

## Chapter 1: TRANSFORMATIONS AND FUNCTIONS

## 1.3 – Combining Transformation

We need to be able to transform graphs with horizontal and vertical translations, reflections and horizontal and vertical stretches/compressions.

General Transformation Model:  $y = a f [b(x-h)] + k$

- The order in which we do the transformations **matters!** \_\_\_\_\_

- Always perform stretches and reflections 1st!
- Work left to right

## Example #1

Describe, in words and in order, the transformations represented by each equation.

a)  $y = 4f(-x) - 5$

Vertical stretch by factor of 4.

Reflect over the y-axis.

Shift vertical 5 units down.

c)  $y = 2f(-3x - 6) - 1$   $y = 2f[-3(x+2)] - 1$

Vertical stretch by a factor of 2

Reflection in the y-axis

Shift left 2 units.

Shift down 1 unit.

↳ ~~left~~ horizontal

horizontal stretch by a factor of 1/3.

b)  $y - 3 = -\frac{1}{2}f(x)$

$y = -\frac{1}{2}f(x) + 3$

Reflection over the x-axis

Vertical compression by a factor of 2.

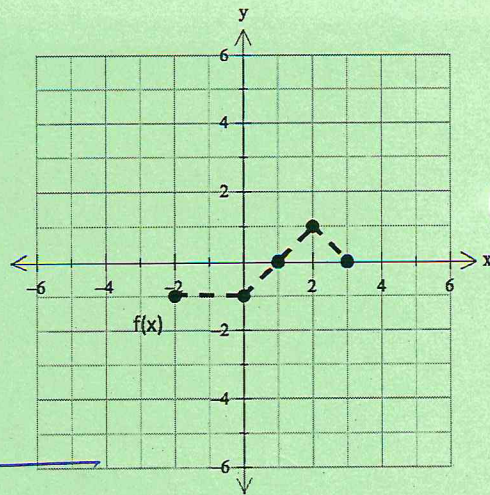
Vertical shift 3 units up.

↳ Vertical stretch by a factor of 1/2.



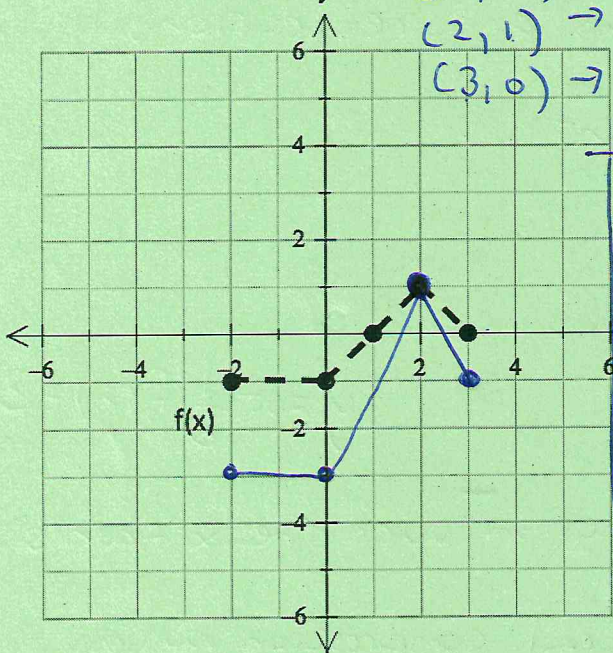
Example #2

Given the graph of  $y = f(x)$ , sketch the graphs given the following transformations.



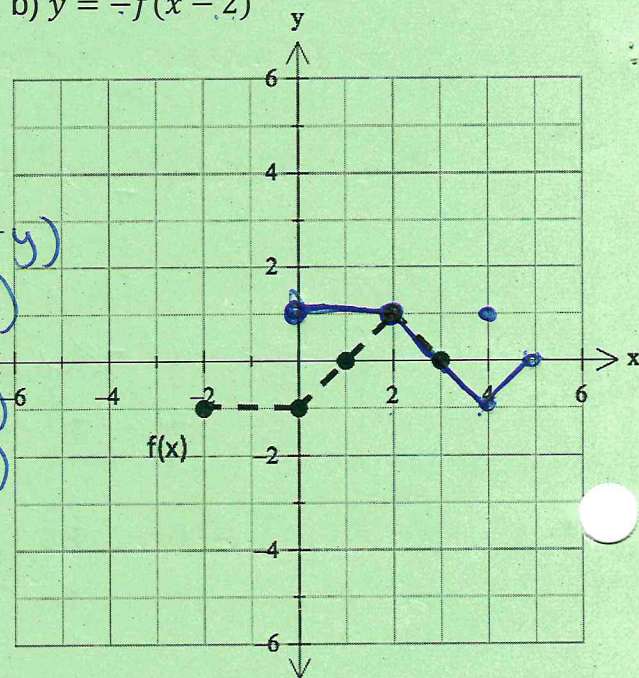
a)  $y = 2f(x) - 1$

$(x, y) \rightarrow (x, 2y - 1)$   
 $(-2, -1) \rightarrow (-2, -3)$   
 $(0, -1) \rightarrow (0, -3)$   
 $(2, 1) \rightarrow (2, 1)$   
 $(3, 0) \rightarrow (3, -1)$



b)  $y = -f(x - 2)$

$(x, y) \rightarrow (x + 2, -y)$   
 $(-2, -1) \rightarrow (0, 1)$   
 $(0, -1) \rightarrow (2, 1)$   
 $(2, 1) \rightarrow (4, -1)$   
 $(3, 0) \rightarrow (5, 0)$

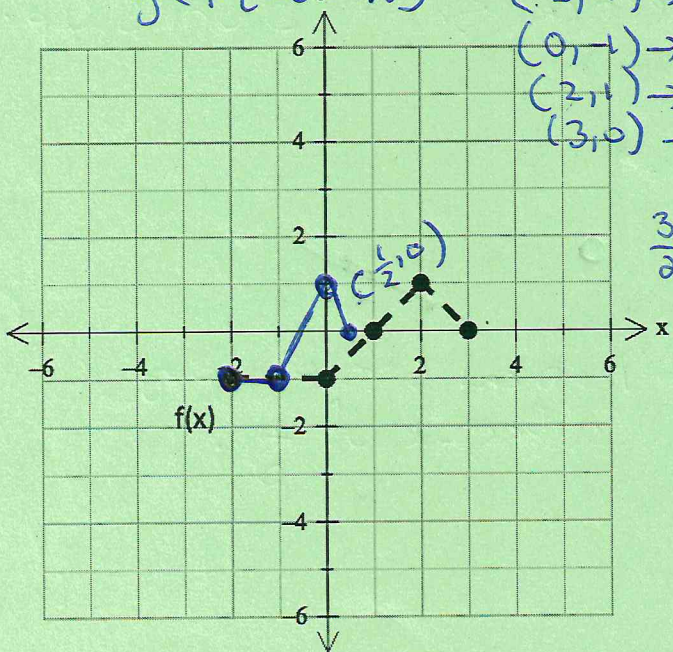


c)  $y = f(2x + 2)$

$y = f[2(x + 1)]$

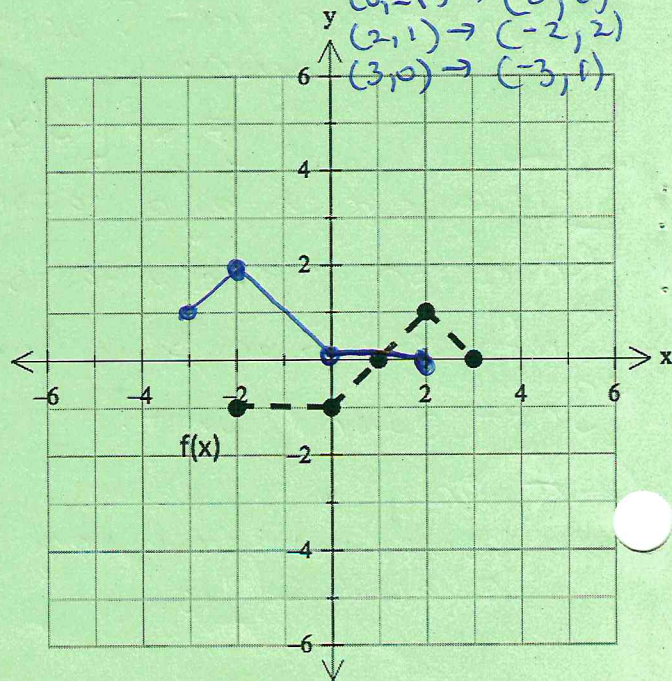
$(x, y) \rightarrow (\frac{1}{2}x - 1, y)$   
 $(-2, -1) \rightarrow (-2, -1)$   
 $(0, -1) \rightarrow (-1, -1)$   
 $(2, 1) \rightarrow (0, 1)$   
 $(3, 0) \rightarrow (\frac{1}{2}, 0)$

$\frac{3 - 2}{2} = \frac{1}{2}$



d)  $y = f(-x) + 1$

$(x, y) \rightarrow (-x, y + 1)$   
 $(-2, -1) \rightarrow (2, 0)$   
 $(0, -1) \rightarrow (0, 0)$   
 $(2, 1) \rightarrow (-2, 2)$   
 $(3, 0) \rightarrow (-3, 1)$





## Example #3

The graph of  $y = f(x)$  is shown below. Graph the transformation

$y = \frac{1}{2}f(-2x + 6) - 1$  on the same grid below.

$$y = \frac{1}{2} f[-a(x-3)] - 1$$

$$(x, y) \rightarrow \left(-\frac{1}{2}x + 3, \frac{1}{2}y - 1\right)$$

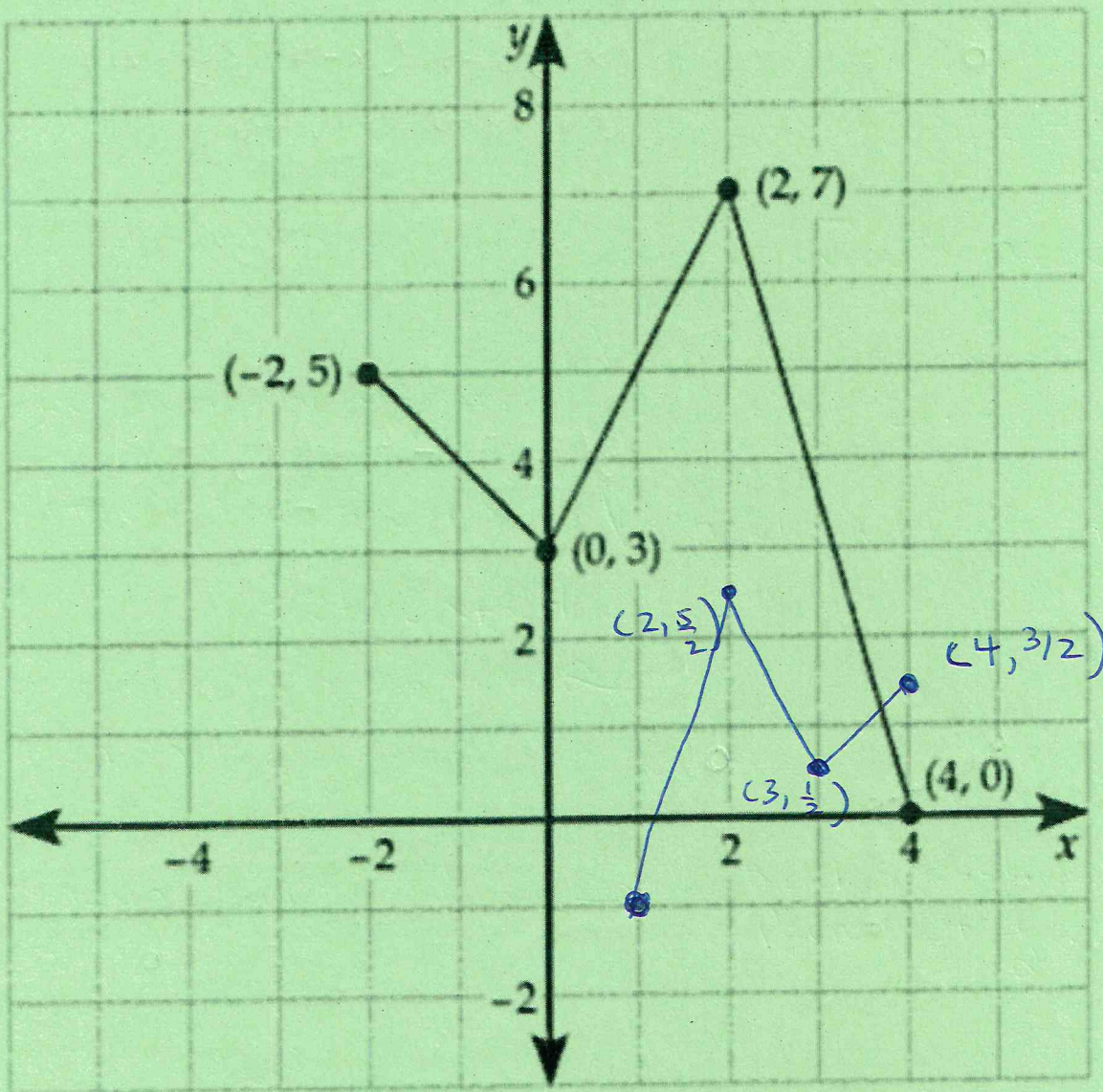
$$(-2, 5) \rightarrow \left(4, \frac{3}{2}\right)$$

$$(0, 3) \rightarrow \left(3, \frac{1}{2}\right)$$

$$(2, 7) \rightarrow \left(2, \frac{5}{2}\right)$$

$$(4, 0) \rightarrow (1, -1)$$

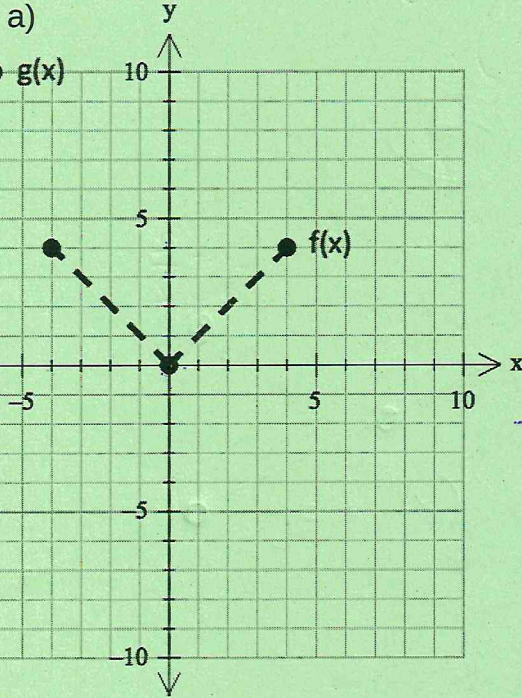
$$\frac{5}{2} - \frac{2}{2}$$





## Example #4

The graph of  $f(x)$  has been transformed into the graph  $g(x)$ . Write the **equation** of the transformed graph, in terms of  $f(x)$ .



$$g(x) = 2f[(x+7)] + 2$$

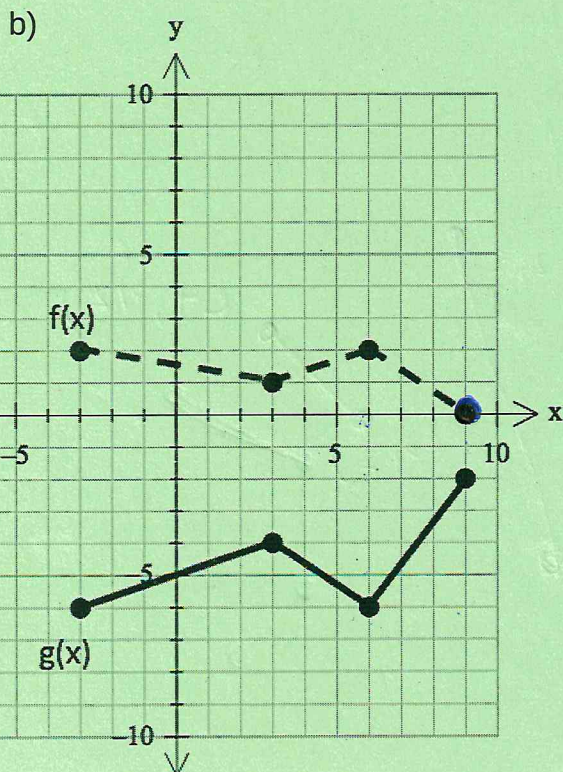
$$(x, y) \rightarrow (x, 2y)$$

$$(0, 0) \rightarrow (0, 0)$$

7 units left. 2 units up

$$(x, y) \rightarrow (x-7, 2y+2)$$

$$(-4, 4) \rightarrow (-11, 10)$$



$$g(x) = -4f(x) - 4$$