$

$y = 2$

when $x = 1$

$y = \dfrac{3(1)-4}{1-2}$

$= \dfrac{-1}{-1}$

$= 1$

$(1, 1)$

when $x = 3$

$y = \dfrac{3(3)-4}{3-2}$

$= \dfrac{5}{1}$ $(3, 5)$

vertical asymptote	$x = 2$
horizontal asymptote	$y = 3$
domain $\{x \in R, x \neq 2\}$	
range $\{y \in R, y \neq 3\}$	

Example #2 $\lim_{x \to \infty}$

Sketch the graph of $y = \dfrac{x}{x^2 - 4}$. $y = \dfrac{x}{(x-2)(x+2)}$

$x = -3$

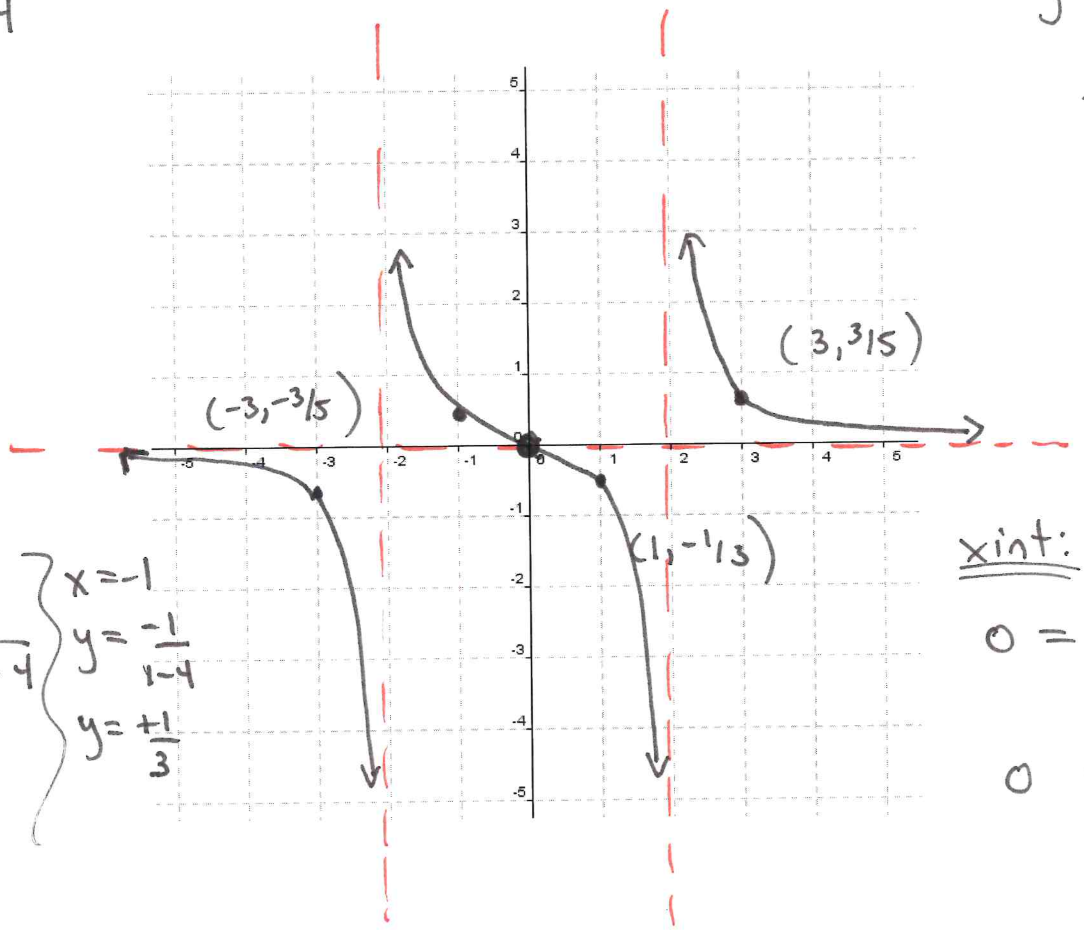
$y = \dfrac{-3}{9-4}$

$y = -\dfrac{3}{5}$

$x = 3$

$y = \dfrac{3}{9-4}$

$= \dfrac{3}{5}$



$x = \pm 1$

$y = \dfrac{1}{1-4}$

$y = -\dfrac{1}{3}$

$x = -1$

$y = \dfrac{-1}{1-4}$

$y = +\dfrac{1}{3}$

x int:

$0 = \dfrac{x}{x^2-4}$

$0 = x$

vertical asymptote(s)	$x = \pm 2$
horizontal asymptote	$y = 0$
domain $\{x \in R,\ x \neq$	$\pm 2\}$
range $\{y \in \mathbb{R}\}$	

NPV $x \neq \pm 2$

Example #3

Sketch the graph of $y = \frac{-1}{x^2+1}$.

$x = 1$

$y = \frac{1}{2}$

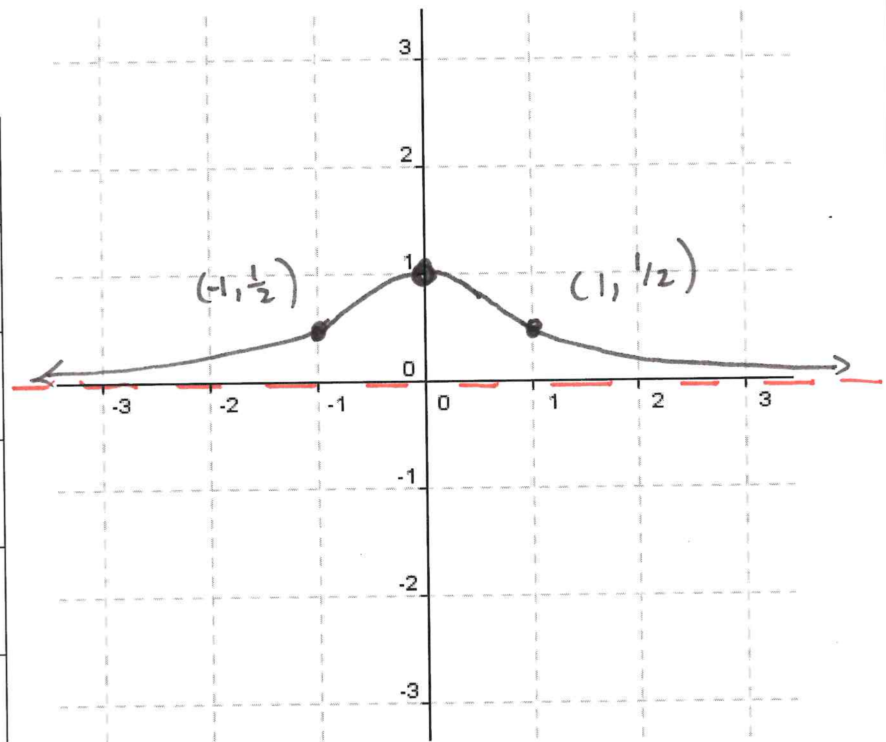
x int: $0 = \frac{1}{x^2+1}$

$0 = 1$

y int: $y = \frac{1}{1}$

non-permissible value	none!
x-intercept	none!
y-intercept	1
vertical asymptote	none!
horizontal asymptote	$y=0$
domain $x \in \mathbb{R}$.	$(-\infty, \infty)$
range $(0, 1]$	

$0 < y \leq 1$



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Ex4. Sketch the graph of $f(x)=\dfrac{x+1}{x^2-2x-3}$.

Remember to first factor the numerator and denominator if possible.

$$f(x) = \frac{\cancel{(x+1)}}{(x-3)\cancel{(x+1)}}$$

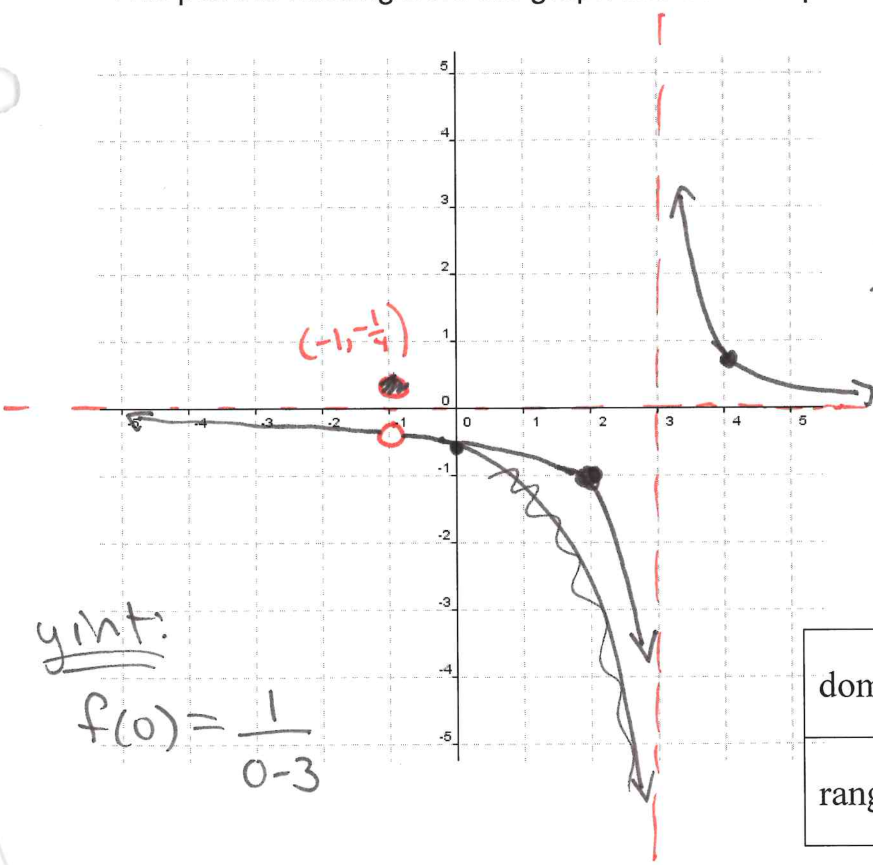
$$f(x) = \frac{1}{x-3} \qquad x \neq -1$$

when. $x=-1$

$$f(-1) = \frac{1}{-1-3} = \frac{-1}{4}$$

A point of discontinuity (Removable discontinuity) is a point at which the graph of a function is not continuous.

This point is missing from the graph and so we represent it with an open circle.



$$f(x) = \frac{1}{x-3} \qquad x=-1$$

V.A $\quad x=3$

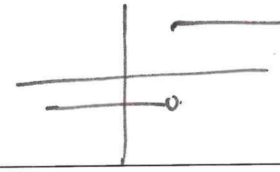
H.A. $\quad y=0$

$$f(4) = \frac{1}{4-3} = 1$$

y-int:

$$f(0) = \frac{1}{0-3}$$

domain	$\{x \in \mathbb{R},$	$x \neq -1, -3\}$.
range	$\{y \in \mathbb{R},$	$y \neq -\frac{1}{4}, 0\}$



Example #5

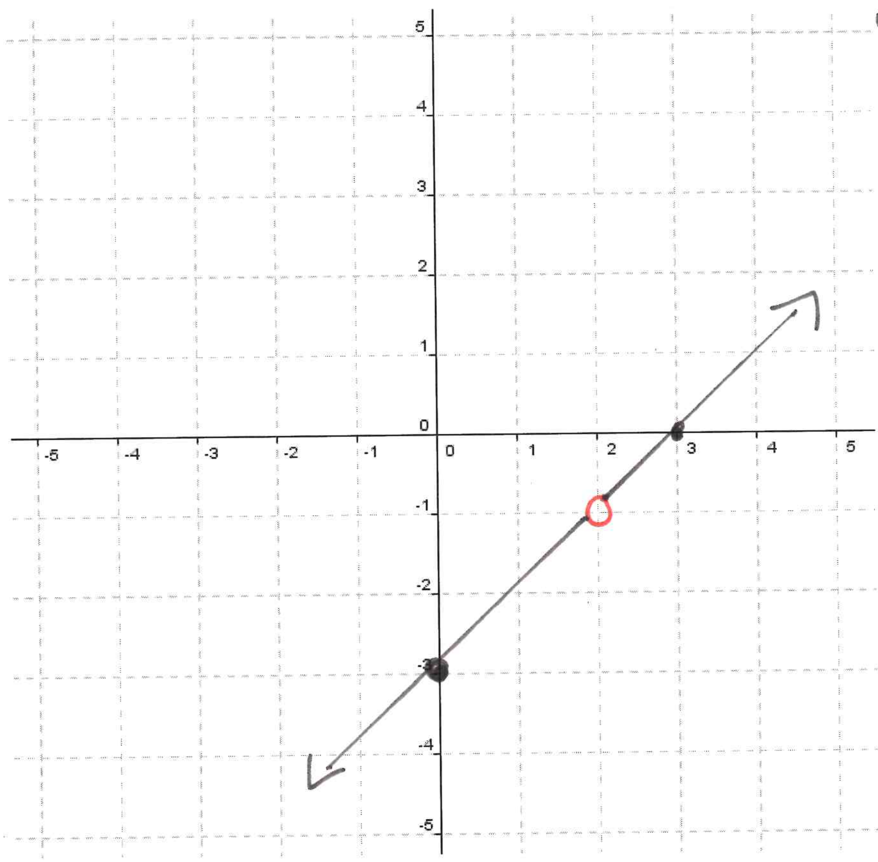
Sketch the following:

$$y = \frac{x^2 - 5x + 6}{x - 2}$$

$$y = \frac{\cancel{(x-2)}(x-3)}{\cancel{(x-2)}}$$

$$y = \frac{(x-3)}{1}, \quad x \neq 2$$

when $x = 2$
$y = -1$



Example #6

What type of discontinuity will this function have?

$$y = \frac{x-2}{x^2 - 3x + 2}$$

$$y = \frac{\cancel{x-2}}{(\cancel{x-2})(x-1)}$$

$$y = \frac{1}{x-1} \qquad x \neq 2$$

asymptotic dis. @ $x=1$

point discontinuity (removable discontinuity) @ $x=2$

$$y = \frac{1}{x-1}$$

$$y = \frac{1}{2-1}$$

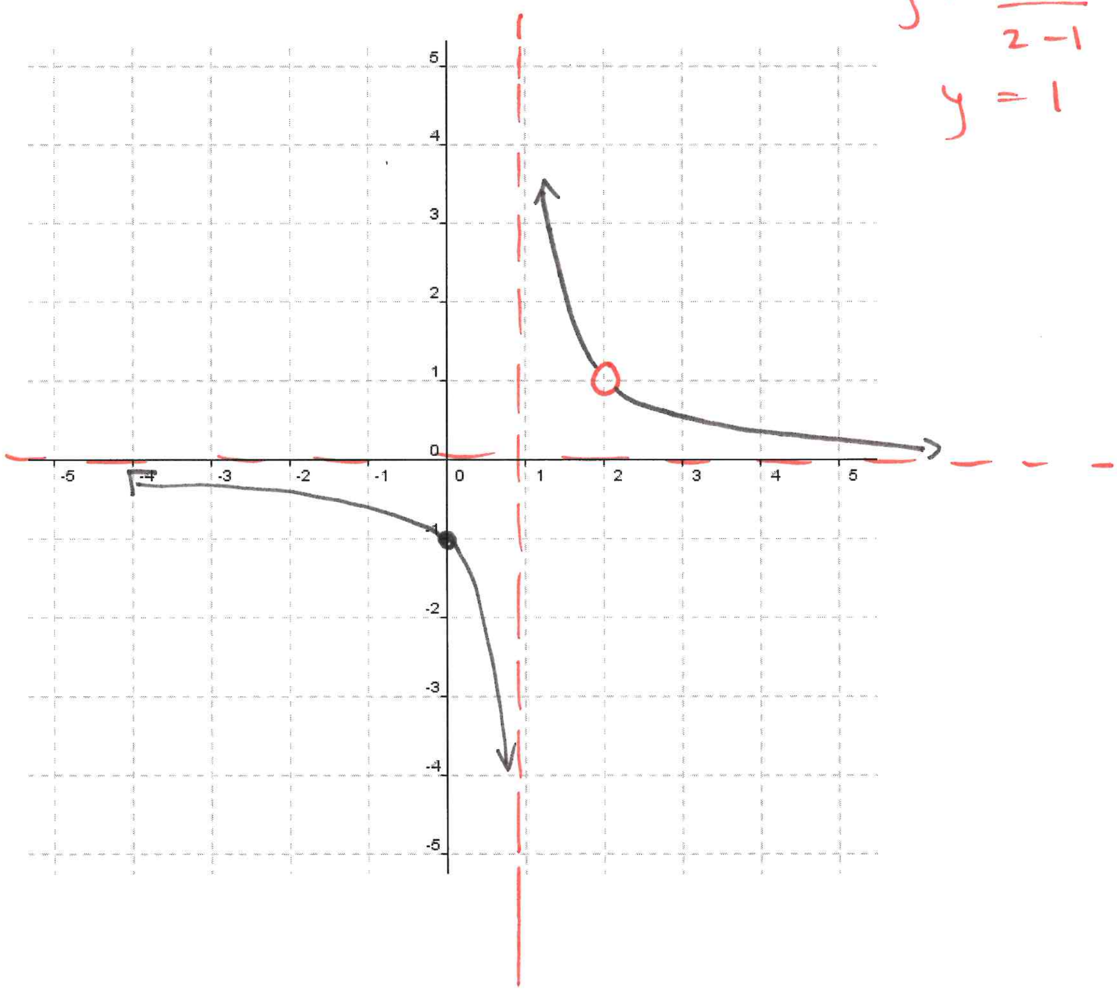
$$y = 1$$

Graph the function.

$x=0$

$$y = \frac{1}{-1}$$

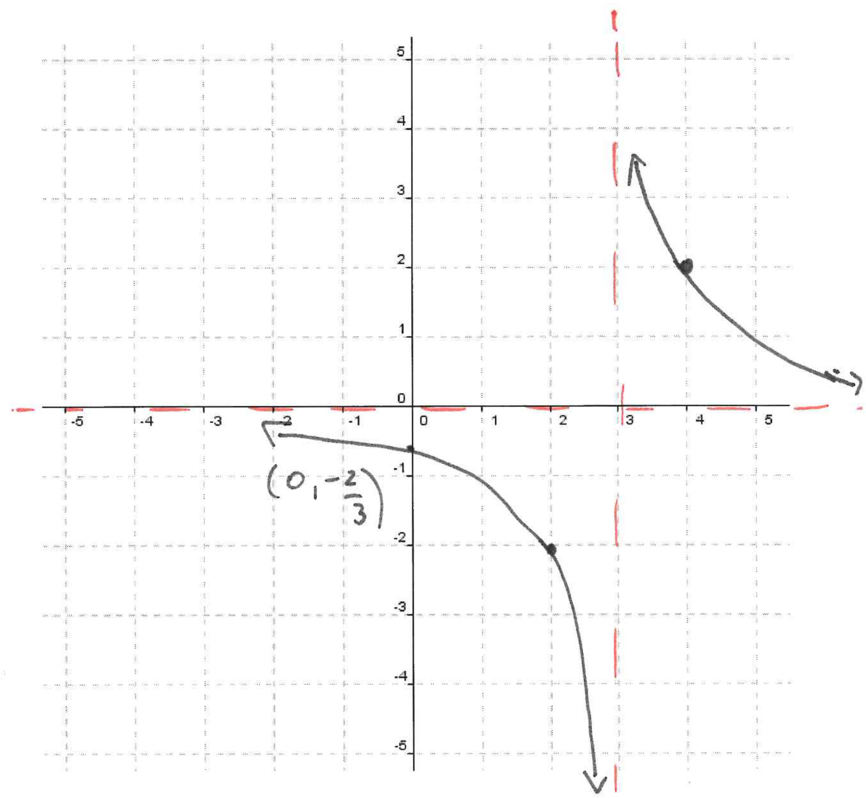
$(0,-1)$



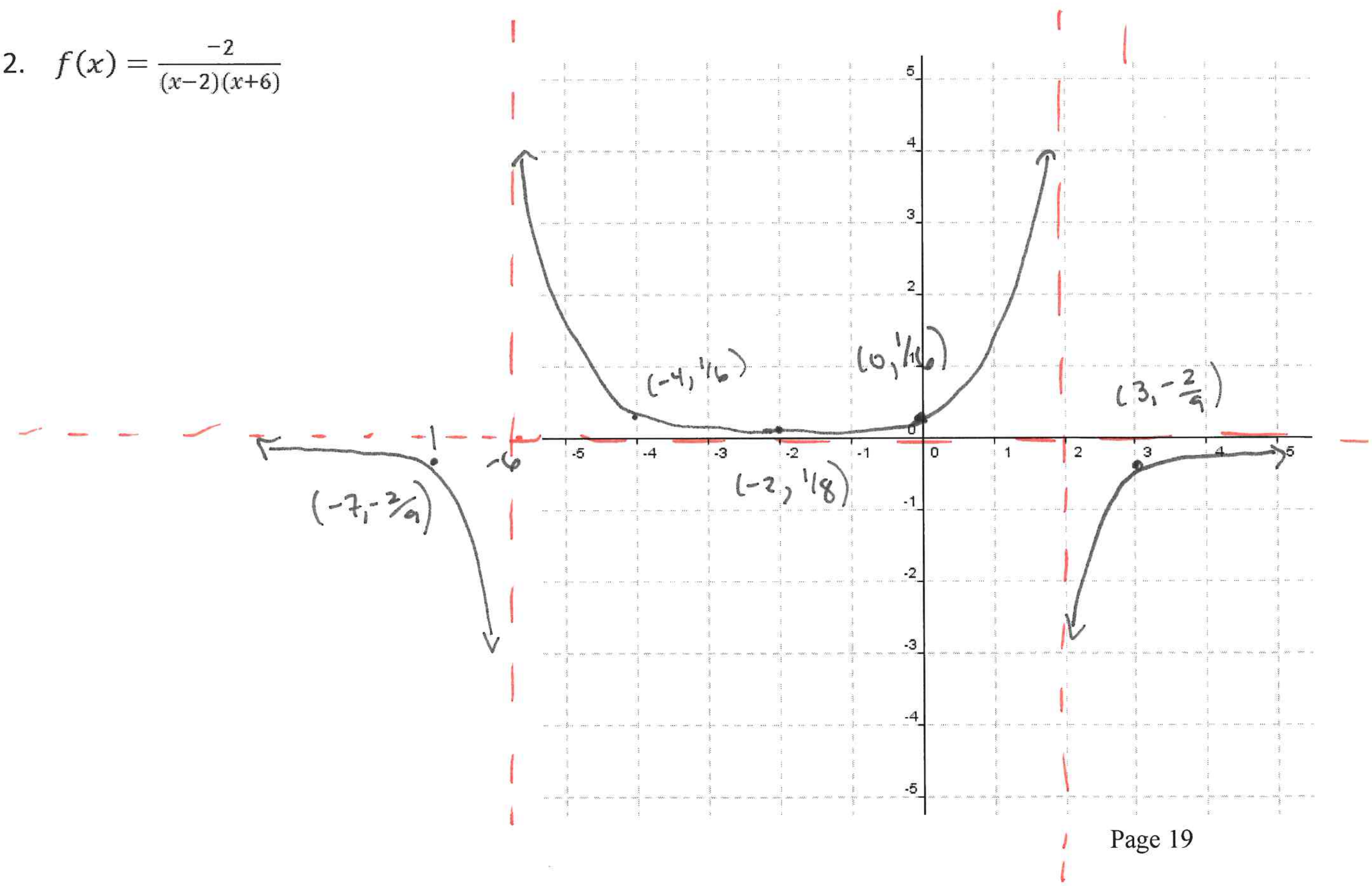
SOLUTIONS

9.2 Homework Graphing rational functions – R14

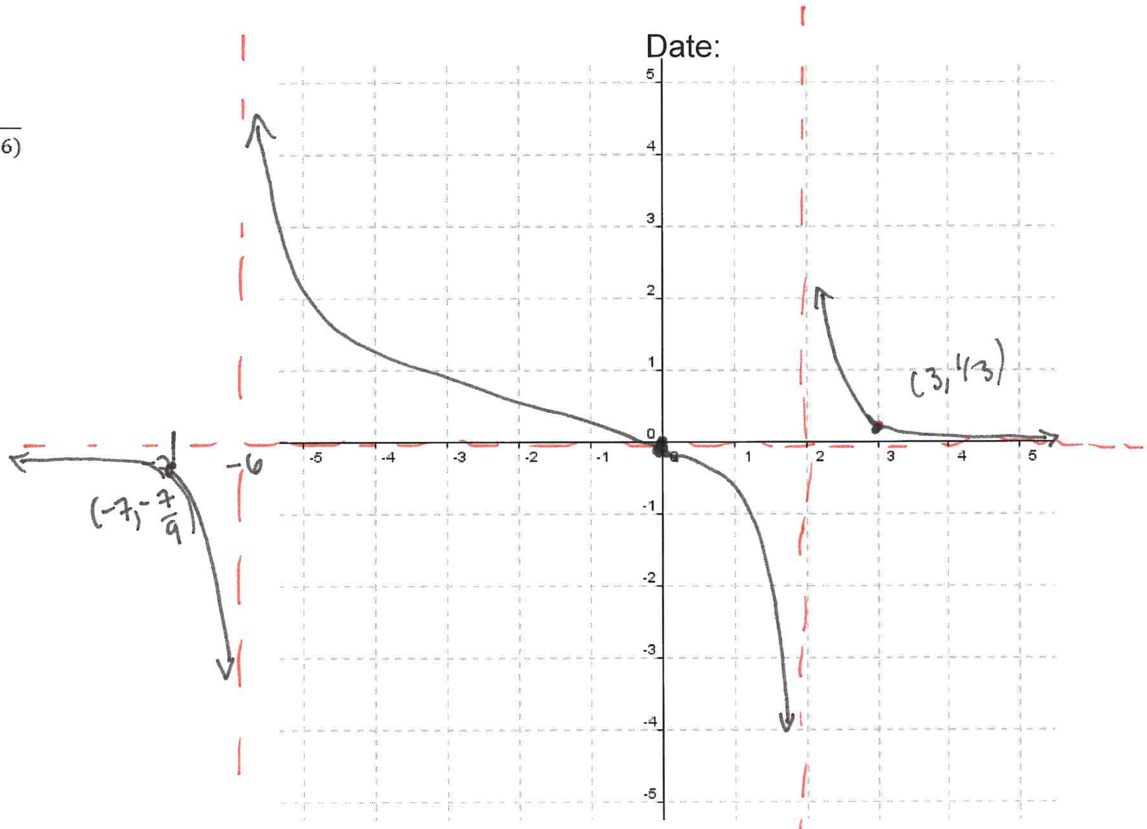
1. $y = \dfrac{2}{x-3}$



2. $f(x) = \dfrac{-2}{(x-2)(x+6)}$

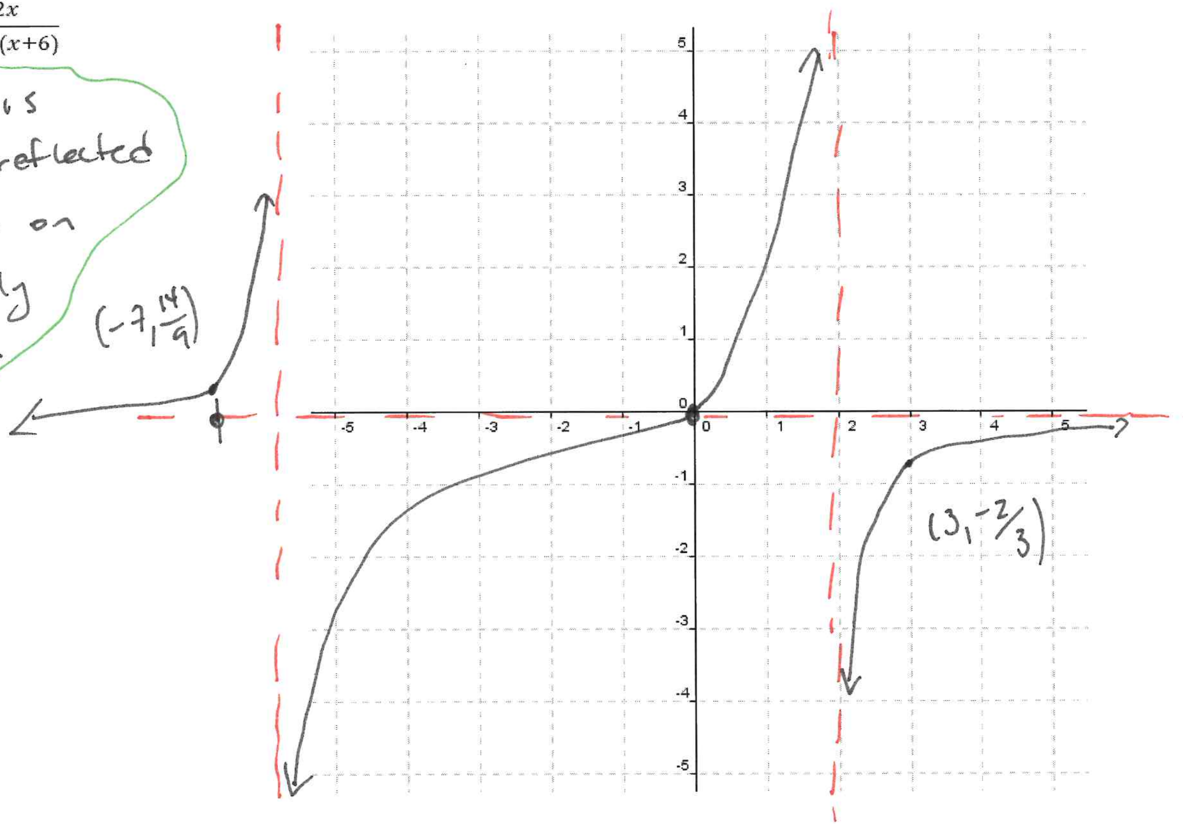


3. $f(x) = \frac{x}{(x-2)(x+6)}$



4. $f(x) = \frac{-2x}{(x-2)(x+6)}$

Note: This graph is the above graph reflected over the x-axis and stretched vertically by a factor of 2.



5. $f(x) = \frac{(x-4)\cancel{(x+5)}}{(x-2)\cancel{(x+5)}}$

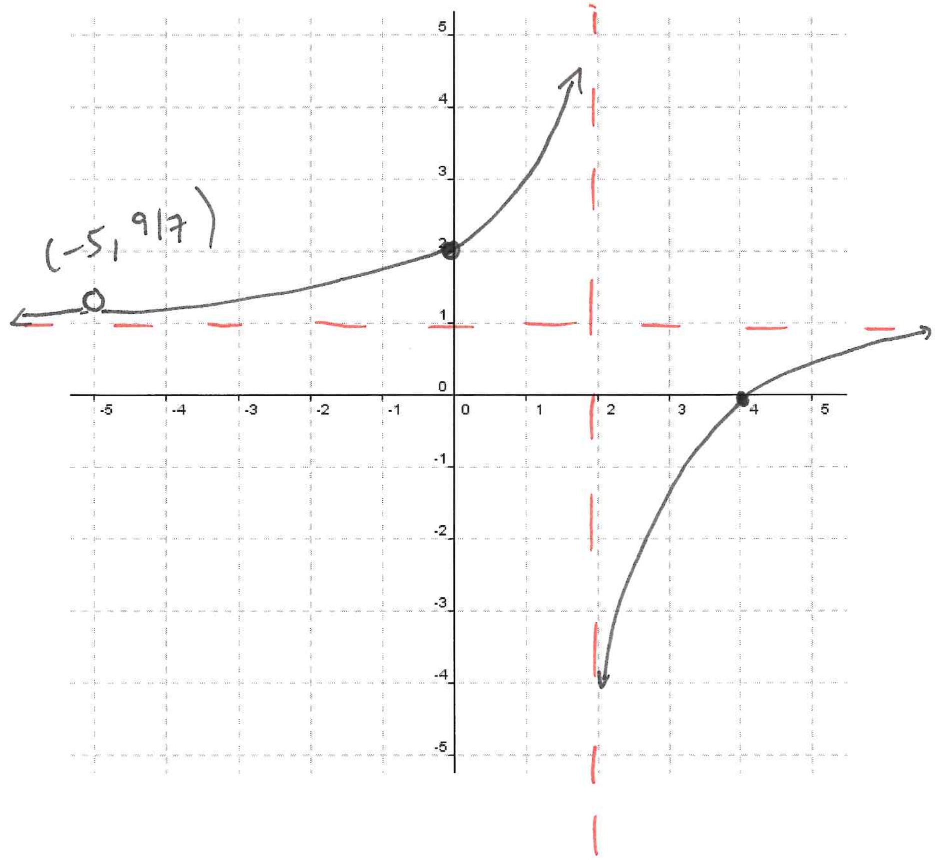
$f(x) = \frac{x-4}{x-2}$, $x \neq -5$

point of discontinuity

$f(-5) = \frac{-9}{-7}$

$= 9/7$

$(-5, 9/7)$



6. $f(x) = \frac{x^2-5x+6}{x-3}$

$f(x) = \frac{(x-3)(x-2)}{(x-3)}$

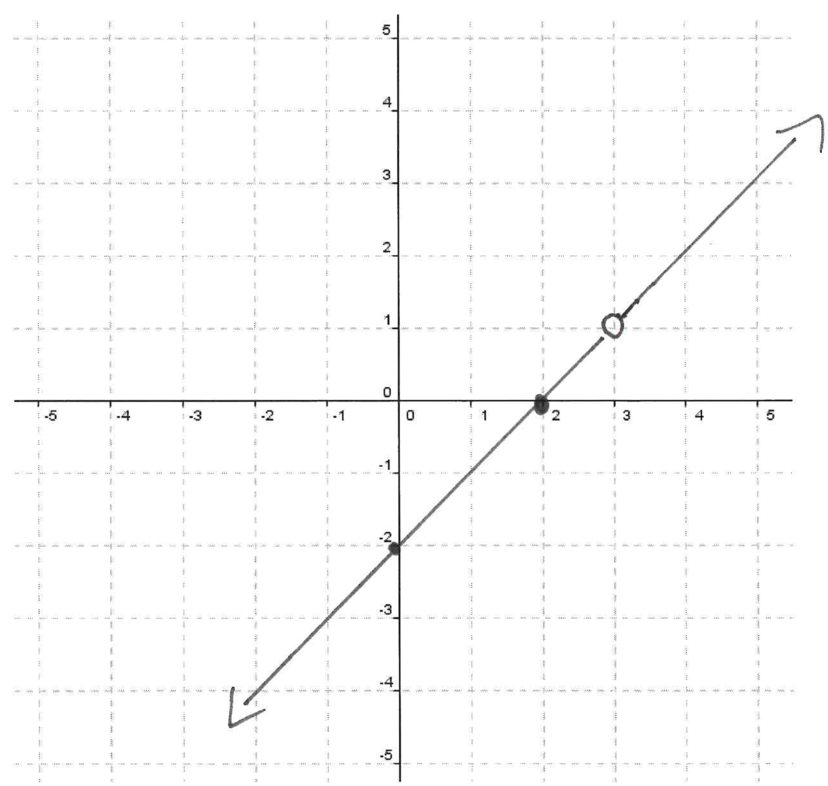
$f(x) = x-2$, $x \neq 3$

when $x = 3$

$f(3) = 1$

point of discontinuity

@ $(3,1)$



Chapter 9: RATIONAL FUNCTIONS

9.3 – Connecting Graphs and Rational Equations

Example #1

Ex1: Solve the following equations algebraically and graphically.

$y = \frac{1}{x}$

a) $\frac{1}{x+2} - 4 = 0$

Algebraically LCD: $x+2$

$\frac{1}{x+2}(x+2) - 4(x+2) = 0(x+2)$

$1 - 4(x+2) = 0$

$1 - 4x - 8 = 0$

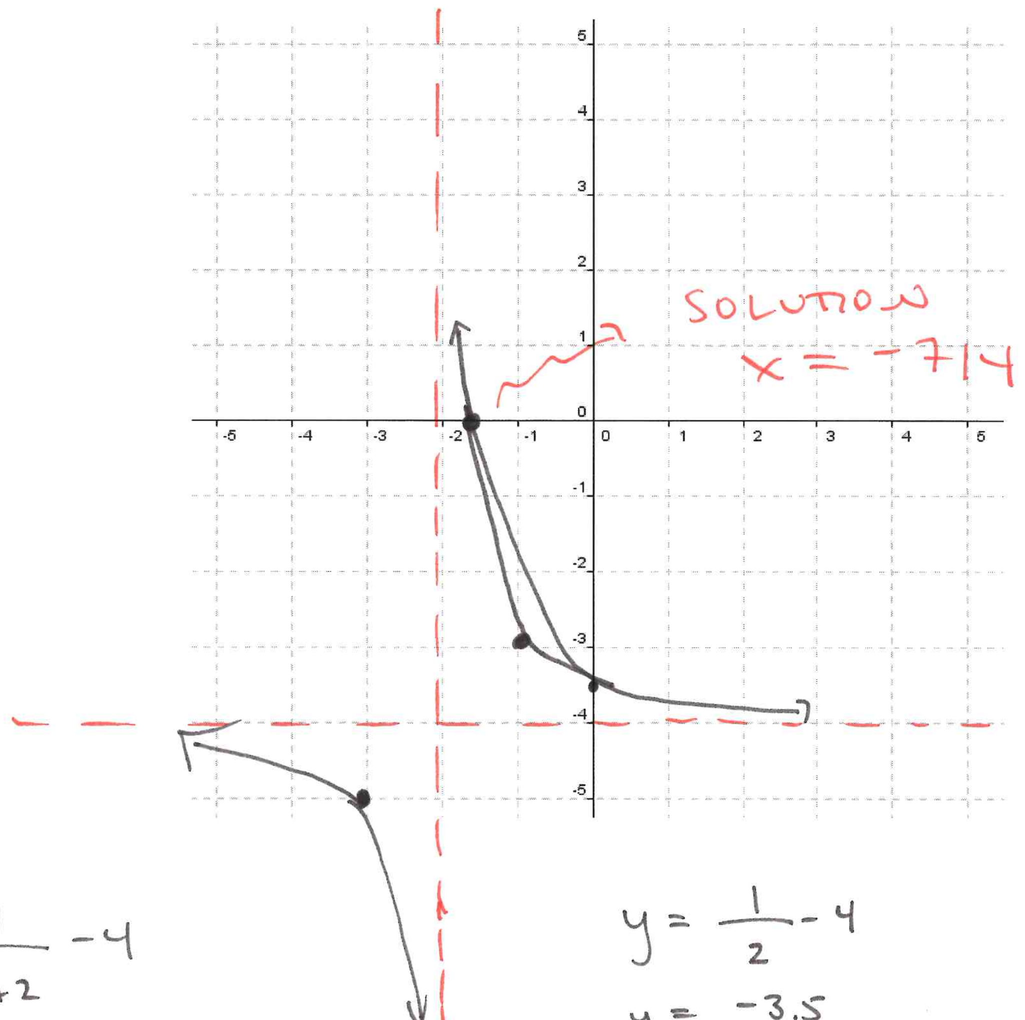
$-7 = 4x$

$-\frac{7}{4} = x$

$x = -1\,3/4$

Graphically $y = \frac{1}{x+2} - 4$

Find xint → it's the solution.



$y = \frac{1}{-3+2} - 4$

$y = -1 - 4$

$(-3, -5)$

$y = \frac{1}{2} - 4$

$y = -3.5$

$x \neq 1$

LCD: $(x-1)$

b) $(x-1)\dfrac{2}{\cancel{x-1}} = (x-2)(x-1)$

Algebraically

$2 = x^2 - 3x + 2$

$0 = x^2 - 3x$

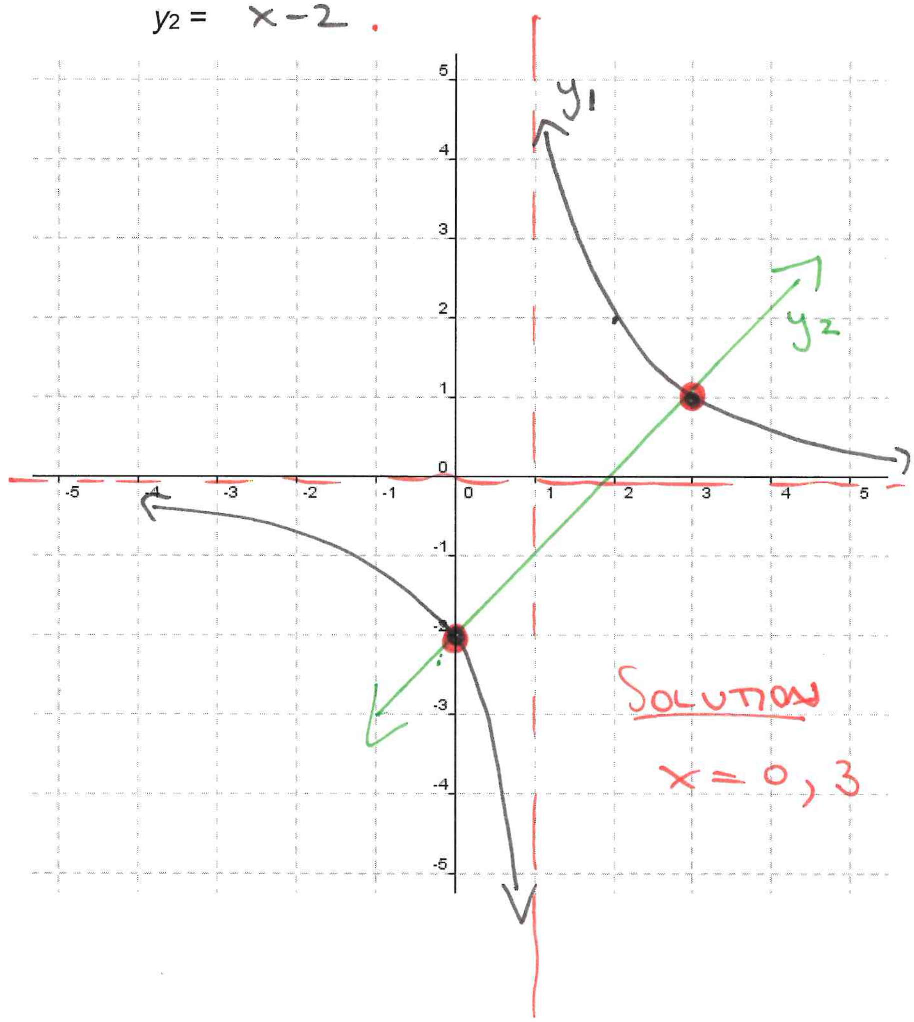
$0 = x(x-3)$

$x = 0 \qquad x = 3$

Graphically

$\boxed{y_1 = \dfrac{2}{x-1}}$ $+0$

$y_2 = x - 2$



Solution

$x = 0, 3$