MPC40S Date: Answer key

6.3 – Proving Trigonometric Identities (Part 2) Practice

The following trigonometric identities are from old Provincial Exams.

Prove each of the trigonometric identities.

1. $\cos 2\theta = \frac{1-\tan^2\theta}{\sec^2\theta}$ From January 2008 (3 Marks)

LHS	RHS
$\cos 2\theta$	$\frac{1-\tan^2\theta}{\sec^2\theta}$
	$= \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}$
	$= \frac{\frac{\cos^2\theta}{\cos^2\theta}-\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}$
	$= \frac{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}$
	$= \frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{1}$
	$= \cos^2\theta-\sin^2\theta$
	$= \cos 2\theta$
	LHS = RHS

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2. $\cos 2\theta = \frac{\cot^2\theta - \tan^2\theta}{\sec^2\theta \csc^2\theta}$ From January 2009 (4 Marks)

LHS	RHS
$\cos 2\theta$	$\frac{\cot^2\theta - \tan^2\theta}{\sec^2\theta \csc^2\theta}$
	$= \frac{\frac{\cos^2\theta}{\sin^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}}{\sec^2\theta \csc^2\theta}$
	$= \frac{\frac{\cos^4\theta - \sin^4\theta}{\cos^2\theta \sin^2\theta}}{\frac{1}{\cos^2\theta \sin^2\theta}}$
	$= \frac{\cos^4\theta - \sin^4\theta}{\cancel{\cos^2\theta \sin^2\theta}} \times \frac{\cos^2\theta \cancel{\sin^2\theta}}{1}$
	$= \cos^4\theta - \sin^4\theta$
	$= (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$
	$= (\cos^2\theta - \sin^2\theta)(1)$
	$= \cos 2\theta$
	LHS = RHS

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3. $\sec x + \tan x = \frac{\cos x}{1-\sin x}$ From January 2010 (4 Marks)

LHS	RHS
$\sec x + \tan x$	$\frac{\cos x}{1-\sin x}$
$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$	$= \frac{\cos x}{1-\sin x} \cdot \left(\frac{1+\sin x}{1+\sin x}\right)$
$= \frac{1+\sin x}{\cos x}$	$= \frac{\cos x(1+\sin x)}{1+\sin x-\sin x-\sin^2 x}$
	$= \frac{\cos x(1+\sin x)}{1-\sin^2 x}$
	$= \frac{\cos x(1+\sin x)}{\cos^2 x}$
	$= \frac{1+\sin x}{\cos x}$
	LHS = RHS

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4. $\frac{2}{1-\tan^2\theta} = \frac{2-2\sin^2\theta}{\cos 2\theta}$ From January 2011 (4 Marks)

LHS	RHS
$\frac{2}{1-\tan^2\theta}$	$\frac{2-2\sin^2\theta}{\cos 2\theta}$
$= \frac{2}{1-\frac{\sin^2\theta}{\cos^2\theta}}$	$= \frac{2(1-\sin^2\theta)}{\cos 2\theta}$
$= \frac{2}{\frac{\cos^2\theta}{\cos^2\theta}-\frac{\sin^2\theta}{\cos^2\theta}}$	$= \frac{2\cos^2\theta}{\cos 2\theta}$
$= \frac{2}{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}$	
$= 2 \times \frac{\cos^2\theta}{\cos^2\theta-\sin^2\theta}$	
$= \frac{2\cos^2\theta}{\cos^2\theta-\sin^2\theta}$	
$= \frac{2\cos^2\theta}{\cos 2\theta}$	
LHS = RHS	

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5. $\frac{2\tan x}{1+\tan^2 x} = \sin 2x$ From June 2011 (3 Marks)

LHS	RHS
$\frac{2\tan x}{1+\tan^2 x}$ $= \frac{2\tan x}{\sec^2 x}$ $= \frac{\frac{2\sin x}{\cos x}}{\frac{1}{\cos^2 x}}$ $= \frac{2\sin x}{\cos x} \cdot \frac{\cos^2 x}{1}$ $= 2\sin x \cos x$ $= \sin 2x$ LHS = RHS	$\sin 2x$