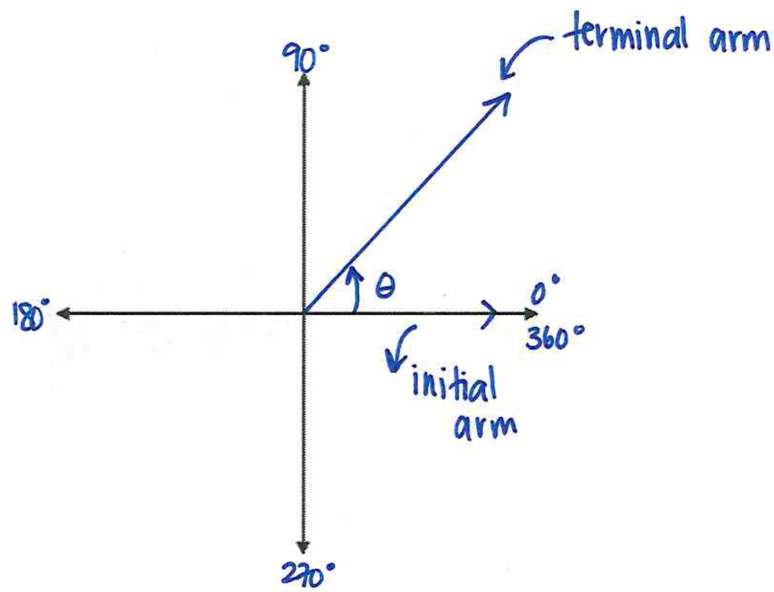
Chapter 4: TRIGONOMETRY AND THE UNIT CIRCLE

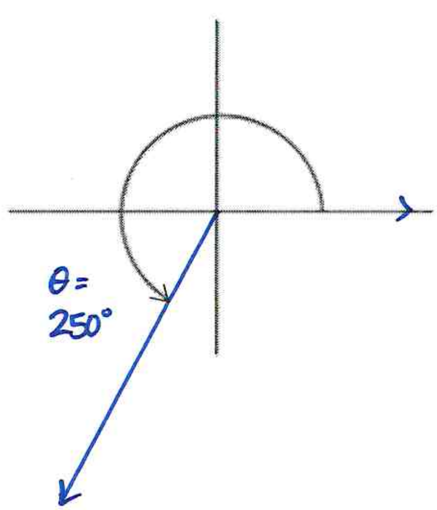
4.1 – Angles and Angle Measure

An angle in standard position has its centre at the origin and its initial arm along the positive x-axis

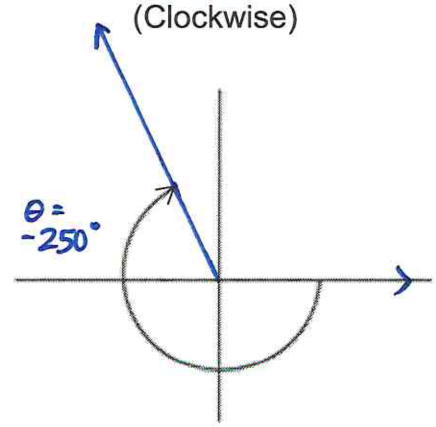
There are positive and negative angles.



Positive Angles
(Counter-clockwise)



Negative Angles
(Clockwise)



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Example #1

In which **quadrant** is the terminal arm of each angle located?

a) 400° I

b) 700° IV

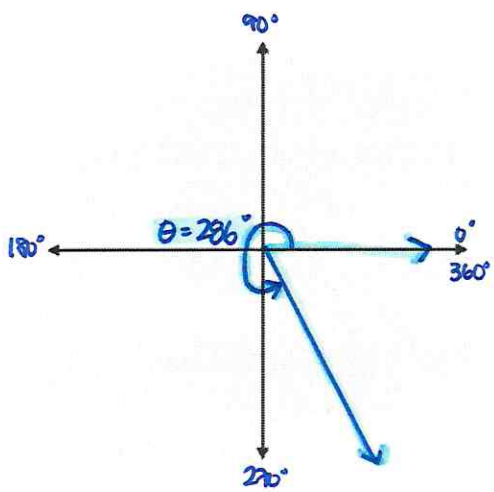
c) – 65° IV

d) – 150° III

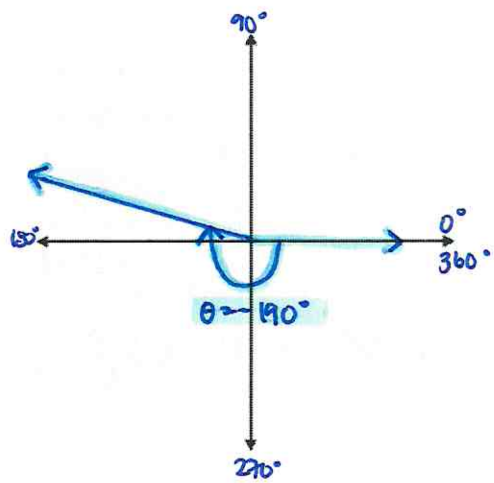
Example #2

Sketch each angle in **standard position**.

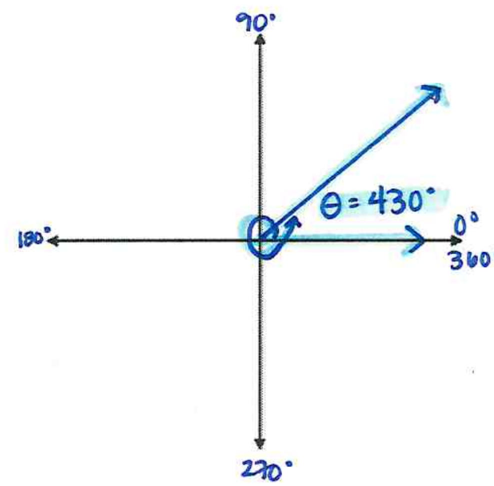
a) $286°$



b) $-190°$



c) $430°$



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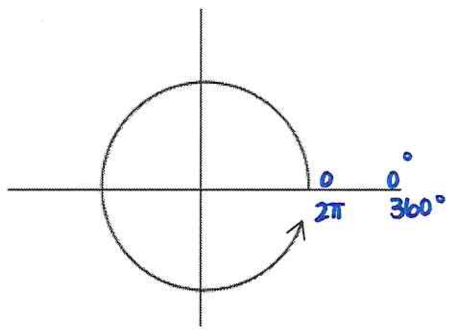
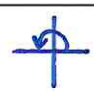
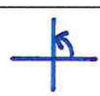
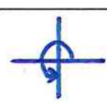
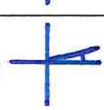
Radian Measure of an Angle

- The formula for the **circumference** of a circle is $C = 2\pi r$
- The **unit circle** has a radius = 1
- Therefore, the circumference of the unit circle is $C = 2\pi$

$$2\pi = 6.283185\ldots$$

This means that the distance traveled from the initial arm all around the circle and back again is 6.283185…



Revolutions	Degrees	Radian Measure	
1 revolution 	360°	2π radians	6.283185… radians
$\frac{1}{2}$ revolution 	180°	π radians	3.141592… radians
$\frac{1}{4}$ revolution 	90°	$\frac{\pi}{2}$ radians	1.570796… radians
$\frac{3}{4}$ revolution 	270°	$\frac{3\pi}{2}$ radians	4.712388… radians
$\frac{1}{360}$ revolution 	1°	$\frac{\pi}{180}$ radians	0.017453… radians

Note that 1 radian = $\left(\frac{180°}{\pi}\right) \approx 57.3°$

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Converting Degrees to Radians: Multiply by $\left(\frac{\pi}{180^\circ}\right)$

Example #3

Express the following angle measures in **radians**.

a) $30°$

$$30^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{30^\circ\pi}{180^\circ}$$

$$= \frac{\pi}{6}$$

b) $225°$

$$225^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{225^\circ\pi}{180^\circ}$$

$$= \frac{5\pi}{4}$$

c) $720°$

$$720^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{720^\circ\pi}{180^\circ}$$

$$= 4\pi$$

Converting Radians to Degrees: Multiply by $\left(\frac{180^\circ}{\pi}\right)$

Example #4

Express the following angle measures in **degrees**

a) $\frac{2\pi}{3}$

$$\frac{2\pi}{3}\left(\frac{180^\circ}{\pi}\right) = \frac{360^\circ\pi}{3\pi}$$

$$= 120^\circ$$

b) 1.6

$$1.6\left(\frac{180^\circ}{\pi}\right) = \frac{288^\circ}{\pi}$$

$$= 91.7^\circ$$

c) $\frac{5\pi}{6}$

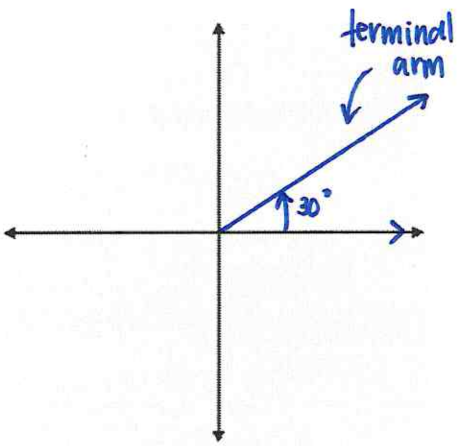
$$\frac{5\pi}{6}\left(\frac{180^\circ}{\pi}\right) = \frac{900^\circ\pi}{6\pi}$$

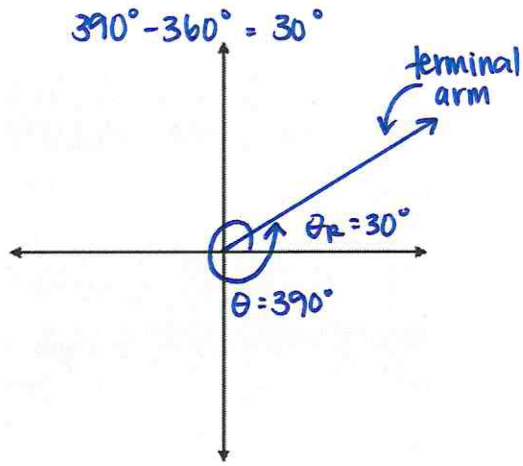
$$= 150^\circ$$

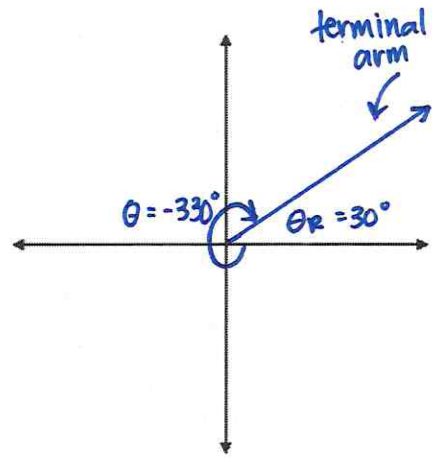
Coterminal Angles

Coterminal Angles are angles in standard position that share the same terminal arm.

Example: Sketch $\theta = 30°$ as an angle in standard position, and show that $\theta = 390°$ and $\theta = -330°$ are **coterminal angles**.



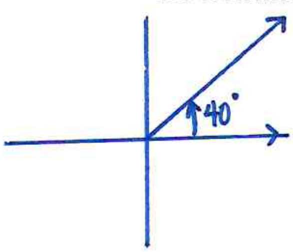




The coterminal angle can be found by **adding** or **subtracting** revolutions; either $\pm 360°$ when given degree measure or $\pm 2\pi$ when given radian measure. There are an infinite number of coterminal angles.

Example #5

Determine 3 **coterminal angles** for $40°$.



$40° + 360° = 400°$

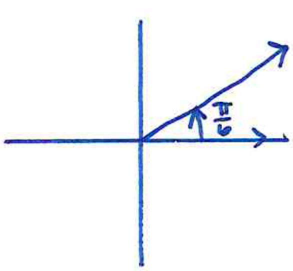
$40° - 360° = -320°$

$40° + 360° + 360° = 760°$

$400°$, $-320°$ and $760°$ are all coterminal with $40°$

Example #6

Determine 3 **coterminal angles** for $\frac{\pi}{6}$



$\frac{\pi}{6} + 2\pi = \frac{13\pi}{6}$

$\frac{\pi}{6} - 2\pi = -\frac{11\pi}{6}$

$\frac{\pi}{6} + 2\pi + 2\pi = \frac{25\pi}{6}$

$\frac{13\pi}{6}$, $-\frac{11\pi}{6}$, $\frac{25\pi}{6}$ are all coterminal with $\frac{\pi}{6}$

General Form of Coterminal Angles

Degrees: $\theta \pm 360°n$, $n \in N$

Radians: $\theta \pm 2\pi n$, $n \in N$

Represents all the possible coterminal angles.

↳ n is a Natural # (1, 2, 3, ...)

Example #7

Express the angles **coterminal** with 50° in general form.

$50° \pm 360°(n)$, $n \in N$

Example #8

Express a general form for all **coterminal angles** of $\frac{5\pi}{3}$

$\therefore \frac{5\pi}{3} \pm 2\pi(n)$, $n \in N$

Example #9

Determine a **coterminal angle** to 740° over the interval $\underbrace{-360° < \theta < 0°}_{\text{Restriction!}}$ only angles between are acceptable.

$740° - 360°(1) = 380°$ ✗

$740° - 360°(2) = 20°$ ✗

$740° - 360°(3) = -340°$ ✓

$740° - 360°(4) = -700°$ ✗

$\therefore$ $-340°$ is the only coterminal angle over this interval.

Example #10

Determine all **coterminal angles** to $\frac{5\pi}{3}$ over the interval $[-4\pi, 2\pi]$ → Think with denominator of 3

$\left[-\frac{12\pi}{3}, \frac{6\pi}{3}\right]$

$\frac{5\pi}{3} + 2\pi(1) = \frac{11\pi}{3}$ ✗

$\frac{5\pi}{3} - 2\pi(1) = -\frac{\pi}{3}$ ✓

$\frac{5\pi}{3} - 2\pi(2) = -\frac{7\pi}{3}$ ✓

$\frac{5\pi}{3} - 2\pi(3) = -\frac{13\pi}{3}$ ✗

$\therefore -\frac{\pi}{3}$ & $-\frac{7\pi}{3}$ are the coterminal angles over this interval

Arc Length

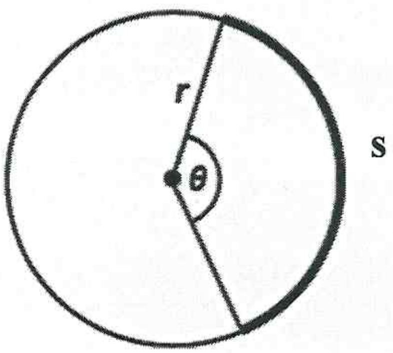
The central angle is the relationship between the length of the arc and the radius of the circle.

The equation that represents this relationship is:

$S = \theta r$ where: $S =$ arc length, $r =$ radius — must be the same units

$\theta =$ central angle

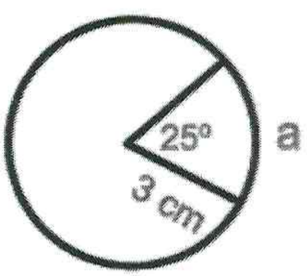
↳ Must use radians



Note: If there is no unit attached to the angle measure (ex: $\theta = 2.5$) it is assumed to be in **radians**.

Example #11

Determine the **arc length**.



$\theta = 25°$

↳ Must be radians

$25°\left(\frac{\pi}{180°}\right)$

$= \frac{5\pi}{36}$

$S = \theta r$

$S = \left(\frac{5\pi}{36}\right)(3)$

$S = 1.309$ cm

Example #12

A bicycle tire has a radius of 0.5 m and travels a distance of 1.5 m. Determine the rotated angle, in degrees.

$S = \theta r$

$\frac{1.5}{0.5} = \frac{(\theta)(0.5)}{0.5}$

$3 = \theta$

↳ radians

$3\left(\frac{180°}{\pi}\right) = 171.887°$

Example #13

Given the following information determine the missing value.

a) $r = 8.7$ cm, $\theta = 75°$ determine arc length

$S = \theta r$

$75°\left(\frac{\pi}{180°}\right) = \frac{5\pi}{12}$

$S = \left(\frac{5\pi}{12}\right)(8.7)$

$S = 11.388$ cm

b) $\theta = 1.8$, $S = 4.7$ mm, determine the radius

$S = \theta r$

$\frac{4.7}{1.8} = \frac{(1.8)r}{1.8}$

2.611 mm $= r$

c) $r = 5$ m, $S = 13$ m, determine the measure of the central angle

$S = \theta r$

$\frac{13}{5} = \frac{(\theta)(5)}{5}$

$2.6 = \theta$

↳ radians, so no units

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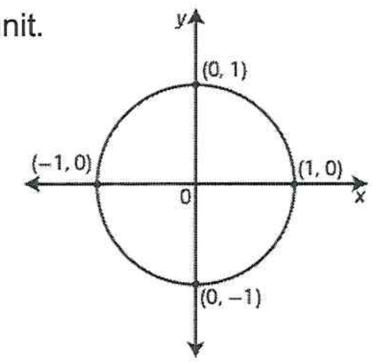
4.2 – The Unit Circle

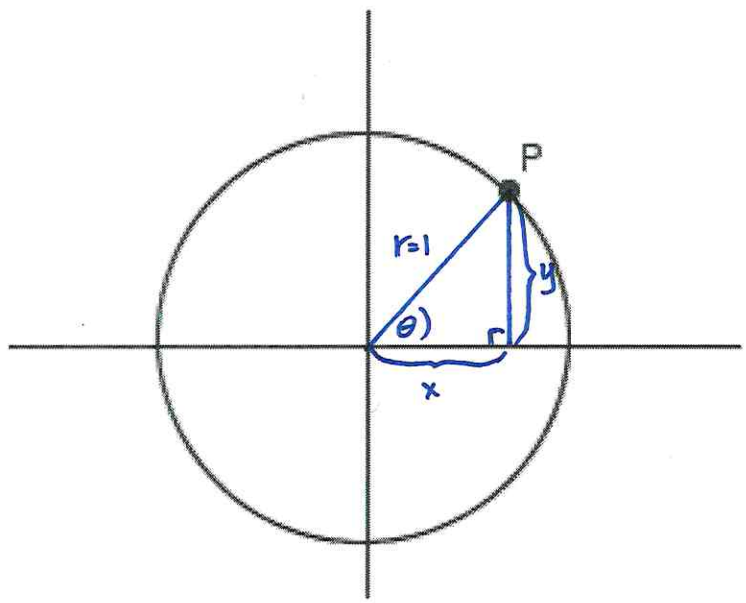
The unit circle is centered at the origin and has a radius of 1 unit.

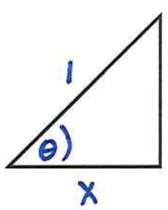


We use the notation $P(\theta)$ to indicate a point on the circle.

θ = arc length

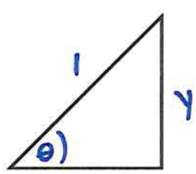
$P(\theta)$ = defined by a point (x, y)





$\cos\theta = \frac{x}{1}$

$\therefore \cos\theta = x$



$\sin\theta = \frac{y}{1}$

$\therefore \sin\theta = y$

Since the radius is 1, then the equation of the unit circle is $x^2 + y^2 = 1$

Important ideas:

$P(\theta) = (x, y) \rightarrow P(\theta) = (\cos\theta, \sin\theta)$

$x^2 + y^2 = 1 \rightarrow \cos^2\theta + \sin^2\theta = 1$

Example #1

Determine whether or not the point $\left(\frac{2}{5},\frac{3}{5}\right)$ is on the unit circle. Justify your reasoning.

LHS	RHS
x^2+y^2	1
$=\left(\frac{2}{5}\right)^2+\left(\frac{3}{5}\right)^2$	
$=\frac{4}{25}+\frac{9}{25}$	
$=\frac{13}{25}$	
	LHS $\neq$ RHS

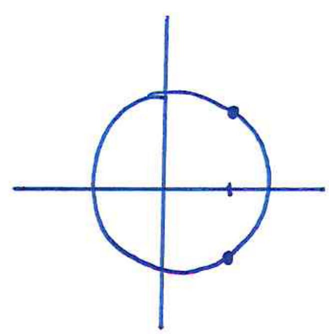
$\therefore$ The point $\left(\frac{2}{5},\frac{3}{5}\right)$ is not on the unit circle

Example #2

A point $\left(\frac{2}{3},y\right)$ is on the unit circle. Determine the value of y.

$$x^2+y^2=1$$
$$\left(\frac{2}{3}\right)^2+y^2=1$$
$$\frac{4}{9}+y^2=1$$
$$y^2=1-\frac{4}{9}$$
$$y^2=\frac{9}{9}-\frac{4}{9}$$
$$y^2=\frac{5}{9}$$
$$y=\pm\sqrt{\frac{5}{9}}$$
$$y=\pm\frac{\sqrt{5}}{3}$$

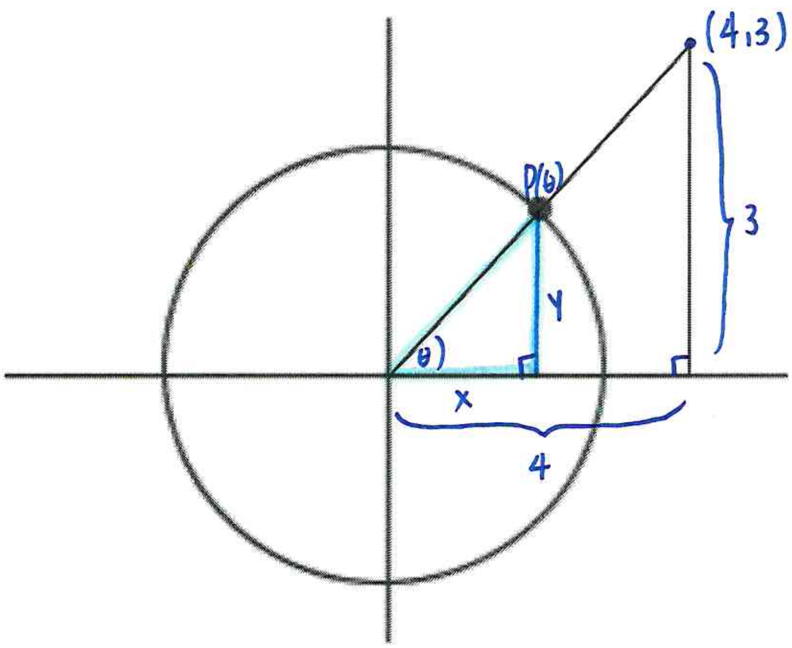
$\therefore$ $y=+\frac{\sqrt{5}}{3}$ & $y=-\frac{\sqrt{5}}{3}$

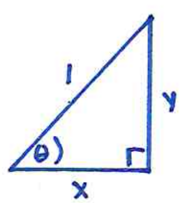


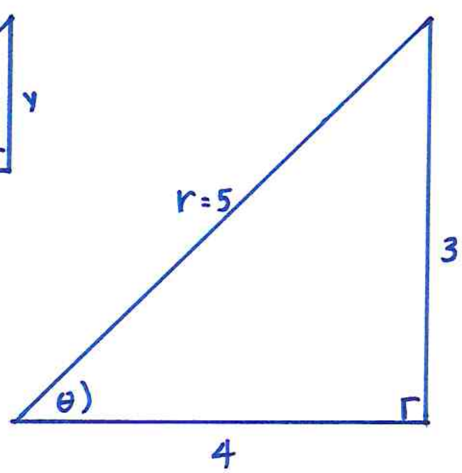
Example #3

The point $P(\theta)$ lies on the intersection of the unit circle and a line joining the origin to the point $(4, 3)$.

Determine the coordinates of $P(\theta)$.







$(3)^2 + (4)^2 = r^2$

$9 + 16 = r^2$

$25 = r^2$

$\sqrt{25} = r$

$5 = r$

$P(\theta) = (\cos\theta, \sin\theta)$

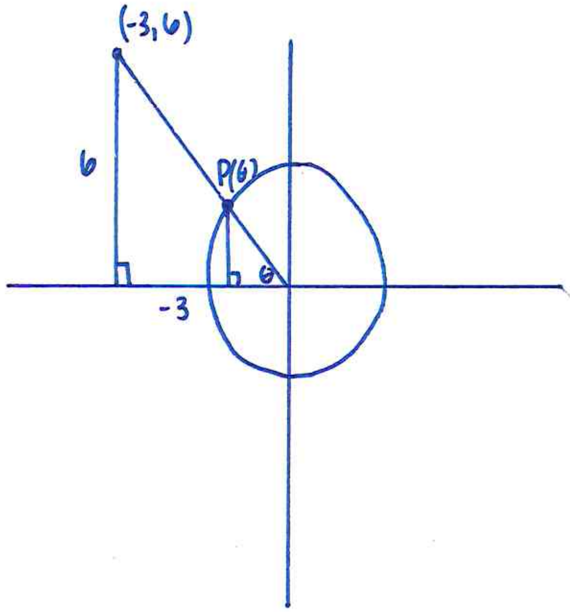
$\cos\theta = \frac{4}{5}$ $\quad \sin\theta = \frac{3}{5}$

$\therefore P(\theta) = \left(\frac{4}{5}, \frac{3}{5}\right)$

Example #4

The point $P(\theta)$ lies on the intersection of the unit circle and a line joining the origin to the point (–3, 6).

Determine the coordinates of $P(\theta)$.



$$x^2 + y^2 = r^2$$
$$(-3)^2 + (6)^2 = r^2$$
$$9 + 36 = r^2$$
$$45 = r^2$$
$$\sqrt{45} = r$$

$\cos\theta = -\frac{3}{\sqrt{45}}$ $\sin\theta = \frac{6}{\sqrt{45}}$

$$P(\theta) = (\cos\theta, \sin\theta)$$

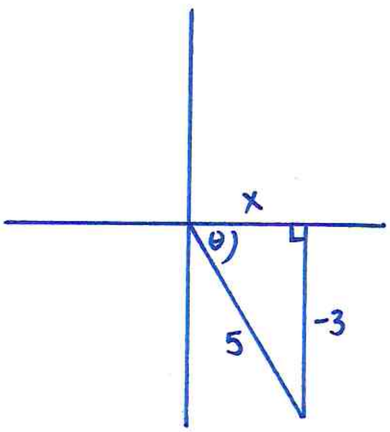
$$P(\theta) = \left(-\frac{3}{\sqrt{45}}, \frac{6}{\sqrt{45}}\right)$$

Q4

Example #5

Determine the values of $\cos\theta$ and $\tan\theta$ over the interval $\frac{3\pi}{2} \leq \theta \leq 2\pi$ when $\sin\theta = -\frac{3}{5}$.

$\cos\theta = \frac{\text{adj}}{\text{hyp}}$ $\tan\theta = \frac{\text{opp}}{\text{adj}}$ $\sin\theta = \frac{\text{opp}}{\text{hyp}}$



$$x^2 + y^2 = r^2$$

$$x^2 + (-3)^2 = (5)^2$$

$$x^2 + 9 = 25$$

$$x^2 = 16$$

$$x = \pm\sqrt{16}$$

$$x = \pm 4$$

↳ Q4, x is positive

$$x = 4$$

$$\cos\theta = \frac{4}{5}$$

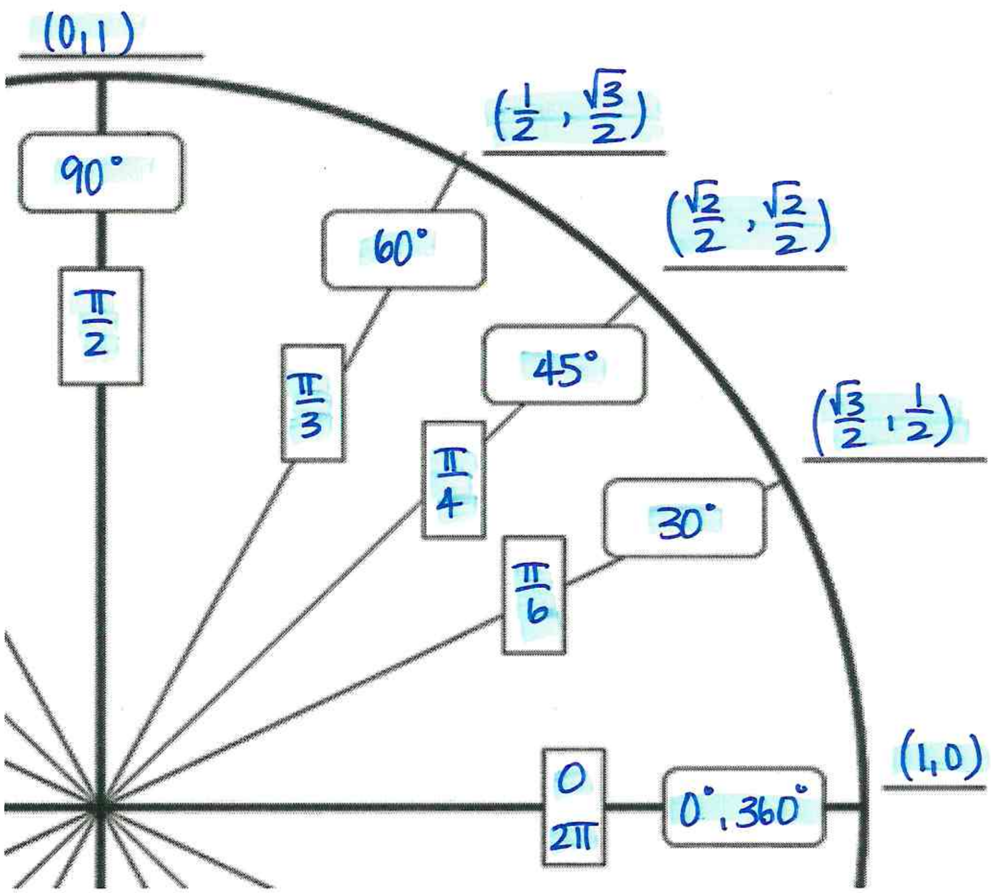
&

$$\tan\theta = -\frac{3}{5}$$

Chapter 4: TRIGONOMETRY AND THE UNIT CIRCLE

The Unit Circle

QUADRANT 1



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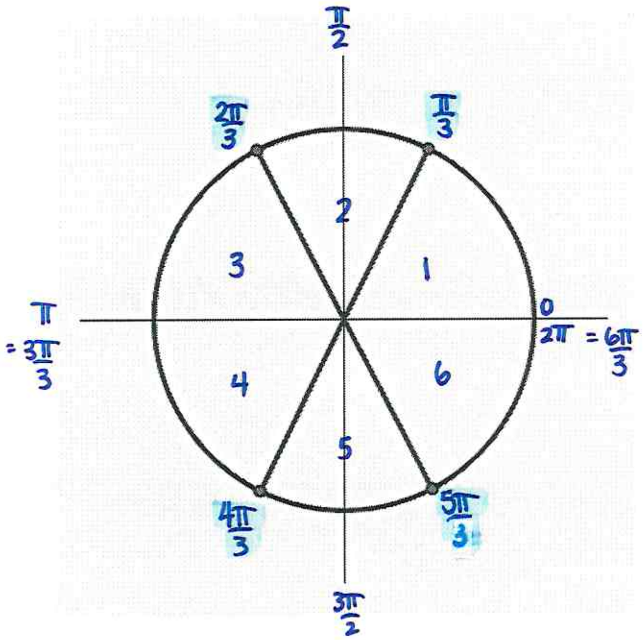
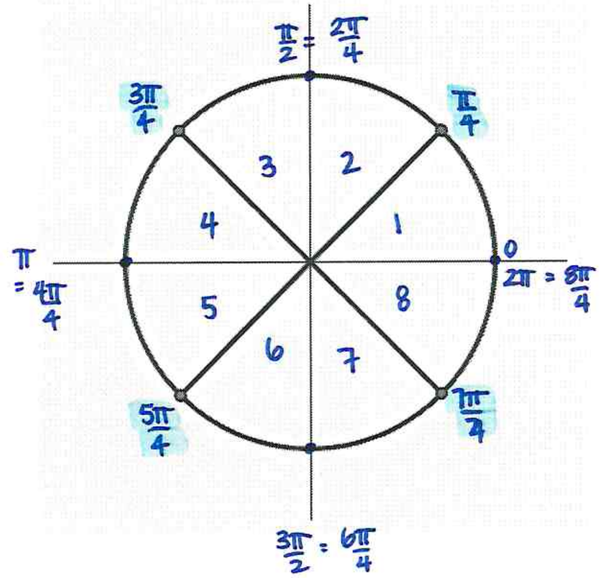
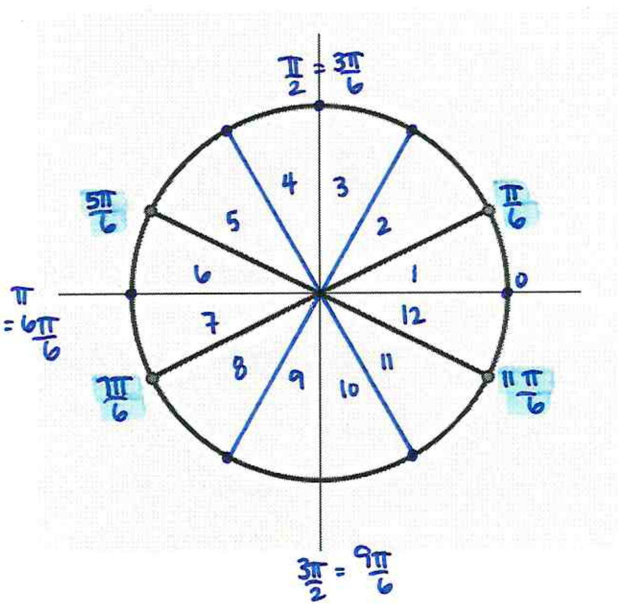
$\frac{\pi}{3}$ Family



$\frac{\pi}{4}$ Family



$\frac{\pi}{6}$ Family



↳ How many 60° fit in 360°?

↳ How many 45° fit in 360°?

↳ How many 30° fit in 360°?

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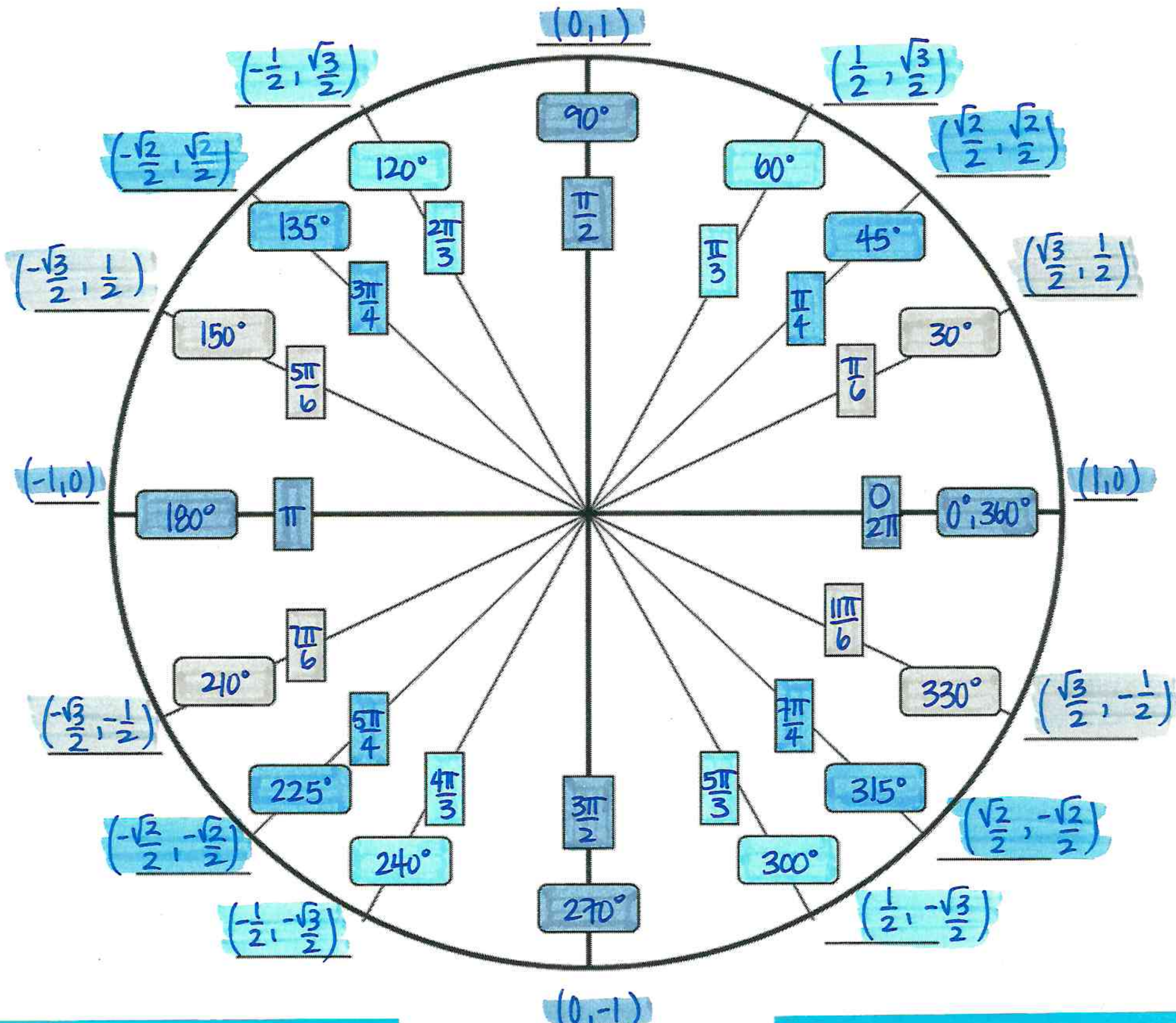
THE UNIT CIRCLE



(−, +)	(+, +)
(−, −)	(+, −)

↳ Note:

$$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

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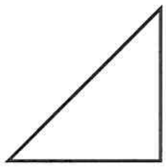
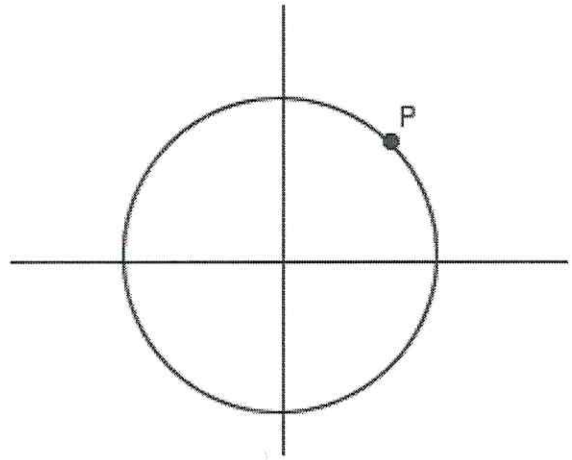
Chapter 4: TRIGONOMETRY AND THE UNIT CIRCLE
4.3 – Trigonometric Ratios

Recall:

θ = arc length

$P(\theta)$ = defined by a point (x, y)





If we use the trigonometric rations SOH CAH TOA, then

$\sin\theta = \frac{y}{1}$ → $\sin\theta = y$

$\cos\theta = \frac{x}{1}$ → $\cos\theta = x$

$\tan\theta = \frac{y}{x}$ → $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Thus, any point on the unit circle can be described as: $P(\theta) = (\cos\theta, \sin\theta)$

PRIMARY FUNCTIONS

$\sin\theta = y$

$\cos\theta = x$

$\tan\theta = \frac{y}{x}$

RECIPROCAL FUNCTIONS $\left(\frac{1}{f(x)}\right)$

cosecant $\csc\theta = \frac{1}{\sin\theta} = \frac{1}{y}$

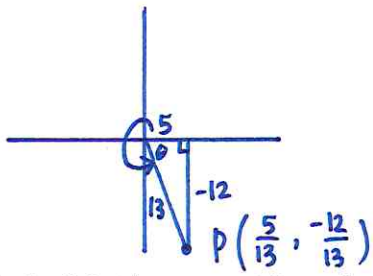
secant $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{x}$

cotangent $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$

Example #1

The point $\left(\frac{5}{13}, -\frac{12}{13}\right)$ lies on the terminal arm of an angle θ in standard position.

a) Draw a diagram to represent this situation.



$\left(\frac{5}{13}, \frac{-12}{13}\right)$

$\downarrow$ $\downarrow$

$\cos\theta$ $\sin\theta$

b) Find all 6 trigonometric ratios for θ.

$\cos\theta = \frac{5}{13}$ $\sec\theta = \frac{13}{5}$

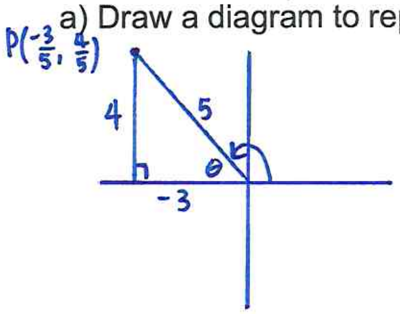
$\sin\theta = \frac{-12}{13}$ $\csc\theta = -\frac{13}{12}$

$\tan\theta = \frac{-12}{5}$ $\cot\theta = -\frac{5}{12}$

Example #2

The point $\left(-\frac{3}{5}, \frac{4}{5}\right)$ lies on the terminal arm of an angle θ in standard position.

a) Draw a diagram to represent this situation.



b) Find all 6 trigonometric ratios for θ.

$\cos\theta = -\frac{3}{5}$ $\sec\theta = -\frac{5}{3}$

$\sin\theta = \frac{4}{5}$ $\csc\theta = \frac{5}{4}$

$\tan\theta = -\frac{4}{3}$ $\cot\theta = -\frac{3}{4}$

Determining Exact Values

↳ Determine quadrant

↳ Find ratio

S	A
T	C

Example #3

Determine the **exact** value of the following trigonometric ratios.

a) $\cos\frac{\pi}{3} = \frac{1}{2}$

b) $\sec\frac{\pi}{3} = \frac{1}{\cos\frac{\pi}{3}}$

$= \frac{1}{\frac{1}{2}}$

$= 2$

c) $\sin\left(-\frac{5\pi}{6}\right) = -\frac{1}{2}$

d) $\cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2}$

e) $\cot(270°) = \frac{1}{\tan(270°)}$

$= \frac{0}{-1}$

$= 0$

f) $\csc\left(\frac{2\pi}{3}\right) = \frac{1}{\frac{\sqrt{3}}{2}}$

$= \frac{2}{\sqrt{3}}$

g) $\tan\frac{11\pi}{4} = \frac{\sin\frac{11\pi}{4}}{\cos\frac{11\pi}{4}}$

$= \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$

$= 1$

h) $\sec 5\pi = \frac{1}{-1}$

$= -1$

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Example #4

Determine the **exact** value of the following expressions.

a) $\cos(120°) - \tan(-135°)$

$= -\frac{1}{2} - 1$

$= -\frac{1}{2} - \frac{2}{2}$

$= -\frac{3}{2}$

b) $\cot\left(-\frac{3\pi}{4}\right) + \csc\left(\frac{\pi}{2}\right)$

$= 1 + 1$

$= 2$

c) $\sin^2\left(\frac{7\pi}{6}\right) + \cos^2\left(\frac{7\pi}{6}\right)$

$= \left(-\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2$

$= \frac{1}{4} + \frac{3}{4}$

$= \frac{4}{4}$

$= 1$

d) $\tan^2\left(\frac{-\pi}{3}\right)\sec\left(\frac{4\pi}{3}\right)$

$= (-\sqrt{3})^2(-2)$

$= (3)(-2)$

$= -6$

Chapter 4: TRIGONOMETRY AND THE UNIT CIRCLE

4.4 – Trigonometric Equations

We can solve trigonometric equations just like we have been solving equations from previous units.

Note: If interval/domain is given in **radians**, your answer must be in **radians**.
If interval/domain is given in **degrees**, your answer must be in **degrees**.

Example #1

Solve the following trigonometric equation, over the given domain.

$\sin\theta = \frac{1}{2}, \quad 0 \le \theta \le 2\pi$

$\theta_R = \frac{\pi}{6}$ $\qquad$ $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Example #2

Solve the following trigonometric equations, over the given domain.

a) $2\cos\theta + 3 = 1, \quad 0° \le \theta \le 540°$

$\frac{2\cos\theta}{2} = \frac{-2}{2}$

$\cos\theta = -1$

$\theta = 180°, 540°$

b) $4\sec x + 8 = 0, \quad 0 \le x \le 2\pi$

$\frac{4\sec x}{4} = \frac{-8}{4}$

$\sec x = -2$

$\cos x = -\frac{1}{2}$

$x_R = \frac{\pi}{3}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

Example #3

Solve the following trigonometric equations, over the given intervals.

a) $3\tan^2 x - 9 = 0, \quad 0° \le x \le 360°$

$$\frac{3\tan^2 x}{3} = \frac{9}{3}$$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$\tan x = \sqrt{3}$	$\tan x = -\sqrt{3}$
$x = 60°, 240°$	$x = 120°, 300°$

$\therefore \theta = 60°, 120°, 240°, 300°$

b) $2\cos^2\theta + \cos\theta = 1, \quad 0 \le \theta \le 2\pi$

$2\cos^2\theta + \cos\theta - 1 = 0$

↳ Factor

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$2\cos\theta - 1 = 0$	$\cos\theta + 1 = 0$
$2\cos\theta = 1$	$\cos\theta = -1$
$\cos\theta = \frac{1}{2}$	$\theta = \pi$
$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$	

$\therefore \theta = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

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Example #4

Solve the following trigonometric equations, over the given intervals.

a) $2\sin^2 x - 1 = \sin x, \quad 0 \leq x \leq 270°$

$2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$2\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$

$x_R = 30°$

$x = 210°, \cancel{330°}$

$\sin x - 1 = 0$

$\sin x = 1$

$x = 90°$

$\therefore x = 90°, 210°$

---

b) $\sin^2 x + \sin x - 12 = 0, \quad 0 \leq x \leq 2\pi$

$(\sin x + 4)(\sin x - 3) = 0$

$\sin x + 4 = 0$

$\cancel{\sin x = -4}$

no solution

$\sin x - 3 = 0$

$\cancel{\sin x = 3}$

no solution

$\therefore$ There is no solution

* Note: for $\cos\theta$ & $\sin\theta$ only, they can only be between ± 1

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Example #5

Solve the following trigonometric equations, over the given intervals.

a) $\csc^2 x + \csc x - 12 = 0,\ \ 0 \le x \le 2\pi$

$(\csc x + 4)(\csc x - 3) = 0$

$\csc x + 4 = 0$	$\csc x - 3 = 0$
$\csc x = -4$	$\csc x = 3$
$\sin x = -\frac{1}{4}$	$\sin x = \frac{1}{3}$
$x_R = \sin^{-1}(1/4)$	$x_R = \sin^{-1}(1/3)$
$x_R = 0.253$	$x_R = 0.340$

Q3: $\theta = \pi + \theta_R$, $\theta = 3.395$

Q4: $\theta = 2\pi - \theta_R$, $\theta = 6.030$

Q1: $\theta = \theta_R$, $\theta = 0.340$

Q2: $\theta = \pi - \theta_R$, $\theta = 2.802$

$\therefore \theta = 3.395, 6.030, 0.340, 2.802$

---

b) $\tan^2\theta - 5\tan\theta + 4 = 0\,,\ -2\pi \le x \le 2\pi$ → think $[-2\pi, 0]$ and $[0, 2\pi]$

$(\tan\theta - 4)(\tan\theta - 1) = 0$

$\tan\theta - 4 = 0$	$\tan\theta - 1 = 0$
$\tan\theta = 4$	$\tan\theta = 1$
$\theta_R = \tan^{-1}(4)$	$\theta_R = \frac{\pi}{4}$
$\theta_R = 1.326$	$\theta = \frac{\pi}{4}, \frac{5\pi}{4}, -\frac{3\pi}{4}, -\frac{7\pi}{4}$

Q1+: $\theta = \theta_R$, $\theta = 1.326$

Q3+: $\theta = \pi + \theta_R$, $\theta = 4.468$

Q1−: $\theta = -2\pi + \theta_R$, $\theta = -4.957$

Q3−: $\theta = -\pi + \theta_R$, $\theta = -1.816$

$\therefore \theta = -\frac{3\pi}{4}, -\frac{7\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4},$

$1.326, 4.468, -4.957, -1.816$

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$\_\_\_ + \_\_\_ = -4$
$\_\_\_ \times \_\_\_ = -10$

Example #6

Solve the following trigonometric equations, over the given interval.

$$2\cos^2\theta - 4\cos\theta - 5 = 0, \quad 0 \leq \theta \leq 2\pi$$

↳ When equation does not factor, use quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\cos\theta = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-5)}}{2(2)}$$

$$\cos\theta = \frac{4 \pm \sqrt{16+40}}{4}$$

$$\cos\theta = \frac{4 \pm \sqrt{56}}{4}$$

$$\cos\theta = \frac{4+\sqrt{56}}{4}$$

$$\cos\theta = 2.870\ldots$$

no solution

$$\cos\theta = \frac{4-\sqrt{56}}{4}$$

$$\cos\theta = -0.871\ldots$$

$$\theta_R = \cos^{-1}(0.871)$$

$$\theta_R = 0.514$$

Q2

$\theta = \pi - \theta_R$

$\theta = 2.628$

Q3

$\theta = \pi + \theta_R$

$\theta = 3.656$

$\therefore \theta = 2.628, \; 3.656$

General Solution of Trigonometric Equations

If the domain is **real numbers**, there are an **infinite** number of rotations on the unit circle in both a positive and negative direction.

To determine a **general solution**, find the solutions in one positive rotation. Then use the concept of coterminal angles to write an expression that identifies all possible measures.

There are different was to request the **general solution** answers. They are:

- Domain is all real numbers
- $x \in R$ or $\theta \in R$
- General solution

Example #7

a) **Solve** $\cot\theta = \frac{1}{\sqrt{3}}$ over the interval $0 \leq \theta \leq 2\pi$

$\tan\theta = \sqrt{3}$

$\theta_R = \frac{\pi}{3}$

$\theta = \frac{\pi}{3}$

$\theta = \frac{4\pi}{3}$

b) **Solve** the above equation if $\theta \in R$

↳ Asking for the general solution (All possible solutions for θ)

$\theta = \frac{\pi}{3} + 2\pi k,\ k \in Z$

$\theta = \frac{4\pi}{3} + 2\pi k,\ k \in Z$

Z = The set of integers $\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Example #8

Solve each of the following trigonometric equations.

a) Solve $\tan\theta = -4$ if the domain is all real numbers, in radians.

$\tan\theta = -4$

$\theta_R = \tan^{-1}(4)$

$\theta_R = 1.326$

Q2	Q4
$\theta = \pi - \theta_R$	$\theta = 2\pi - \theta_R$
$\theta = 1.816$	$\theta = 4.957$

$\therefore\ \theta = 1.816 + 2\pi k,\ k \in Z$

$\theta = 4.957 + 2\pi k,\ k \in Z$

b) Find the general solution of $\cos\beta = 0$, in degrees.

$\cos\beta = 0$

$\beta = 90^\circ + 360^\circ k,\ k \in Z$

$\beta = 270^\circ + 360^\circ k,\ k \in Z$

Example #9

Solve the following trigonometric equation, where $\theta \in R$. (In radians)

$$2\tan^2\theta - \tan\theta - 1 = 0$$

$$(2\tan\theta + 1)(\tan\theta - 1) = 0$$

$2\tan\theta + 1 = 0$

$\tan\theta = -1/2$

$\theta_R = \tan^{-1}(1/2)$

$\theta_R = 0.464$

Q2

$\theta = \pi - \theta_R$

$\theta = 2.678$

Q4

$\theta = 2\pi - \theta_R$

$\theta = 5.819$

$\tan\theta - 1 = 0$

$\tan\theta = 1$

$\theta = \frac{\pi}{4}$ $\quad \theta = \frac{5\pi}{4}$

The general solution is

$\theta = 2.678 + 2\pi k,\ k \in Z$

$\theta = 5.819 + 2\pi k,\ k \in Z$

$\theta = \frac{\pi}{4} + 2\pi k,\ k \in Z$

$\theta = \frac{5\pi}{4} + 2\pi k,\ k \in Z$