Chapter 9: RATIONAL FUNCTIONS

9.1 – Exploring Rational Functions: $y = \frac{a}{x-h} + k$

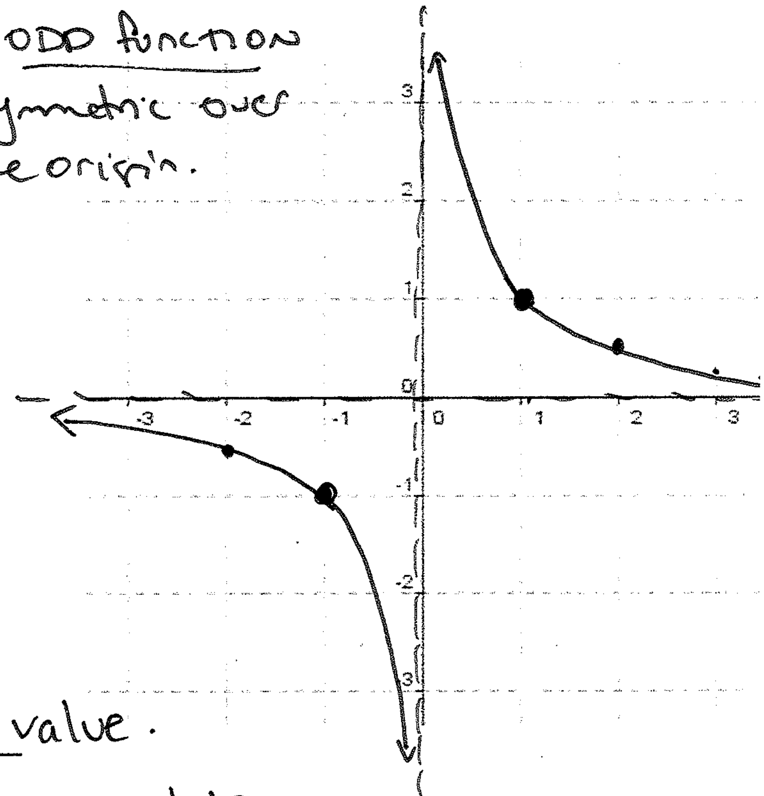
Sketch the graph of $f(x) = \frac{1}{x}$.

ODD function

Symmetric over the origin.

1/2 2

x	y
1	1
2	1/2
3	1/3
-1	
-2	



Note, $x = 0$ is a nonpermissible value.

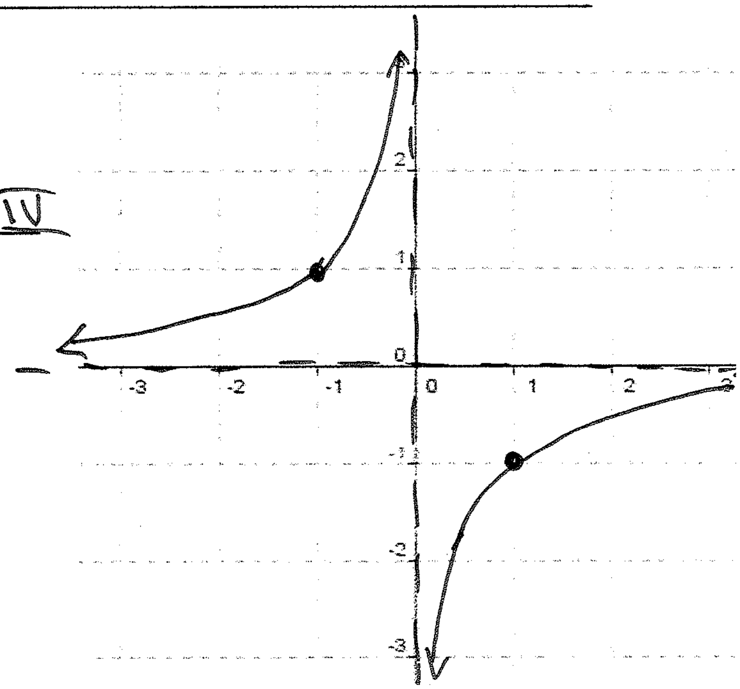
Graphically this creates a vertical asymptote

When $x \to \infty$, $y \to 0$ and when $x \to -\infty$, $y \to 0$

Thus, $y = 0$ is a horizontal asymptote. The curves are in Quadrants I, III

Sketch the graph of $f(x) = -\frac{1}{x}$.

The curves are now in Quadrants II, IV

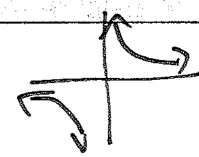


Note: $-\frac{1}{x}$ is the same as $\frac{-1}{x}$.

$\frac{1}{-x}$

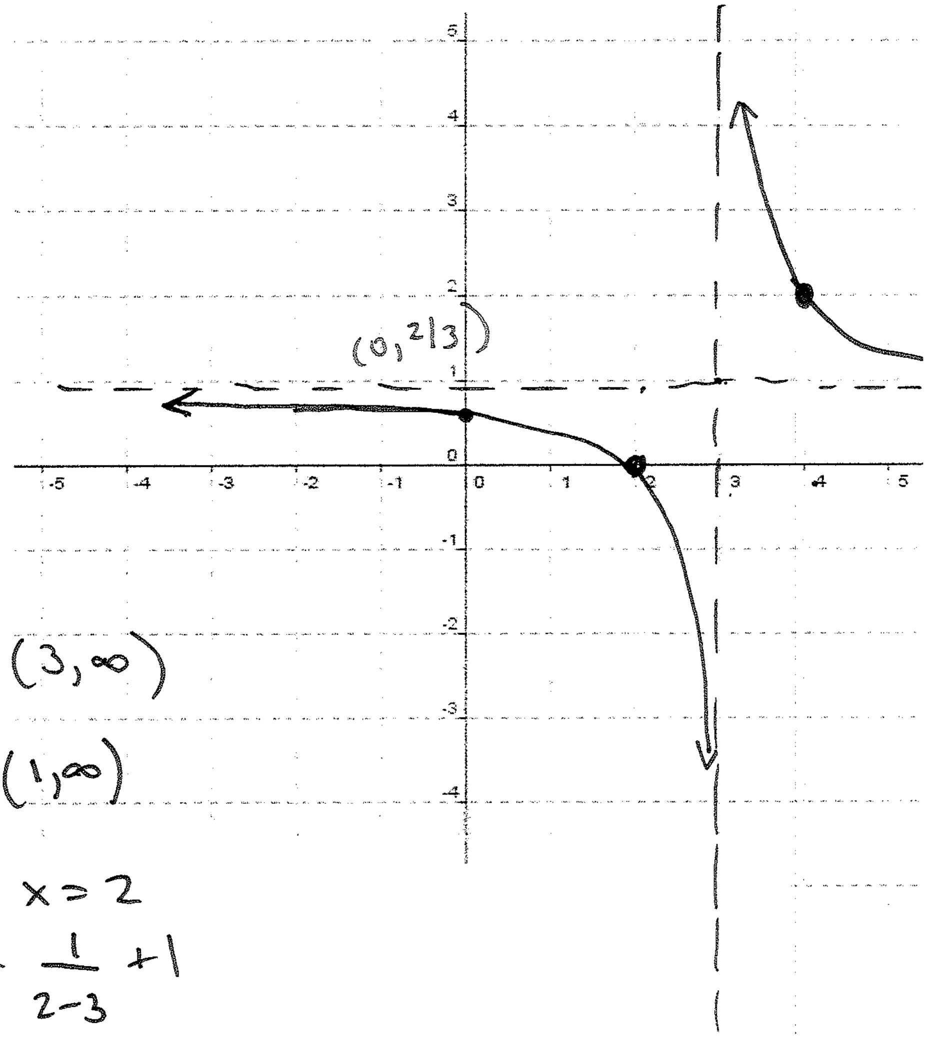
Example #1

Sketch the graph of $y=\dfrac{1}{x-3}+1$ $y=\dfrac{1}{x}$



Note : You must label a point in each section of the graph

non-permissible value	$x \neq +3$
x-intercept	2
y-intercept	$2/3$
vertical asymptote	$x=+3$
horizontal asymptote	$y=1$
domain $\{x \in R, x \neq 3\}$	$(-\infty,3) \cup (3,\infty)$
range $\{y \in R, y \neq \phi\}$	$(-\infty,1) \cup (1,\infty)$



when $x=4$

$y=\dfrac{1}{4-3}+1$

$y=2$

when $x=2$

$y=\dfrac{1}{2-3}+1$

$y=0$

$\underline{y\ \text{int:}}$ $y=\dfrac{1}{0-3}+1$

$=-\dfrac{1}{3}+1$

$=-\dfrac{1}{3}+\dfrac{3}{3}$

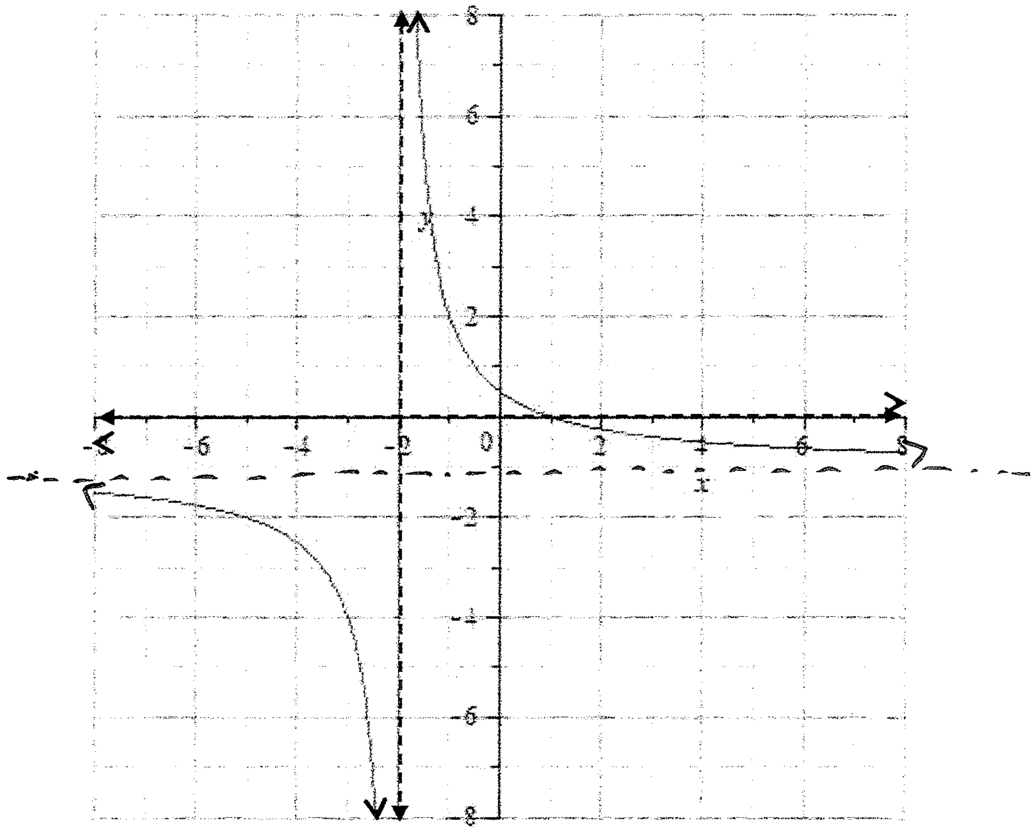
$=2/3$

Given $f(x)=\frac{1}{x}$, we can sketch the graph of $y=3f(x+2)-1$.

Note:

The equation of the transformed graph is $y=\frac{3}{x+2}-1$.

Can you see the connection?



The general equation of a rational function is $\boxed{y=\frac{a}{x-h}+k}$.

This represents a vertical stretch by a factor of *a*, followed by a horizontal shift of *h* units, and a vertical shift of *k* units.

x=h is a vertical Asymptote, *y= k* is a horizontal asymptote.

Explain the behaviour of the graph for values of the ~~variable~~ function around x = -2.

$\lim_{x \to -2^+} f(x) = \infty$

As we approach $x=-2$ from the right the y values approach ∞.

As we approach $x=-2$ from the left the y values approach $-\infty$.

Explain the end behaviour of the graph.

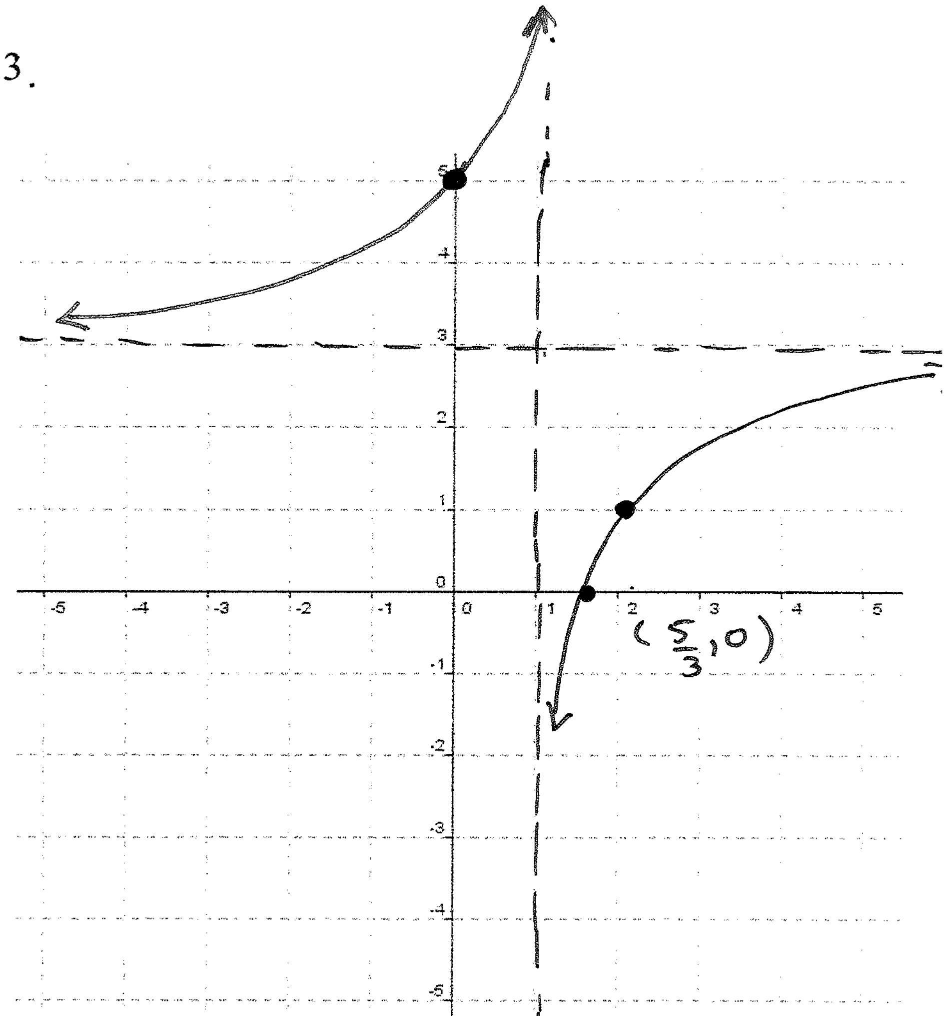
As $|x|$ get infinitely big y approaches -1

$$\lim_{x \to \pm\infty} f(x) = -1$$

Example #2

Sketch the graph of $y = \frac{-2}{x-1} + 3$.

non-permissible value	$x \neq 1$
x-intercept	$5/3$
y-intercept	5
vertical asymptote	$x = 1$
horizontal asymptote	$y = 3$
domain $\{x \in R,\ x \neq 1\}$.	
range $\{y \in R,\ y \neq 3\}$	



$x = 0$

$y = \frac{-2}{-1} + 3$

$y = 5$

$x = 2$

$y = \frac{-2}{2-1} + 3$

$y = 1$

x int:

$0 = \frac{-2}{x-1} + 3$

$-3 = \frac{-2}{x-1}$

$-3(x-1) = -2$

$-3x + 3 = -2$

$-3x = -5$

$x = 5/3$

OR

$x - 1 = \frac{-2}{-3}$

$x = -\frac{2}{3} + 1$

$x = +\frac{2}{3} + \frac{3}{3}$

$x = \frac{5}{3}$

Date:____________

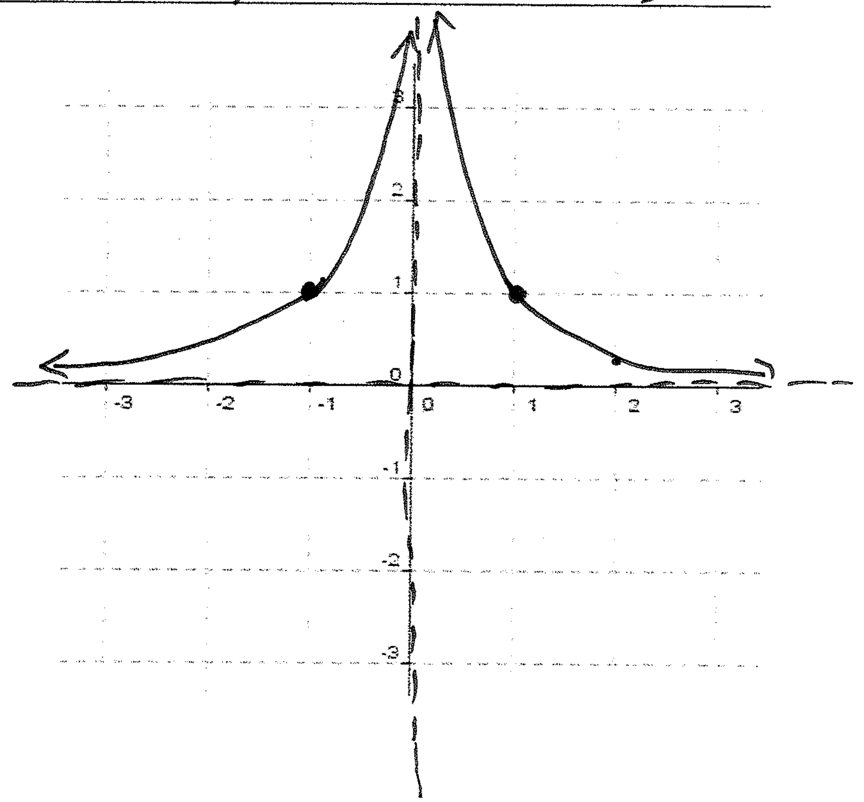
$y = \dfrac{a}{(x-h)^2} + k$

even function

Example #3

Sketch the graph of $f(x) = \dfrac{1}{x^2}$.

x	y

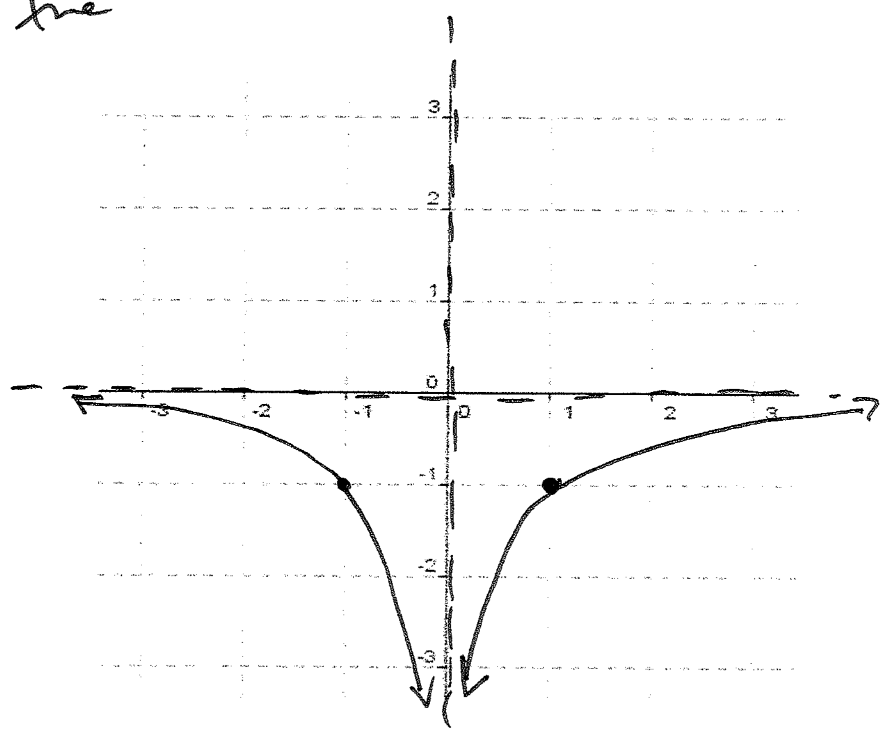


Example #4

Sketch the graph of $f(x) = -\dfrac{1}{x^2}$.

reflects over the x-axis!

x	y



Note: We can use the previous ideas to help us graph the transformed versions of these functions.