Chapter 1: TRANSFORMATIONS AND FUNCTIONS

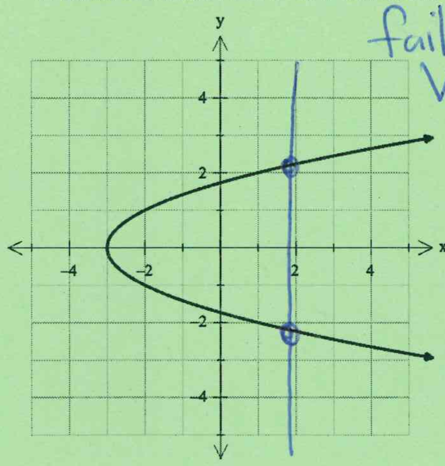
1.4 – Inverse of a Relation

Relation: In mathematics, it is the connection between the x and y variables. One single x-value may have multiple y-values.

Function: A particular relation where each element in the domain (x) has one and only one element in the range (y). (Remember, a function passes the Vertical Line Test)

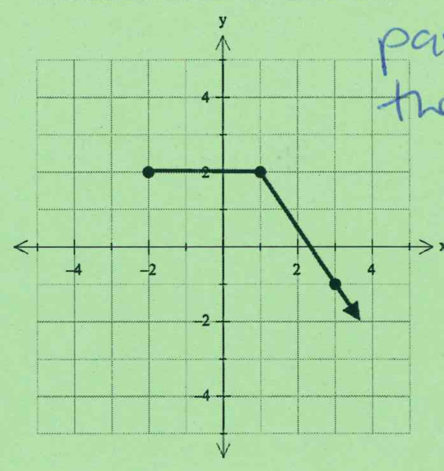
Example

- Relation but NOT a Function



fails the VLT

- Relation AND a Function



passes the VLT

Inverse of a relation

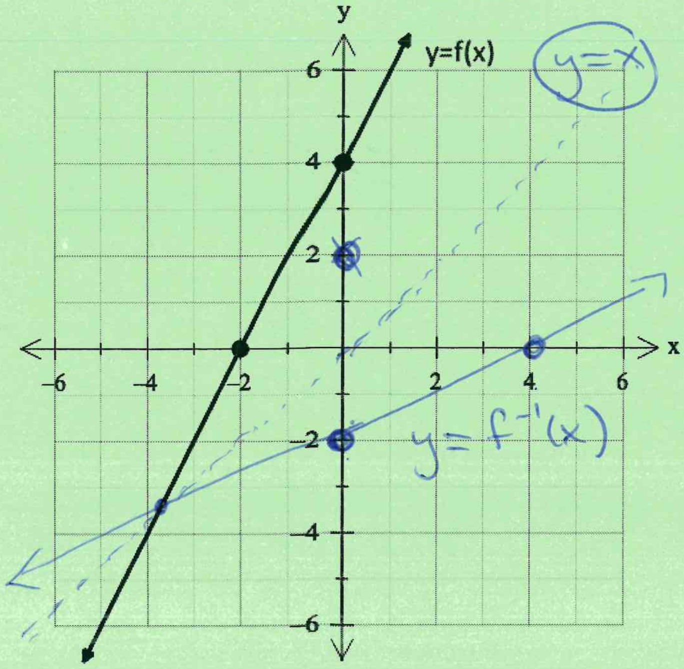
- The **inverse** of a relation is found when the x and y coordinates are switched.
- The mapping notation is $(x, y) \to (y, x)$
- The graph of a relation and its inverse are reflections over the line $y=x$
- If the inverse is a function we denote it as $f^{-1}(x)$
- To see if the inverse of a relation will result in a function, we can use the horizontal line test

Example #1

Given $y = f(x)$ sketch its **inverse** on the same graph.

Does the inverse represent a function?

State the **domain** and **range** for both $y = f(x)$ and the inverse.



* Switch the x & y coordinates of each point.

$(x, y) \to (y, x)$

$(-2, 0) \to (0, -2)$

$(0, 4) \to (4, 0)$

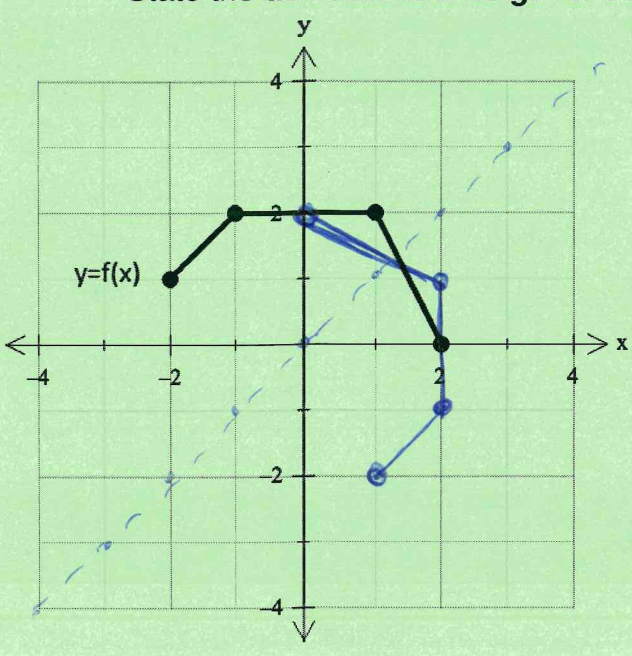
	$f(x)$	Inverse
Domain	$(-\infty, \infty)$ $x \in \mathbb{R}$	$(-\infty, \infty)$
Range	$(-\infty, \infty)$ $y \in \mathbb{R}$	$(-\infty, \infty)$

Example #2

Given $y = f(x)$, sketch its **inverse** on the same graph.

Does the inverse represent a function? No!

State the **domain** and **range** for both $y = f(x)$ and the inverse.



$(x, y) \to (y, x)$

$(-2, 1) \to (1, -2)$

$(-1, 2) \to (2, -1)$

$(1, 2) \to (2, 1)$

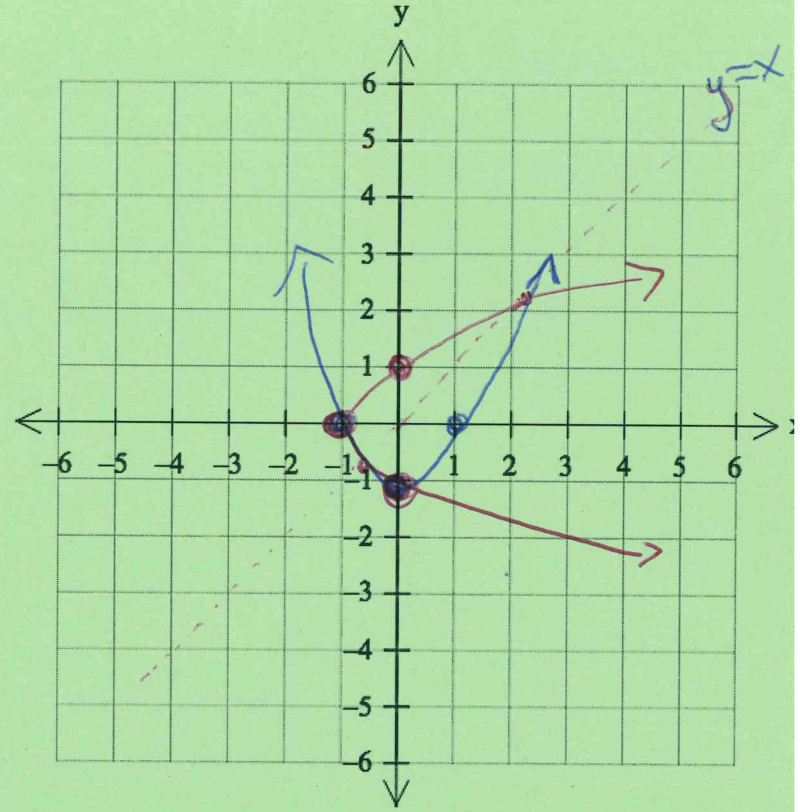
$(2, 0) \to (0, 2)$

	$f(x)$	Inverse
Domain	$[-2, 2]$	$[0, 2]$
Range	$[0, 2]$	$[-2, 2]$

note: the domain of f(x) becomes the range of the inverse.

Example #3

a) Sketch $f(x) = x^2 - 1$.



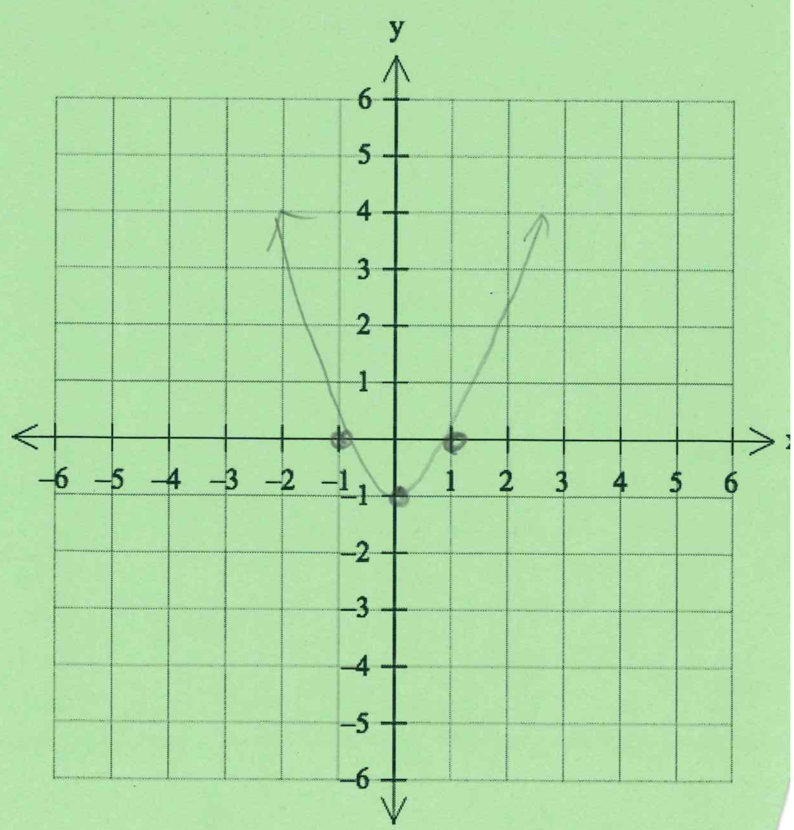
b) Without graphing, will the inverse be a function? Explain.

No! f(x) fails the horizontal line test.

c) Use the graph of $f(x)$ to sketch its **inverse** on the same graph above.

d) Explain how we can **restrict the domain** of the original graph so the inverse is a function.

$x \geq 0$ $[0, \infty)$



e) Is it possible that there is another answer to part d? Explain.

$x \leq 0$

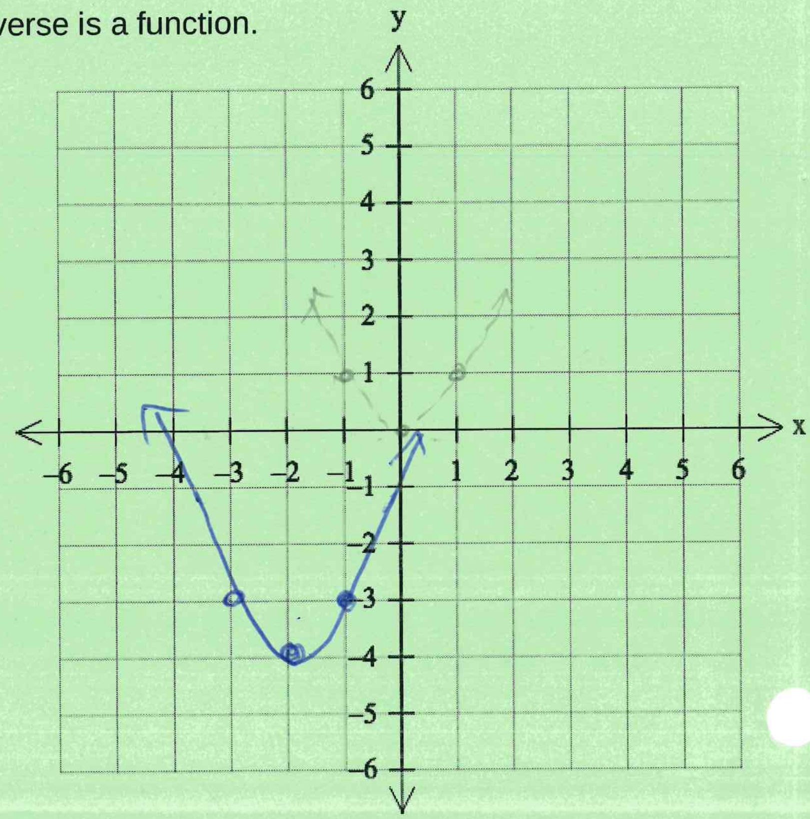
$(-\infty, 0]$

Example #4

Given the $f(x) = (x+2)^2 - 4$

Restrict the domain of $f(x)$ so that its inverse is a function.

$y = x^2$



$x \leq -2$ or $x \geq -2$

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We need to be able to write the **equation of the inverse** of a function.

1) Rewrite f(x) as y.
2) Switch x and y
3) Isolate y.
4) Replace y with $f^{-1}(x)$ * only if. the inverse is a function!

Example #5

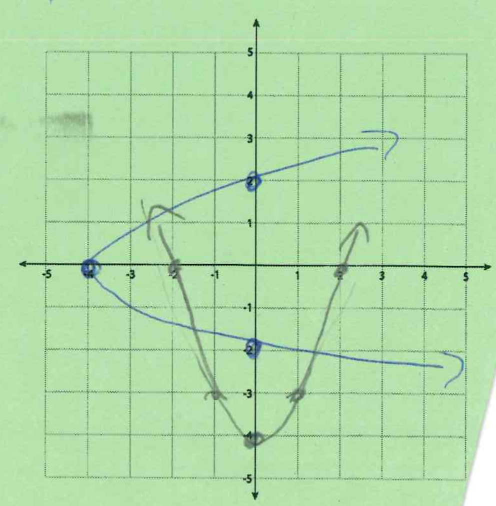
Determine the equation of the inverse of the following function: $f(x) = 2x + 8$

$$y = 2x + 8$$
$$x = 2y + 8$$
$$x - 8 = 2y$$
$$\frac{x-8}{2} = y$$
$$\frac{x}{2} - 4 = y$$
$$\frac{x}{2} - 4 = f^{-1}(x)$$

Example #6

Determine the equation of the inverse of the following function: $f(x) = x^2 - 4$

$$y = x^2 - 4$$
$$x = y^2 - 4$$
$$x + 4 = y^2$$
$$\pm\sqrt{x+4} = y$$



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We can prove functions are inverses of each other, other than just graphically. To prove that two functions are inverses of each other, we must show this algebraically.

If $f(x)$ is the inverse of $g(x)$, then $f(g(x)) = x$

Example #7

Show that $f(x) = 3x - 5$ and $g(x) = \frac{x+5}{3}$ are inverses of each other.

$f(g(x))$

$f\left(\frac{x+5}{3}\right) = 3\left(\frac{x+5}{3}\right) - 5$

$= x+5-5$

$= x$

Since $f(g(x)) = x$, $f(x)$ and $g(x)$ are inverses of each other.

Example #8

Show that $f(x) = x - 4$ and $g(x) = x + 4$ are inverses of each other.

$f(g(x)) = (x+4) - 4$

$= x$

Example #9

Show that $f(x) = \frac{x-2}{2}$ and $g(x) = 2x + 2$ are inverses of each other.

$f(g(x)) = \frac{(2x+2)-2}{2}$

$= \frac{\cancel{2}x}{\cancel{2}}$

$= x$