Chapter 8: LOGARITHMIC FUNCTIONS

8.2 – Transformations of Logarithmic Functions

Recall: The **logarithmic function** is the inverse of the **exponential function.**

Remember, to find the inverse of a function, we switch the x and y values and then solve for y.

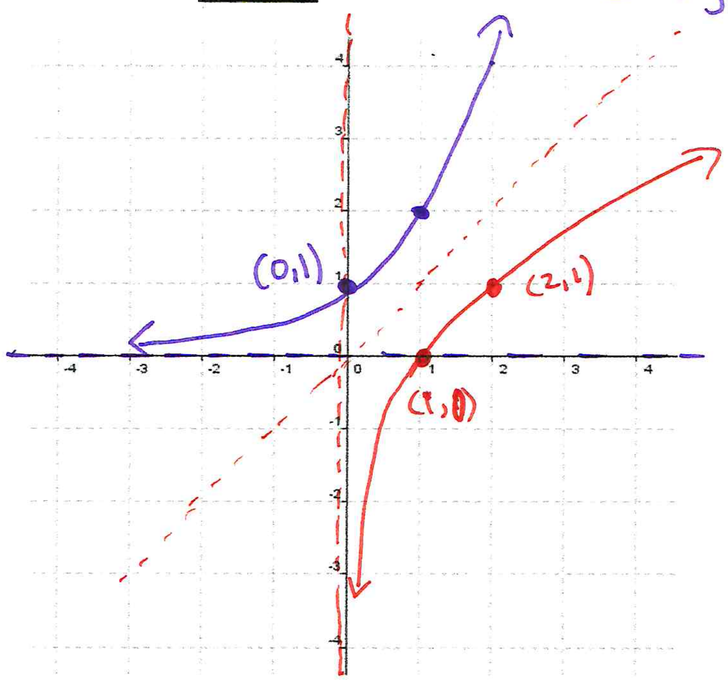
Exponential Function	Inverse Function
$y = b^x$	$x = b^y$
	which is equivalent to
	$y = \log_b x$

Example #1

Sketch the graphs of the functions $y = 2^x$ and $y = \log_2 x$.

Note that these two functions are inverses of each other. $(x,y) \to (y,x)$

x	y



x	y

Note: The graph of $y = \log_2 x$ has a V.A. at $x=0$, x int. (1,0), (2,1)

because $y=2^x$ H.A $y=0$, y int (0,1), point (1,2)

Example #2

Sketch the graphs of the following functions on the same Cartesian plane.

a) $y = \log_3 x$ b) $y = \log_4 x$ c) $y = \log x$ d) $y = \ln x$

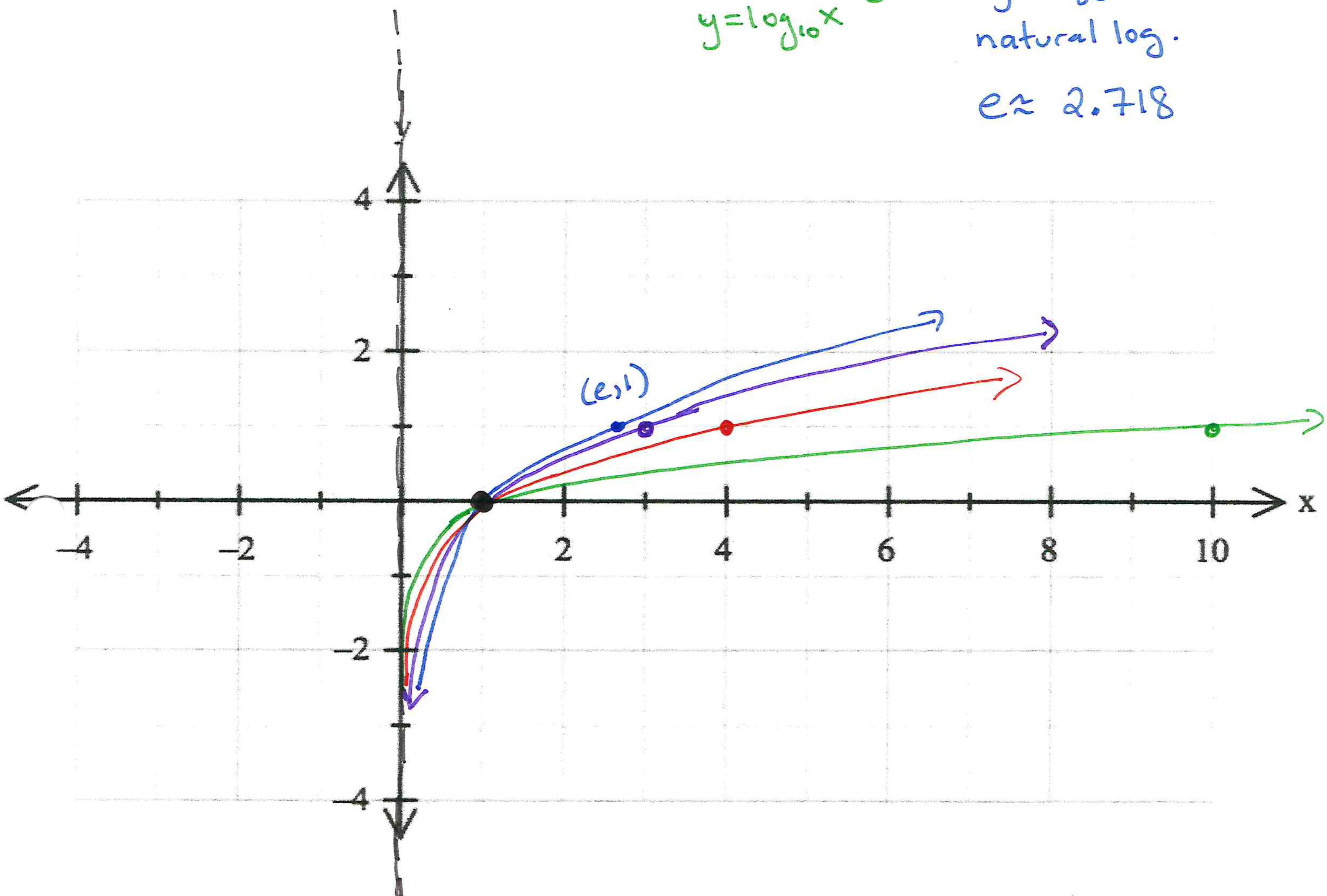
common log

$y = \log_{10} x$

$y = \log_e x$

natural log.

$e \approx 2.718$



Note: All the graphs pass through the point (1,0). The **base** of the logarithm determines the next point.

Base 3 → (3,1) $y = \log_3 x$ → $3^y = x$ → $3^1 = 3$ → (3, 1)

Base 4 → (4,1)

Base e → (e,1)

Vertically stretch by a factor of $|a|$
Reflect in the x-axis if $a < 0$.

Vertically translate k units.

$$y = a \log_c (b(x - h)) + k$$

Horizontally stretch by a factor of $\left|\frac{1}{b}\right|$
Reflect in the y-axis if $b < 0$.

Horizontally translate h units.

Example #3

a) **Sketch** the graph of the function $y = \log_5(x + 2) - 1$

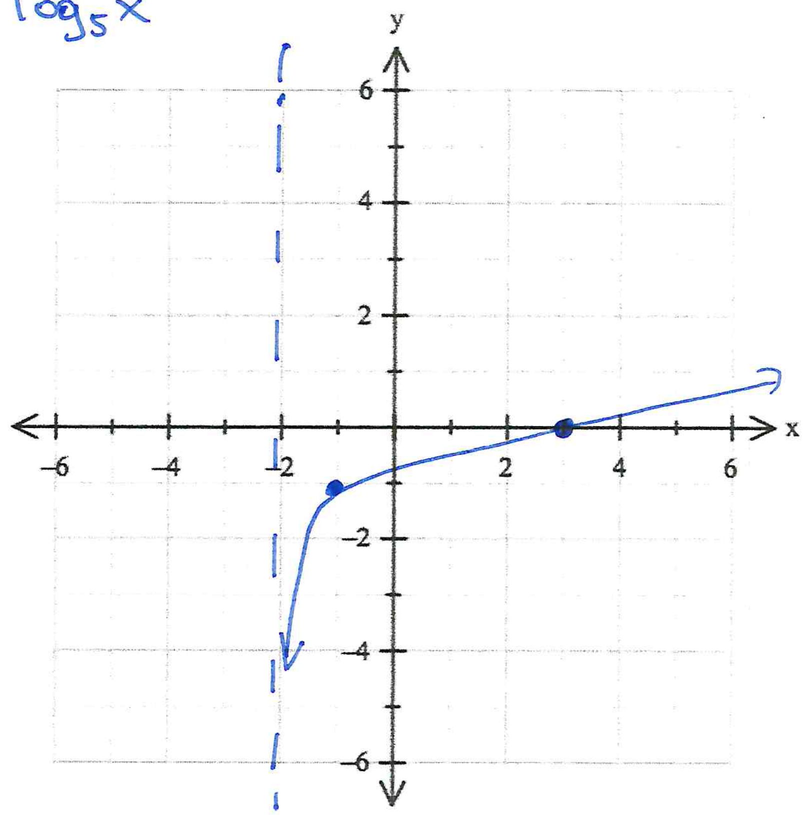
$y = \log_5 x$

$(x, y) \rightarrow (x-2, y-1)$

$(1, 0) \rightarrow (-1, -1)$

$(5, 1) \rightarrow (3, 0)$

$x = 0 \qquad x = -2$



b) Sate the **domain** and **range.**

Domain: $x > -2$

Range: $(-\infty, \infty)$

c) State the equation of the **asymptote.**

$x = -2$

d) Determine the **x – intercept.**

$y = \log_5(x+2) - 1$

$0 = \log_5(x+2) - 1$

$1 = \log_5(x+2)$

$5^1 = x + 2$

$3 = x$

reflection over the y-axis.

Example #4 $y = \log_3 x$

a) **Sketch** the graph of the function $y = \log_3(3-x)+1$

$y = \log_3(-x+3)+1$

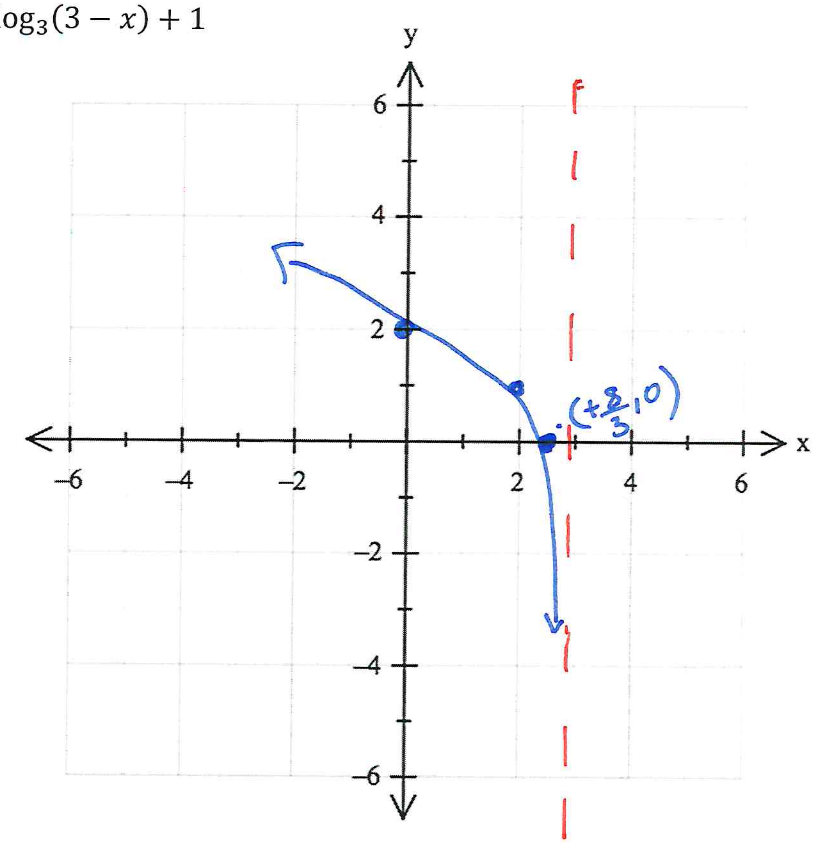
$y = \log_3[-(x-3)]+1$

$(x, y) \rightarrow (-x+3, y+1)$

$(1,0) \rightarrow (2,1)$

$(3,1) \rightarrow (0,2)$

$x=0 \rightarrow x=3$



b) Sate the **domain** and **range.**

Domain: $x<3$ $(-\infty, 3)$

Range: $(-\infty, \infty)$ $y \in \mathbb{R}$

c) State the equation of the **asymptote**.

$x=3$

$y = \log_3(3-x)+1$

d) Determine the **y – intercept** and the **x – intercept**.

x int:

$0 = \log_3(3-x)+1$

$-1 = \log_3(3-x)$

$3^{-1} = 3-x$

$\frac{1}{3} - 3 = -x$

$\frac{1}{3} - \frac{9}{3} = -x$

$-\frac{8}{3} = -x$

$x = 8/3$ $2.\frac{2}{3}$

y int:

$y = \log_3(3-0)+1$

$y = \log_3 3 + 1$

$y = 1+1$

$y = 2$