

p 12  
MS, W  
3, 4, 5, 6, 11

## Chapter 1: TRANSFORMATIONS AND FUNCTIONS

### 1.1 – Horizontal and Vertical Translations

**Transformation:** A change on an original graph.  
 we deal with ① Translations ② reflections  
 ③ stretches/compressions ④ Inverses.

Image points: Points on the original graph correspond to points on the transformed graph. The relationship between these sets of points can be called mapping.  $(x, y) \rightarrow (x, y')$

A translation is one type of transformation. A translation can **shift** the graph of a function up, down, left, right. A translation occurs when the location of a graph changes but the shape remains the same.

#### Vertical Translation

- A **vertical translation** shifts the graph up or down.
- Starting with the original function,  $y = f(x)$

$$y = f(x) + k$$

-If  $k > 0$ , the graph shifts up.

-If  $k < 0$ , the graph shifts down.

- Each point  $(x, y)$  on the **original function** is mapped to  $(x, y+k)$  on the **transformed function**.

$$(x, y) \rightarrow (x, y+k)$$



Example #1

The graph of  $f(x) = x^2$  is shown on the coordinate grid below.

Perform the following **vertical translations** on the same grid.

Describe each transformation **in words** and state the **mapping notation**. Label each graph.

a)  $y = f(x) + 4$

Vertical translation 4  
units up

$(x, y) \rightarrow (x, y+4)$

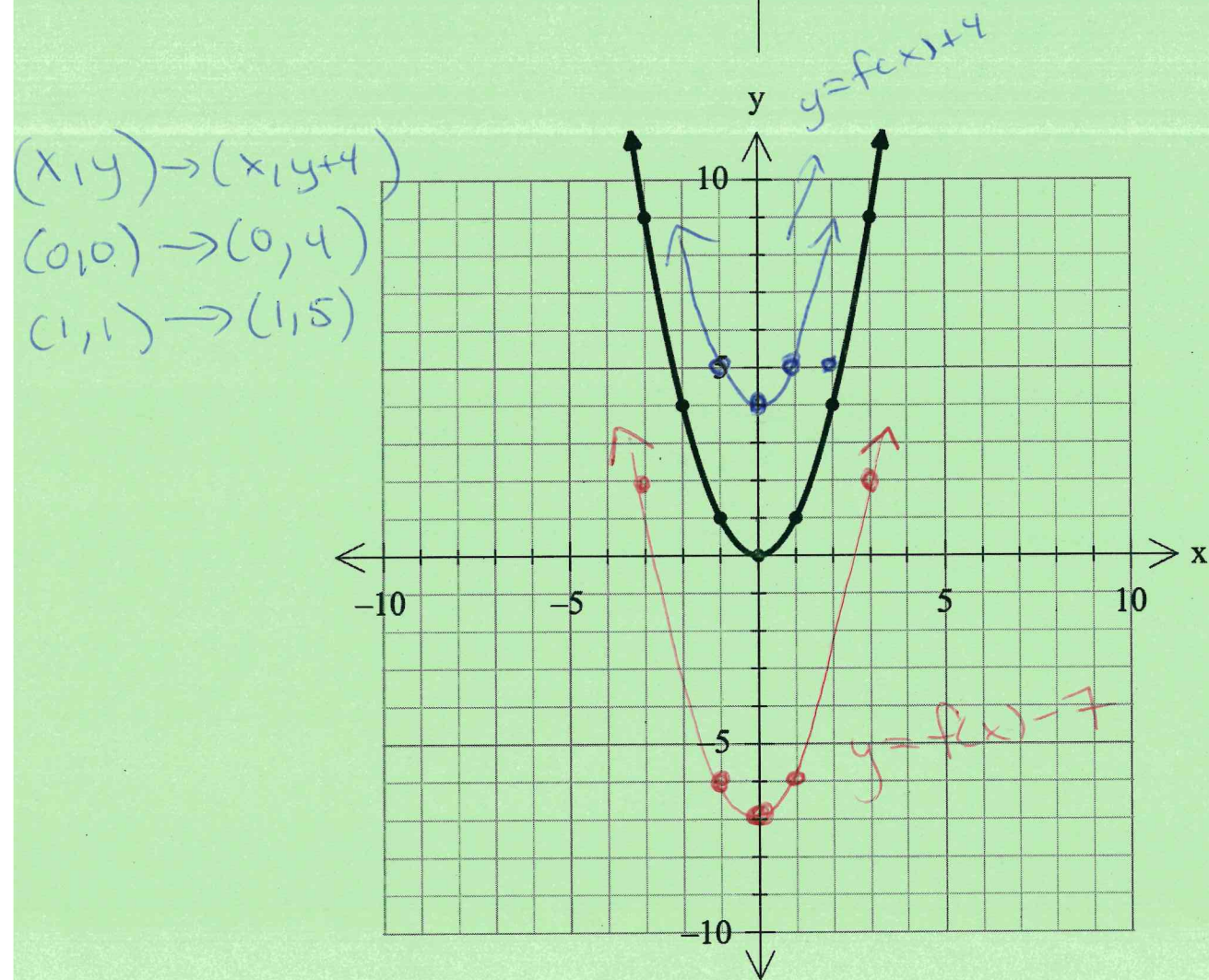
$y = x^2 + 4$

b)  $y = f(x) - 7$

shift down 7

$(x, y) \rightarrow (x, y-7)$

$y = x^2 - 7$



**Horizontal Translation**

- A horizontal translation shifts the graph left or right
- Starting with the original function,  $y = f(x)$

$$y = f(x - h)$$

-If  $h > 0$ , the graph shifts right

-If  $h < 0$ , the graph shifts left

$$y = f(x - (+1))$$

$$y = f(x - 1)$$

$$\downarrow y = f(x + 4)$$

- Each point  $(x, y)$  on the **original function** is mapped to  $(x + h, y)$  on the **transformed function**.

$$(x, y) \rightarrow (x + h, y)$$



Example #2

The graph of  $f(x) = |x - 2|$  is shown on the graph below.

Perform the following **horizontal translations** on the same grid.

Describe each transformation **in words** and state the **mapping notation**. Label each graph.

a)  $y = f(x + 6)$

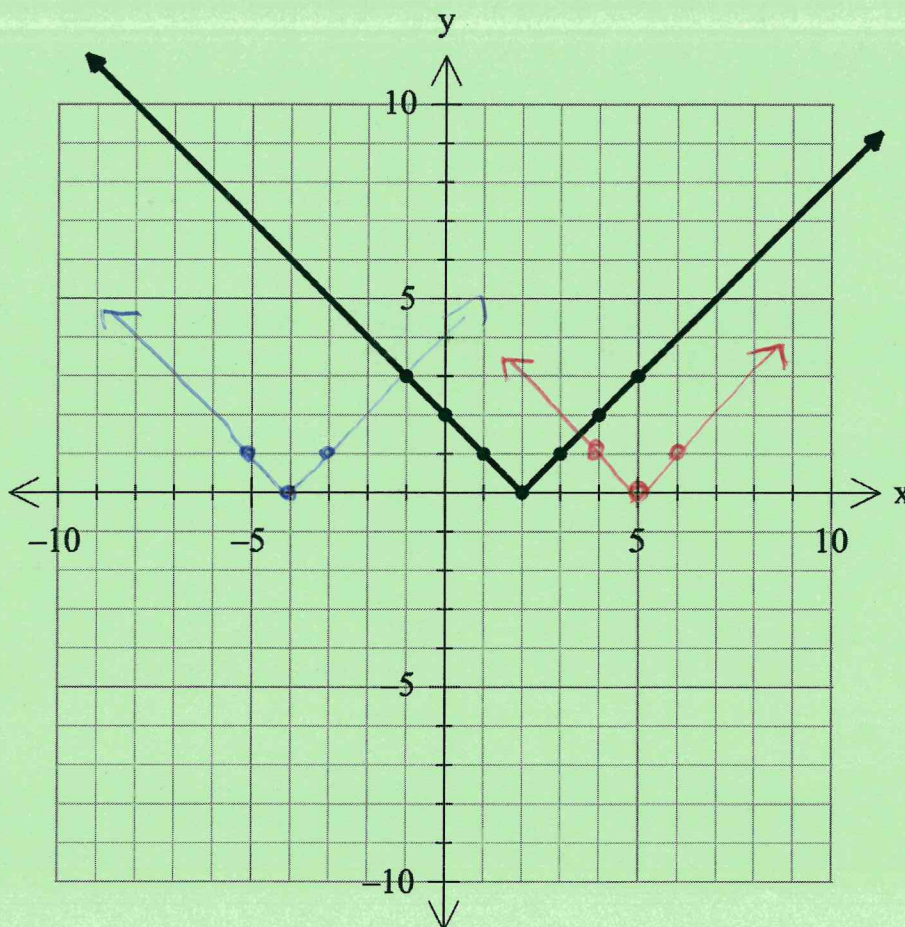
Shift left 6 units

$(x, y) \rightarrow (x - 6, y)$

b)  $y = f(x - 3)$

horizontal translation  
3 units right

~~$(x + 3, y)$~~   
 $(x, y) \rightarrow (x + 3, y)$

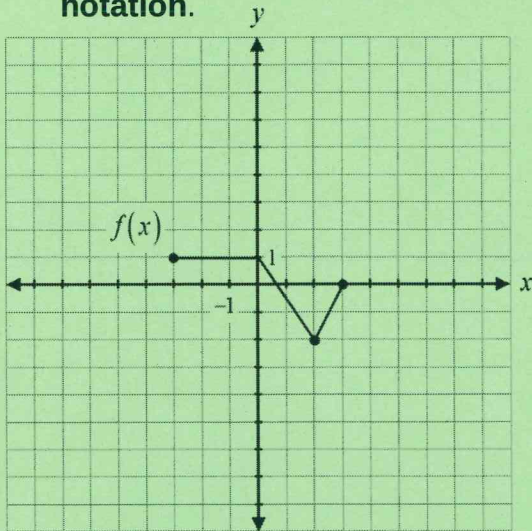




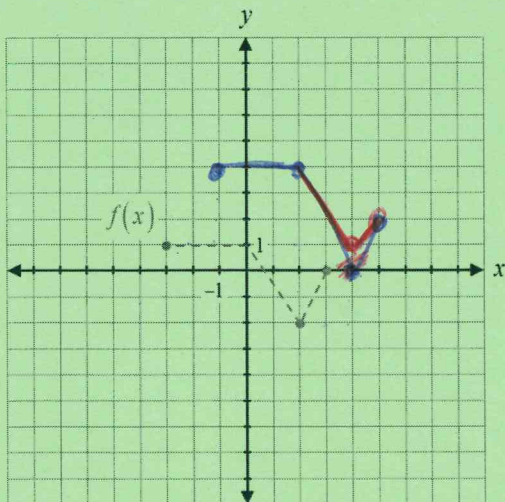
Example #3

Use the given graph of  $f(x)$  to sketch the following functions on the graphs provided.

Describe **in words** how you transformed each function and provide the **mapping notation**.



a)  $g(x) = f(x - 2) + 3$



Shift right 2 and up 3.

$(x, y) \rightarrow (x + 2, y + 3)$

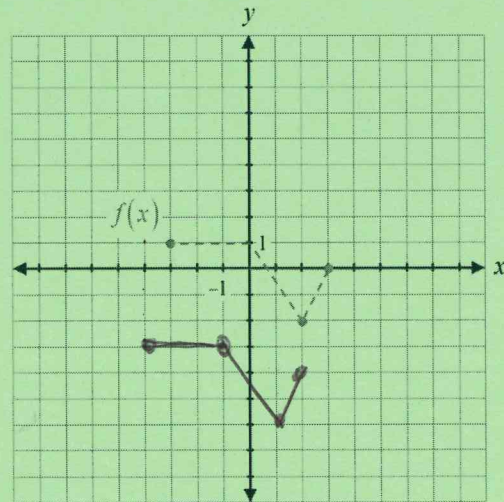
$(-3, 1) \rightarrow (-1, 4)$

$(0, 1) \rightarrow (2, 4)$

$(2, -2) \rightarrow (4, 1)$

$(3, 0) \rightarrow (5, 3)$

b)  $h(x) = f(x + 1) - 4$



left 1 and down 4  
Shift ~~right~~

$(x, y) \rightarrow (x - 1, y - 4)$

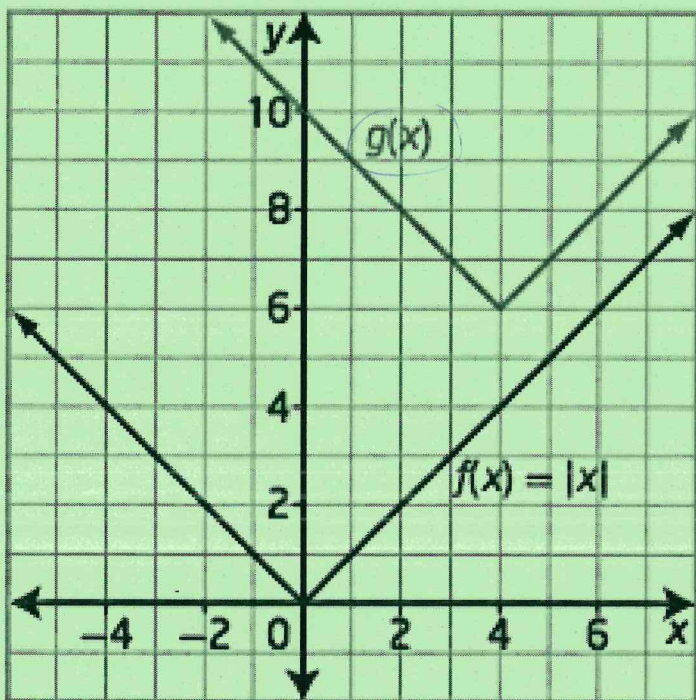
We must also be able to determine the **equation** of a function that has undergone a transformation.

Example #4

The function  $f(x)$  has been transformed into the function  $g(x)$ .

Determine the **equation of the translated function** in the form  $y = f(x - h) + k$

a)



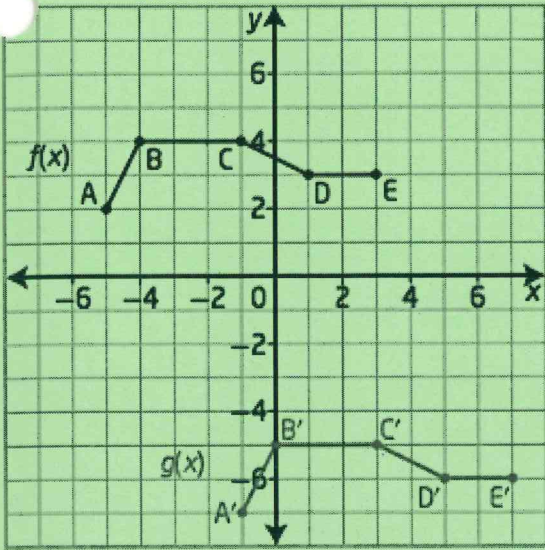
$f(x)$	→	$g(x)$
	→	
	→	
	→	
	→	
	→	

Answer:  $g(x) = f(x - 4) + 6$

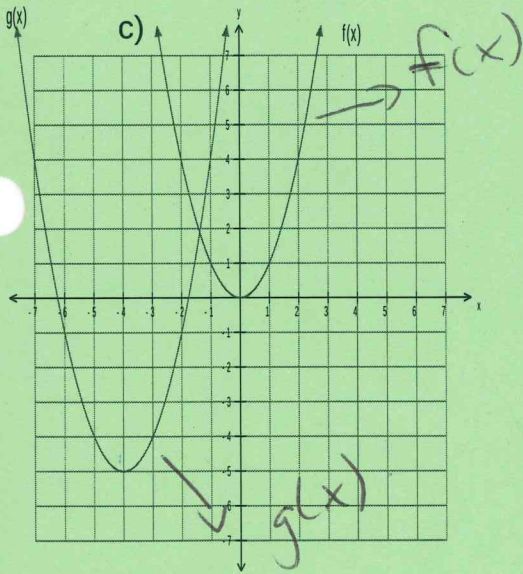
Image Points: New points!  
 $(4,6), (5,7) \dots$



b)

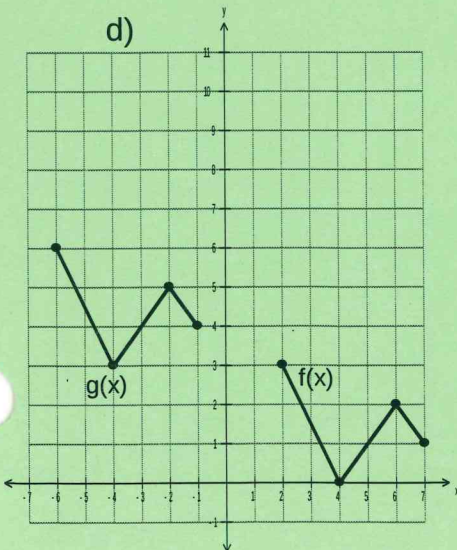


Answer:  $g(x) = f(x-4) - 9$



Answer:  $g(x) = f(x+4) - 5$

d)



Answer:  $g(x) = f(x+8) + 3$

