Chapter 8: LOGARITHMIC FUNCTIONS

8.4 – Applications of Logarithmic & Exponential Functions

Example #1

The Richter magnitude, *M*, of an earthquake is defined as $M = \log\left(\frac{A}{A_0}\right)$ where *A* is the amplitude of the ground motion and A_0 is the amplitude associated with a "standard" earthquake.

a) In 1946, in Haida Gwaii, British Columbia, an earthquake with an amplitude measuring $10^{7.7}$ times A_0 struck. Determine the **magnitude** of this earthquake on the Richter scale.

$$M = \log\left[\frac{(10^{7.7}\cancel{A_0})}{\cancel{A_0}}\right]$$

$$= \log 10^{7.7}$$

$$= 7.7 \log 10$$

$$M = 7.7$$

$$M = \log\left(\frac{A}{A_0}\right)$$

b) The strongest recorded earthquake in Haida Gwaii was in 1949 and had a magnitude of 8.1 on the Richter scale. Determine how many times stronger this earthquake was than the one in 1946. COMPARISON

1949

$$8.1 = \log\left(\frac{A_1}{A_0}\right)$$

exponential form.

$$10^{8.1} = \frac{A_1}{A_0}$$

$$A_0 10^{8.1} = A_1$$

1946

$$7.7 = \log\left(\frac{A_2}{A_0}\right)$$

$$10^{7.7} = \frac{A_2}{A_0}$$

$$A_0 10^{7.7} = A_2$$

COMPARE!

$$\frac{A_1}{A_2} = \frac{\cancel{A_0} 10^{8.1}}{\cancel{A_0} 10^{7.7}}$$

$$\frac{A_1}{A_2} = 10^{8.1-7.7}$$

$$\frac{A_1}{A_2} = 10^{0.4}$$

$$A_1 = 10^{0.4}(A_2)$$

$10^{0.4} \approx 2.512$ 1949 earthquake was apx. 2.512 times stronger!

Example #2

The pH scale is used to measure the acidity or alkalinity of a solution.
It is defined as $pH = -\log(H^+)$ where H^+ is the concentration of hydrogen ions measured in moles per litre (mol/L).

A neutral solution, such as pure water, has a pH of 7. The closer the solution is to 0, the more acidic the solution. The closer the solution is to 14, the more alkaline the solution is.

A cola drink has a pH of 2.5 whereas milk has a pH of 6.6. How many times as acidic as milk is a cola drink? This is calculated by comparing the number of ions in each substance.

COMPARISON!

$$pH = -\log(H^+)$$

COLA

$$2.5 = -\log(H_1^+)$$
$$-2.5 = \log(H_1^+)$$
$$10^{-2.5} = H_1^+$$

MILK

$$6.6 = -\log(H_2^+)$$
$$-6.6 = \log(H_2^+)$$
$$10^{-6.6} = H_2^+$$

COMPARE

Cola / Milk

$$\frac{H_1^+}{H_2^+} = \frac{10^{-2.5}}{10^{-6.6}}$$

$$\frac{H_1^+}{H_2^+} = 10^{-2.5-(-6.6)}$$

$$H_1^+ = 10^{(-2.5+6.6)}(H_2^+)$$

Cola is apx. 12 589 times more acidic!

Example #3

The human ear is able to detect sounds of different intensities. The intensity level, β, in decibels, is defined as $\beta = 10\log\frac{I}{I_0}$ where I is the intensity of the sound measured in watts per square metre (W/m²), and I_0 is 10^{-12} W/m² which is the threshold of hearing (the faintest sound that can be heard by a person of normal hearing).

a) A truck emits a sound intensity of 0.001 W/m². Determine its decibel level.

$$\beta = 10\log\left(\frac{0.001}{10^{-12}}\right)$$
$$= 10\log\left(\frac{10^{-3}}{10^{-12}}\right)$$
$$= 10\log 10^{9}$$
$$= 9(10)\log 10$$
$$= 90\text{ dB}$$

b) It is recommended a person wears protective ear gear when the intensity level is 85 dB or greater. The MTS Centre measures 110 dB when the Jets score a goal.

How many times louder is the MTS Centre than the recommended maximum sound intensity?

Comparison!

$$\beta = 10\log\left(\frac{I}{I_0}\right)$$

MTS

$$110 = 10\log\left(\frac{I_1}{I_0}\right)$$
$$11 = \log\left(\frac{I_1}{I_0}\right)$$
$$10^{11} = \frac{I_1}{I_0}$$
$$I_0 10^{11} = I_1$$

Rec

$$85 = 10\log\left(\frac{I_2}{I_0}\right)$$
$$8.5 = \log\left(\frac{I_2}{I_0}\right)$$
$$10^{8.5} = \frac{I_2}{I_0}$$
$$I_0 10^{8.5} = I_2$$

$$\frac{I_1}{I_2} = \frac{I_0 10^{11}}{I_0 10^{8.5}}$$
$$\frac{I_1}{I_2} = 10^{11-8.5}$$
$$I_1 = 10^{2.5}(I_2)$$

$\approx$ 316 times stronger (louder) than recommended!

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Example #4

Compound interest is a good example of an increasing exponential function.

P = initial value
A = final value
n = # of times the interest is compounded in one year
r = interest rate, as a decimal (ex: 5% = 0.05)
t = time, in years

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

Interest Earned = A – P

An amount of $5000 is invested at a rate of 3.1% compounded monthly.

a) Determine how much interest is earned after 20 years.

$$A = 5000\left(1+\frac{0.031}{12}\right)^{12(20)}$$

$$A = 5000\,(1.00258333)^{240}$$

$$A = \$9287.21$$

$$9287.21 - 5000$$

$$= \$4287.21$$

Note:

Annually	1 time/year
Semi-annually	2 times/year
Quarterly	4 times/year
Monthly	12 times/year
Biweekly	26 times/year
Weekly	52 times/year
Daily	365 times/year

b) Determine how much time is needed for the initial investment to double in size.

$$10000 = 5000\left(1+\frac{0.031}{12}\right)^{12t}$$

$$2 = \left(1+\frac{0.031}{12}\right)^{12t}$$

$$\log 2 = \log(1.0025833)^{12t}$$

$$\log 2 = 12t\,(\log 1.0025833)$$

$$\frac{\log 2}{12\log 1.0025833} = t$$

$$t = \cancel{22.3455}$$

$$22.3887$$

apx 23 yrs.

Example #5

There are 500 mice found in a field on June 1. On June 20, 800 mice are counted.

If the population of mice continues to increase at the same rate, determine how many mice there will be on June 28.

Use $A = Pe^{rt}$ where P = initial value
A = final value
t = time, in days
r = rate of increase or decrease

Note:

If $r > 0$, then the function increases exponentially

If $r < 0$, then the function decreases exponentially

*. Find r

$800 = 500e^{r(19)}$

$\frac{800}{500} = e^{19r}$

$\frac{8}{5} = e^{19r}$

$\ln\left(\frac{8}{5}\right) = \ln\left(e^{19r}\right)$

$\ln\left(\frac{8}{5}\right) = 19r \ln e$ $= 1$

$\frac{\ln(8/5)}{19} = r$

$0.02473703 31 = r$

use r to solve

$A = Pe^{rt}$

$A = 500e^{r(27)}$

≈ 975 mice.