Chapter 1: TRANSFORMATIONS AND FUNCTIONS

1.2 – Reflections and Stretches

Reflection: A mirror image on, over, or about a line, called the line of reflection.

Reflections do not change the shape of the graph, but they may change the orientation.

Invariant Point: A point on a graph that remains **unchanged** after a transformation is applied to it. (i.e. A point that does not move)

Reflection of a graph in the x-axis

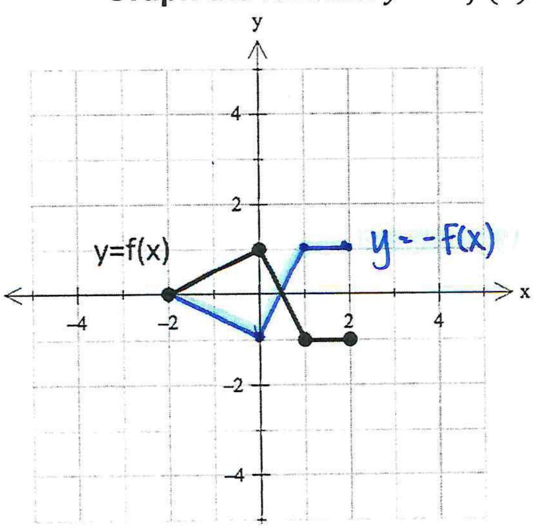
- Given $y = f(x)$, a **reflection of the graph in the x-axis** is described by $y = -f(x)$
- The image point on the graph of $y = -f(x)$ have the same x –coordinates, but different y – coordinates.
- The y – coordinates are multiplied by –1

$(x, y) \rightarrow (x, -y)$

Example #1

The graph of $y = f(x)$ is given.

Graph the function $y = -f(x)$ on the same grid. List any **invariant points**.



↳ Reflection over the x-axis

↳ Multiply all y-values by -1

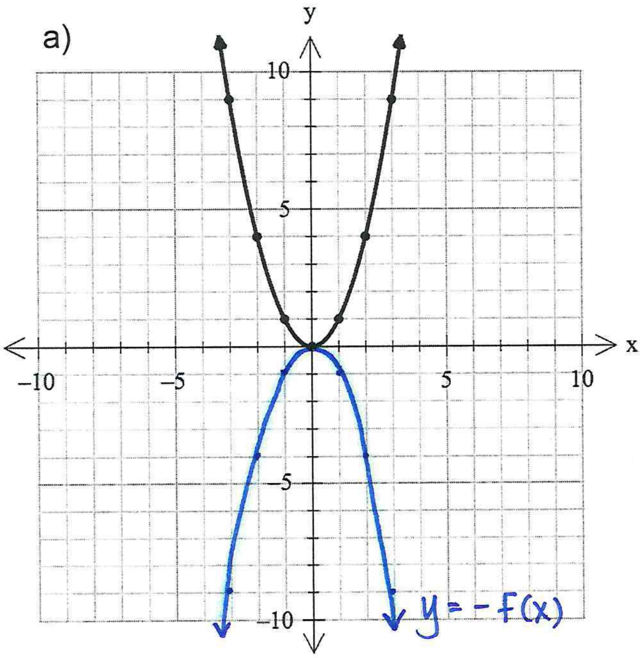
↳ invariant point : (-2,0)

Example #2

The graph of $y = f(x)$ is given.

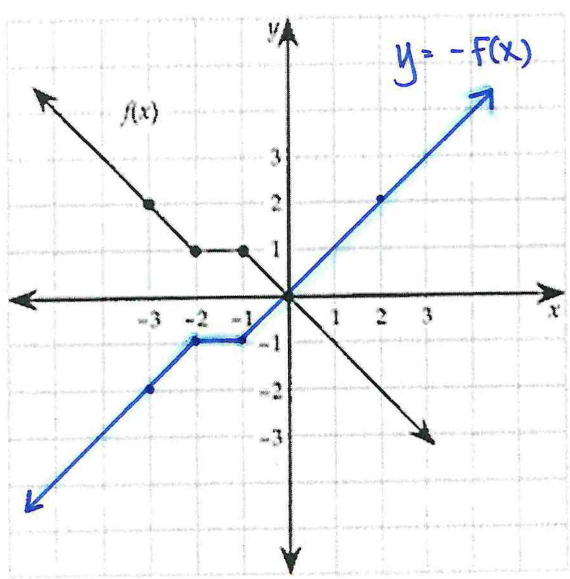
Graph the function $y = -f(x)$ on the same grid. List any **invariant points**.

a)



↳invariant point : (0,0)

b)



↳invariant point : (0,0)

Reflection of a graph in the y-axis

- Given $y = f(x)$, a **reflection of the graph in the y-axis** is described by $y = f(-x)$

- The image point on the graph of $y = f(-x)$ have the same y –coordinates, but different x – coordinates.

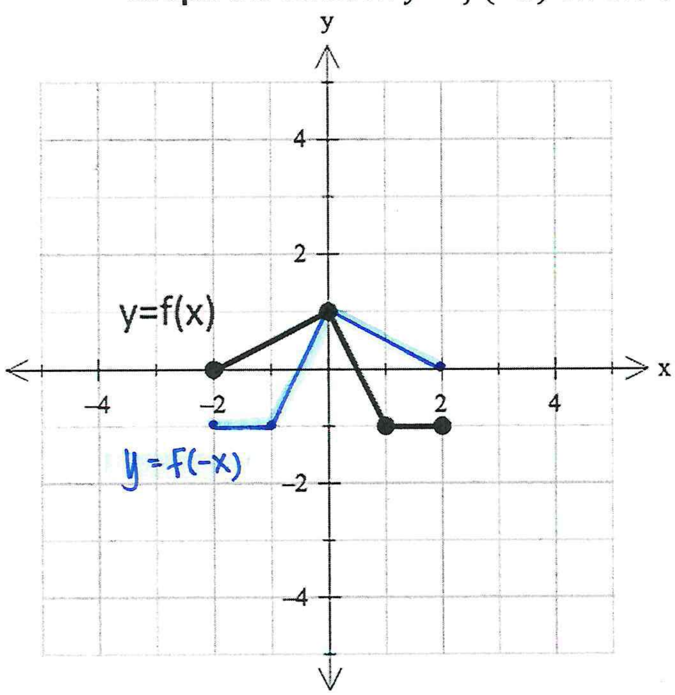
- The x – coordinates are multiplied by –1

$(x, y) \rightarrow (-x, y)$

Example #3

The graph of $y = f(x)$ is given.

Graph the function $y = f(-x)$ on the same grid. List any **invariant points**.



↳ Reflection over the y-axis

↳ Multiply the x-values by -1

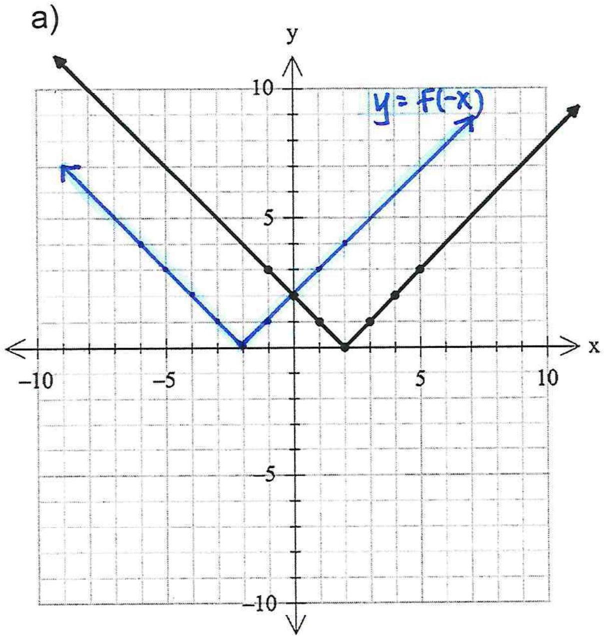
↳ invariant point : (0,1)

Example #4

The graph of $y = f(x)$ is given.

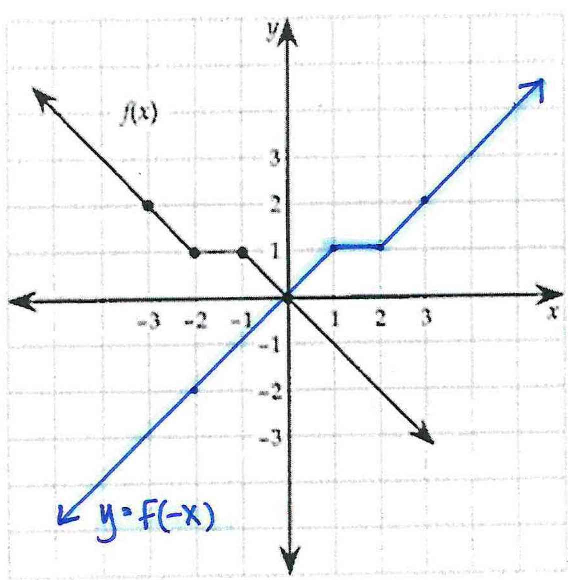
Graph the function $y = f(-x)$ on the same grid. List any **invariant points**.

a)



↳ invariant point : (0,2)

b)



↳ invariant point : (0,0)

Stretch/Compression: changes the shape of the graph

- A transformation where the distance from the line of reflection is multiplied by a scale factor.

↳ scale factors greater than 1 : stretch

↳ scale factors between 0 and 1 : compress

Vertical Stretches and Compressions

- For a function $y = f(x)$, the graph of a function $y = af(x)$ is a **vertical stretch/compression** of the graph about the x – axis by a factor of $|a|$

- $y = af(x)$ is obtained by multiplying the y – value of each coordinate point on the graph of $y = f(x)$ by $|a|$

$$(x, y) \rightarrow (x, ay)$$

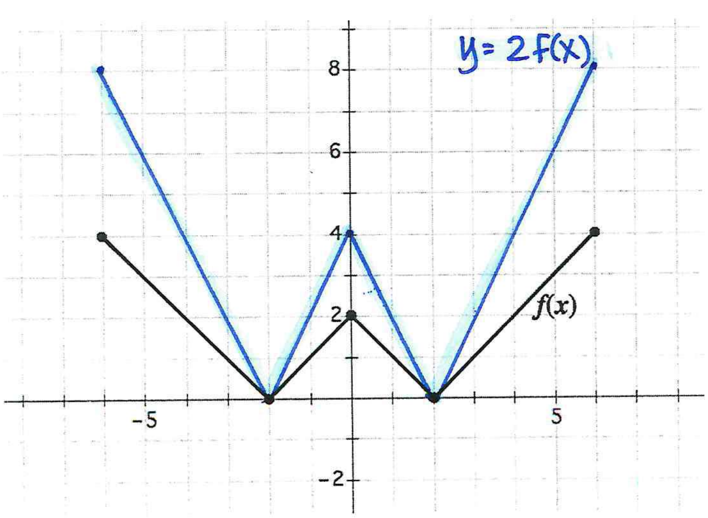
Example #5

The graph of $y = f(x)$ is given.

Sketch the following graphs. List any **invariant points**. State the **domain** and **range**.

a

a) $y = 2f(x)$



↳ Vertical stretch by a factor of 2

$(x, y) \rightarrow (x, 2y)$

↳ invariant points @ $(-2, 0)$ and $(2, 0)$

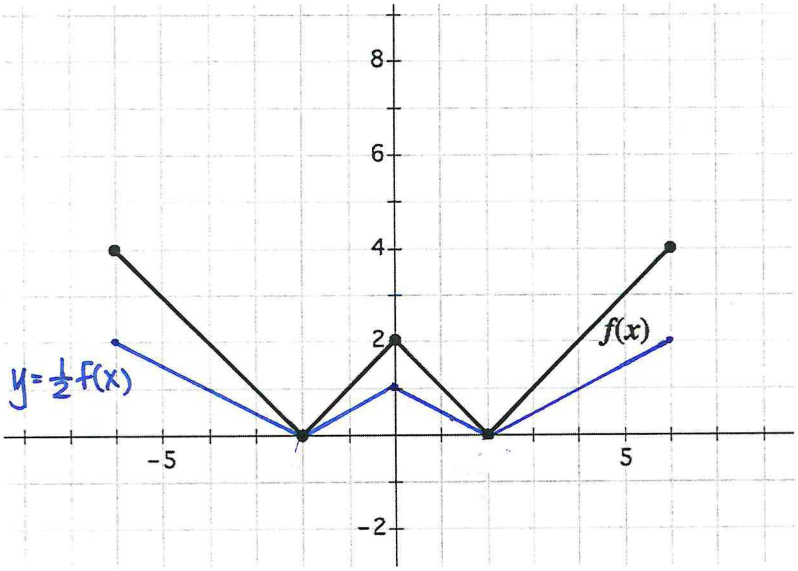
Domain : $[-6, 6]$

Range : $[0, 8]$

Date: ____________

$\vec{a}$

b) $y = \frac{1}{2}f(x)$



↳ Vertical stretch by a factor of 1/2

$(x, y) \rightarrow (x, \frac{1}{2}y)$

↳ invariant points @ $(-2, 0)$ and $(2, 0)$

Domain : $[-6, 6]$

Range : $[0, 2]$

Example #6

The graph of $y = f(x)$ is given.

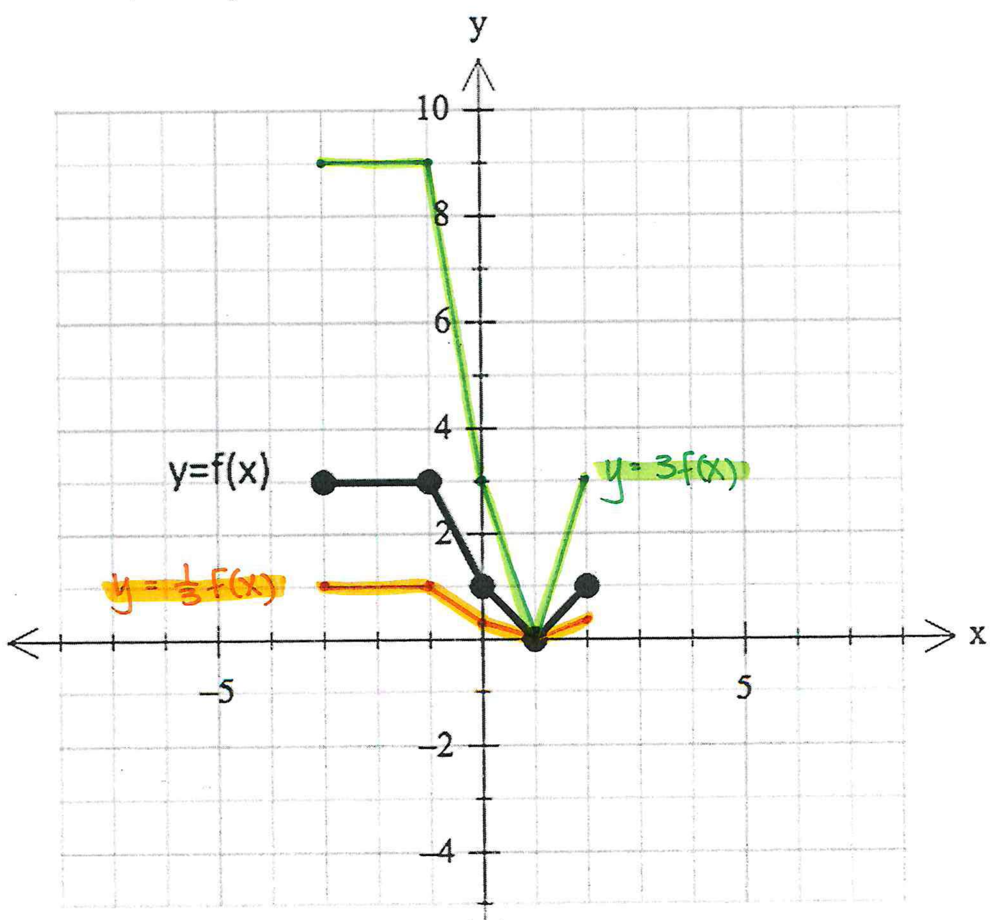
Sketch the following graphs on the same grid. Label each of your functions.

a) $y = 3f(x)$

$(x, y) \rightarrow (x, 3y)$

b) $y = \frac{1}{3}f(x)$

$(x, y) \rightarrow (x, \frac{1}{3}y)$



Horizontal Stretches and Compressions

- For a function $y = f(x)$, the graph of a function $y = f(bx)$ is a **horizontal stretch/compression** of the graph about the y – axis by a factor of $\frac{1}{|b|}$

- $y = f(bx)$ is obtained by multiplying the x – value of each coordinate point on the graph of $y = f(x)$ by $\frac{1}{|b|}$

$$(x,y) \rightarrow \left(\tfrac{1}{b}x, y\right)$$

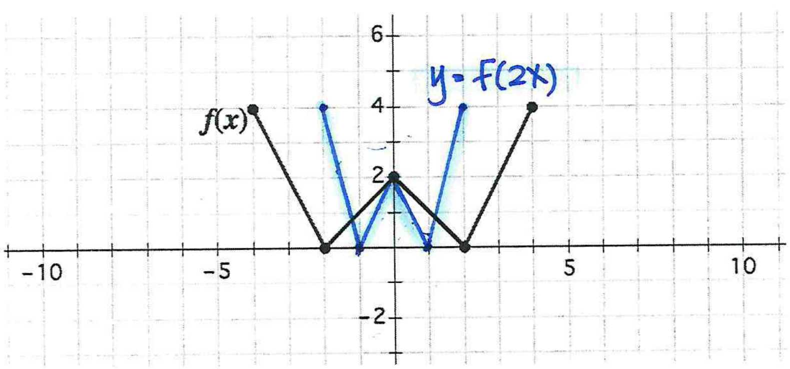
Example #7

The graph of $y = f(x)$ is given.

Sketch the following graphs.

List any **invariant points**. State the **domain** and **range**.

a) $y = f(2x)$



↳ Horizontal stretch by a factor of 1/2

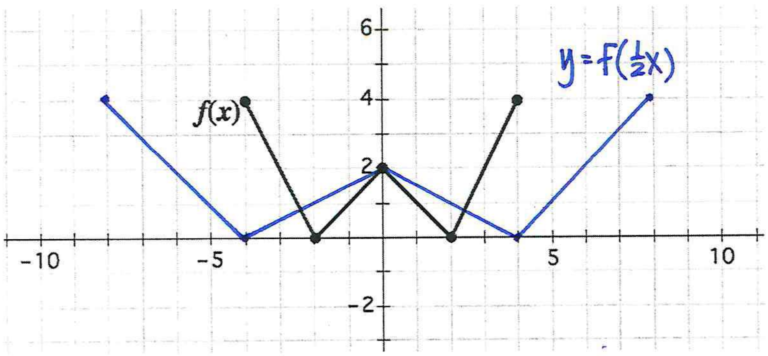
$(x,y) \rightarrow (\frac{1}{2}x, y)$

↳ Invariant point @ $(0,2)$

Domain : $[-2,2]$

Range : $[0,4]$

b) $y = f(\frac{1}{2}x)$



↳Horizontal stretch by a factor of 2

$(x,y) \to (2x,y)$

↳Invariant point @ $(0,2)$

Domain: $[-8,8]$

Range: $[0,4]$

Example #8

The graph of $y = f(x)$ is given.

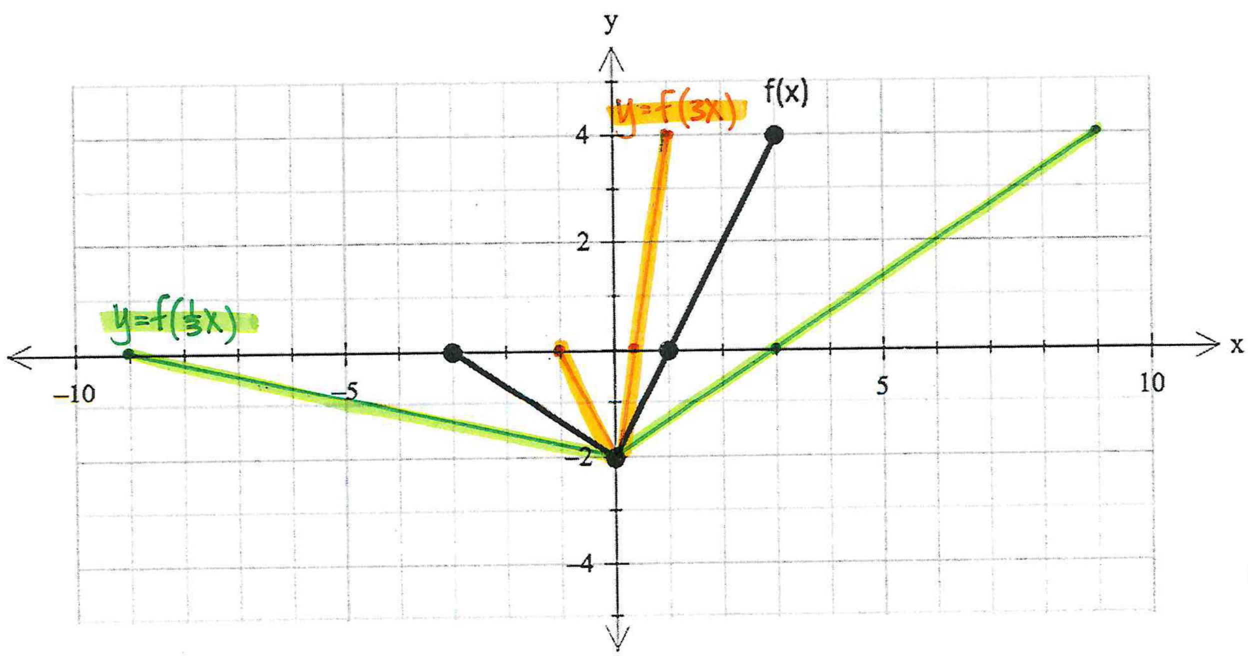
Sketch the following graphs on the same grid. Label each of your functions.

a) $y = f(3x)$

$(x,y) \to (\frac{1}{3}x, y)$

b) $y = f(\frac{1}{3}x)$

$(x,y) \to (3x,y)$

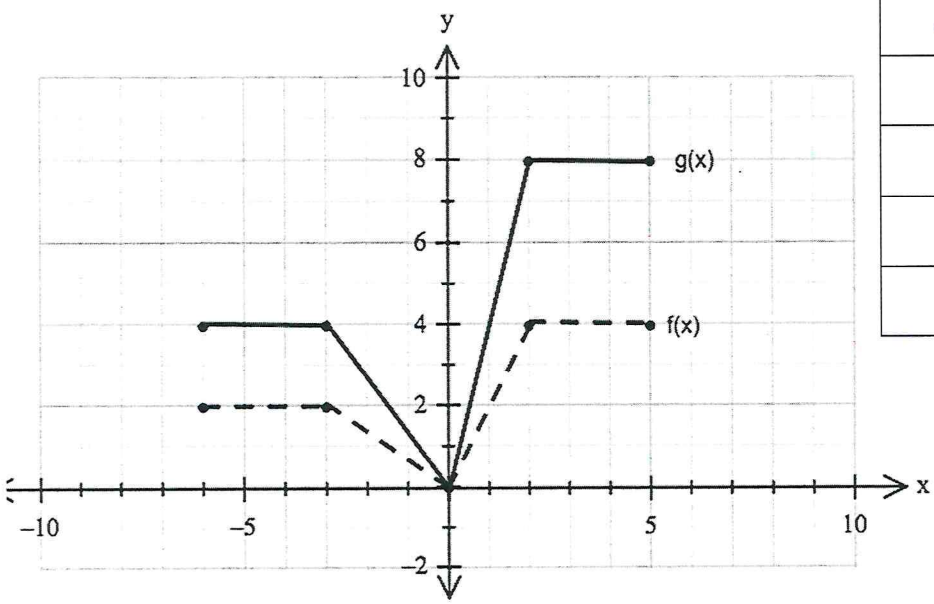


We must also be able to determine the **equation** of a function that has undergone a transformation.

Example #9

The graph of $y = f(x)$ has been transformed.

Write the **equation** of the transformed graph.



x	$y = f(x)$	$y = g(x)$
-6	2	4
3	2	4
0	0	0
2	4	8
5	4	8

↳ y-values multiplied by 2

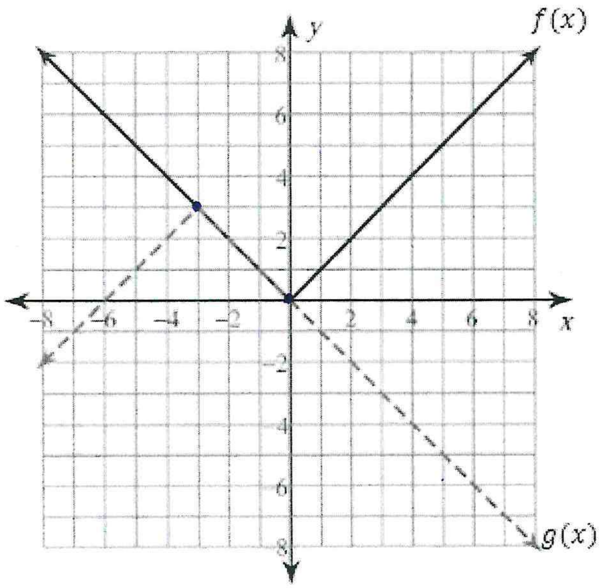
$(x, y) \rightarrow (x, 2y)$

Answer: $g(x) = 2f(x)$

Example #10

The graph of $y = f(x)$ has been transformed.

Write the **equation** of the transformed graph.



x	$y = f(x)$	$y = g(x)$

Answer: $g(x) = -f(x+3) + 3$

↳ Reflection over the x-axis
↳ No stretch/compression

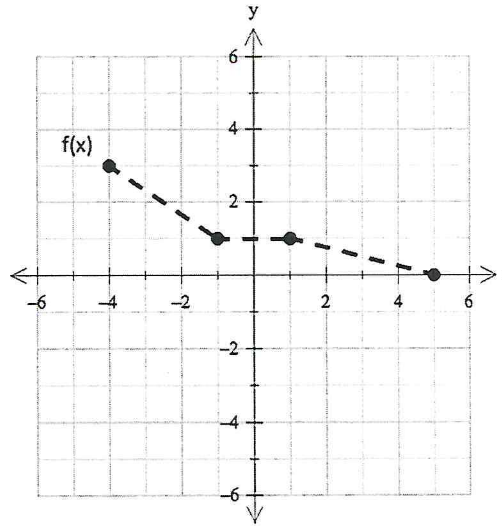
↳ (0,0) not affected by the reflection, moved left 3 units and up 3 units

Example #11

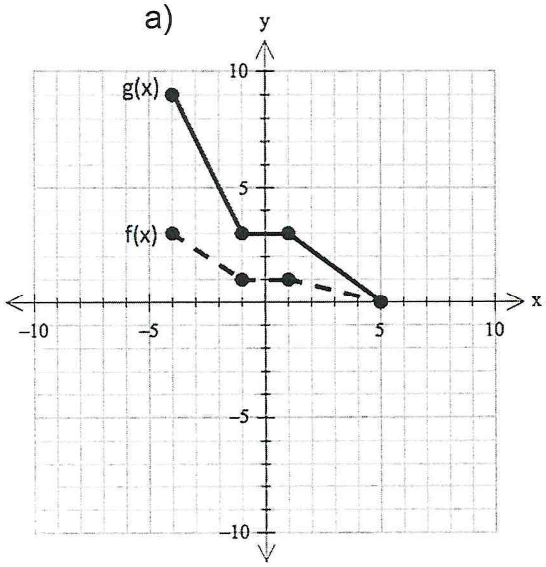
Each of the represent a stretch/compression of the given function $y = f(x)$.

Write the **equation of the transformation** in the form $y = af(bx)$.

↳ work with key points and image points



a)

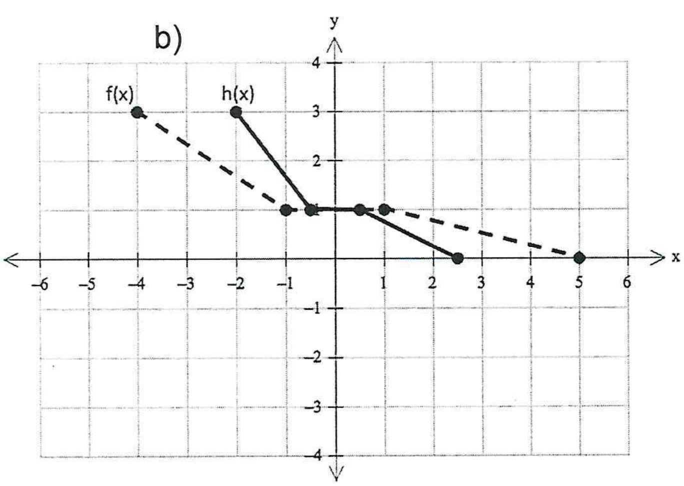


↳ No change in x-values

↳ All y-values are multiplied by 3

Answer: $g(x) = 3f(x)$

b)

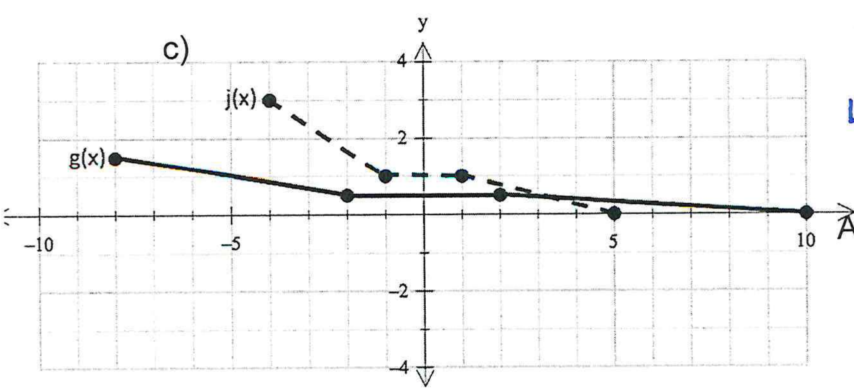


↳ No change in y-values

↳ All x-values are multiplied by ½

Answer: $h(x) = f(2x)$

c)



↳ x-values multiplied by 2

↳ y-values multiplied by ½

Answer: $j(x) = \frac{1}{2}g(\frac{1}{2}x)$