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6.3 – Proving Trigonometric Identities (Part 1) Practice

Prove each of the following identities.

1. $(1+\sin x)(1-\sin x)=\frac{1}{\sec^2 x}$

2. $\cos^2 x-\sin^2 x=1-2\sin^2 x$

3. $\cos^2\theta=(1-\sin\theta)(1+\sin\theta)$

4. $(1-\sec\theta)(1+\sec\theta)=-\tan^2\theta$

5. $2\sec^2 x=\frac{1}{1-\sin x}+\frac{1}{1+\sin x}$

6. $\sec^2 x-\csc^2 x=\tan^2 x-\cot^2 x$

7. $\cos^4 x-\sin^4 x=1-2\sin^2 x$

8. $\frac{\csc^2\theta-1}{\csc^2\theta}=\cos^2\theta$

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1. $(1+\sin x)(1-\sin x) = \frac{1}{\sec^2 x}$

LHS	RHS
$(1+\sin x)(1-\sin x)$	$\frac{1}{\sec^2 x}$
$= 1-\sin x+\sin x-\sin^2 x$	$= \cos^2 x$
$= 1-\sin^2 x$	
$= \cos^2 x$	
LHS = RHS	

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2. $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

LHS	RHS
$\cos^2 x - \sin^2 x$ $= (1 - \sin^2 x) - \sin^2 x$ $= 1 - 2\sin^2 x$ LHS = RHS	$1 - 2\sin^2 x$

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3. $\cos^2\theta = (1 - \sin\theta)(1 + \sin\theta)$

LHS	RHS
$\cos^2\theta$	$(1-\sin\theta)(1+\sin\theta)$ $= 1 + \sin\theta - \sin\theta - \sin^2\theta$ $= 1 - \sin^2\theta$ $= \cos^2\theta$ LHS = RHS

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4. $(1 - \sec\theta)(1 + \sec\theta) = -\tan^2\theta$

LHS	RHS
$(1-\sec\theta)(1+\sec\theta)$ $= 1+\sec\theta-\sec\theta-\sec^2\theta$ $= 1-\sec^2\theta$ $= -\sec^2\theta+1$ $= -(\sec^2\theta-1)$ $= -(\tan^2\theta)$ $= -\tan^2\theta$ LHS = RHS	$-\tan^2\theta$

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5. $2\sec^2 x = \frac{1}{1-\sin x} + \frac{1}{1+\sin x}$

LHS	RHS
$2\sec^2 x$	$\frac{1}{1-\sin x} + \frac{1}{1+\sin x}$
	$= \left[\frac{1+\sin x}{1+\sin x}\right]\left[\frac{1}{1-\sin x}\right] + \left[\frac{1-\sin x}{1-\sin x}\right]\left[\frac{1}{1+\sin x}\right]$
	$= \frac{1+\sin x + 1-\sin x}{(1+\sin x)(1-\sin x)}$
	$= \frac{2}{1-\sin x+\sin x-\sin^2 x}$
	$= \frac{2}{1-\sin^2 x}$
	$= \frac{2}{\cos^2 x}$
	$= 2\sec^2 x$
	LHS = RHS

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6. $\sec^2 x - \csc^2 x = \tan^2 x - \cot^2 x$

LHS	RHS
$\sec^2 x - \csc^2 x$ $= (\tan^2 x + 1) - (1 + \cot^2 x)$ $= \tan^2 x + 1 - 1 - \cot^2 x$ $= \tan^2 x - \cot^2 x$ LHS = RHS	$\tan^2 x - \cot^2 x$

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7. $\cos^4 x - \sin^4 x = 1 - 2\sin^2 x$

LHS	RHS
$\cos^4 x - \sin^4 x$	$1 - 2\sin^2 x$
$= (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$	
$= (\cos^2 x - \sin^2 x)(1)$	
$= \cos^2 x - \sin^2 x$	
$= 1 - \sin^2 x - \sin^2 x$	
$= 1 - 2\sin^2 x$	

LHS = RHS

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8. $\frac{\csc^2\theta-1}{\csc^2\theta} = \cos^2\theta$

LHS	RHS
$\frac{\csc^2\theta - 1}{\csc^2\theta}$ $= \frac{\cot^2\theta}{\csc^2\theta}$ $= \frac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{1}{\sin^2\theta}}$ $= \frac{\cos^2\theta}{\sin^2\theta} \cdot \frac{\sin^2\theta}{1}$ $= \cos^2\theta$ LHS = RHS	$\cos^2\theta$