Homework 8.4 Applications

Question 1

A water filtration system which removes impurities from a sample of water can be modelled by $P = 0.25(0.55)^n$, where:

$P =$ the percentage of impurities remaining, in decimal form

$n =$ the number of filters

$\nearrow 0.01$

Determine, algebraically, how many filters are required so that less than 1% of the impurities remain in the water sample. Express your answer as a whole number.

$$0.01 = 0.25(0.55)^n$$
$$\log 0.04 = n \log 0.55$$
$$\frac{\log 0.04}{\log 0.55} = n$$

$n = 5.384$

$\therefore$ 6 filters.

Question 2

A lake affected by acid rain has a pH of 4.4.
A person suffering from heartburn has a stomach acid pH of 1.2.
The pH of a solution is defined as $\text{pH} = -\log\left[\text{H}^+\right]$ where $\left[\text{H}^+\right]$ is the hydrogen ion concentration.
How many times greater is the hydrogen ion concentration of the stomach than that of the lake?
Express your answer as a whole number.

Rain

$$4.4 = -\log\left[H_1^+\right]$$
$$-4.4 = \log H_1^+$$
$$10^{-4.4} = H_1^+$$

Stomach acid

$$1.2 = -\log\left[H_2^+\right]$$
$$-1.2 = \log H_2^+$$
$$10^{-1.2} = H_2^+$$

$$\frac{H_2^+}{H_1^+} = \frac{10^{-1.2}}{10^{-4.4}} = 10^{-1.2-(-4.4)} = 10^{3.2} \approx 1585$$

1585 times stronger.

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Question 3

An earthquake in Vancouver had a magnitude of 6.3 on the Richter scale. An earthquake in Japan had a magnitude of 8.9 on the Richter scale.

How many times more intense was the Japan earthquake than the Vancouver earthquake?

You may use the formula below:

$$M = \log\left(\frac{A}{A_0}\right)$$

where M is the magnitude of the earthquake on the Richter scale

A is the intensity of the earthquake

A_0 is the intensity of a standard earthquake

Express your answer as a whole number.

$$\frac{A_1}{A_2} = 10^{8.9-6.3}$$

$$= 10^{2.6}$$

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$= 398$ times stronger

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Question 4

A population of 500 bacteria will triple in 20 hours.

Using the formula given below,

$A = Pe^{rt}$

$A =$ population after t hours

$P =$ initial population

$r =$ rate of growth

$t =$ time in hours

a) Determine the rate of growth, r.

$1500 = 500e^{r(20)}$

$3 = e^{20r}$

$\ln 3 = \ln e^{20r}$

$\ln e = 1$

$\ln 3 = 20r \ln e$

$\frac{\ln 3}{20} = r$

$r = 0.054930614$

b) Determine how many hours it will take for the initial population to double with the same rate of growth.

$1000 = 500e^{rt}$

$2 = e^{rt}$

$\ln 2 = \ln e^{rt}$

$\ln 2 = rt$

$\frac{\ln 2}{r} = t$

$t = 12.619$ hrs.

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Question 5

Jess invests $12 000 at a rate of 4.75% compounded monthly. How long will it take for Jess to triple her investment?

Express your answer in years, correct to 3 decimal places.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$36000 = 12000\left(1 + \frac{0.0475}{12}\right)^{12t}$$

$$3 = (1.003958333)^{12t}$$

$$\log 3 = 12t \log 1.003958333$$

$$\frac{\log 3}{12 \log 1.003958333} = t$$

$$t = 23.174 \text{ yrs}$$

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Question 6

The number of times a website is visited can be modeled by the function:

$$A = 800(e)^{rt}$$

where A = the total number of visitors at time t

t = the time in days $(t \geq 0)$

r = the rate of growth

After 5 days, 40 000 people have visited the site.
Determine the number of visitors expected after 9 days.

Express your answer as a whole number.

Find r

$40000 = 800 e^{r(5)}$

$50 = e^{5r}$

$\ln 50 = 5r \ln e$

$\frac{\ln 50}{5} = r$

$0.782404601 = r$

After 9 days

$A = 800 e^{r(9)}$

$A = 914610$ people

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Question 7

The population of Oakfalls can be described by the formula:

$$P = P_0 e^{rt}$$

where P_0 = the initial population

P = the final population

r = the rate of growth

t = the time in years

The population of Oakfalls was 15 125 on January 1, 1976.
On January 1, 1986 the population fell to 13 780.

Assuming that the population decreased at a constant rate, find the value of r.

Express your answer correct to 3 decimal places.

$$13780 = 15125 e^{r(10)}$$

$$\ln\left(\frac{13780}{15125}\right) = \ln e^{10r}$$

$$\ln\left(\frac{13780}{15125}\right) = 10r$$

$$\frac{\ln\left(\frac{13780}{15125}\right)}{10} = r$$

$$r = -0.0093$$